Iberian meeting on numerical semigroups - Granada 2010

Organizers:
M. Delgado, Universidade de Porto
P. A. García-Sánchez, Universidad de Granada

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Maria Bras-Amorós, Universitat Rovira i Virgili
David Llena, Universidad de Almería
José Ignacio Farrán, Universidad de Valladolid
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February, 3-5,
Edificio Mecenas,
Universidad de Granada

Sponsors:
ASA - Automata, Semigroups and Applications (PTDC/MAT/65481/2006)
Métodos Algebraicos Aplicados a la Física, Geometría No Conmutativa y Computación (P06-FQM1889)
Acción complementaria MTM2009-07987-E/MTM del Ministerio de Ciencia e Innovación
Semigrupos Numéricos (MTM2007-62346)
Plan propio de la Universidad de Granada
I2 SINGACOM (GR 135)

Francisco Aguiló-Gost *Universitat Politècnica de Catalunya*

PLANE TESSELLATIONS RELATED TO THREE GENERATED NUMERICAL SEMIGROUPS

It is well known that each three generated numerical semigroup admits several plane tessellations by L-shaped tiles. Many properties of the semigroup can be extracted from his related tiles. Two main properties of these tiles will be exposed: Every 3-semigroup has related two different tiles at most, and how these two tiles are both linked.

From these facts, it follows that each tile can be described by her couple. In particular, only one tile is needed for characterizing symmetric 3-semigroups.

Joint work with Carlos Marijuán.

Paul Baginski *Université Claude Bernard Lyon 1*

FACTORIZATION IN SEMIGROUP RINGS

Given a ring, $(M,+,\cdot)$, and a semigroup, $(S,+)$, one can construct a new ring, $M[X;S]$, called the semigroup ring. This ring generalizes the standard polynomial ring $M[X]$. Semigroup rings have been studied extensively, especially in the case where $M=\mathbb{Z}$ or a field. The semigroup ring, in an intuitive sense, carries some of the factorization structure of S, but due to the interaction with M, one often gets new behavior for factorization. We will discuss to what extent the factorization properties of M and S can be separated within the semigroup ring when S is a numerical monoid, paying particular attention to the Krull case.

Valentina Barucci *Università di Roma "La Sapienza"*

CONDITIONS WHICH MAKE A ONE-DIMENSIONAL ANALYTICALLY IRREDUCIBLE RING CLOSE TO A NUMERICAL SEMIGROUP RING

The study of a one-dimensional analytically irreducible ring R is strictly related to the study of its value semigroup, which is a numerical semigroup. The relation between the ring and the semigroup is very strict if the ring is a semigroup k-algebra, i.e. a ring of the form $k[[t^{n_1},\dots,t^{n_h}]]$, where k is a field and $\langle n_1,\dots,n_h\rangle$ a numerical semigroup.

The talk deals with some conditions on the ring R which make its behaviour similar to the behaviour of a numerical semigroup k-algebra.

Víctor Blanco *Universidad de Granada*

GENERATING FUNCTIONS FOR COUNTING THE NUMBER OF NUMERICAL SEMIGROUPS OF A GIVEN GENUS

We present a new methodology to count the number of numerical semigroups of given genus using generating function of rational polytopes. This work is based on the results by Selmer (1977) and Rosales et al (2002) that identify the set of numerical semigroups with given genus and multiplicity with the set of integer points inside a polytope. With this characterization and the use of generating funtions we are able to give an unknown result stating the polynomial-time complexity of counting numerical semigroups. Also MED-semigroups are analyzed with this methodology. In the cases where the multiplicity is three or four formulas for the number of numerical semigroups for any genus are presented.

Manuel B. Branco *Universidade de Évora*

THE FROBENIUS PROBLEM FOR NUMERICAL SEMIGROUPS WITH MULTIPLICITY FOUR

We study the gender, Frobenius number and pseudo-Frobenius numbers for embedding dimension three, multiplicity four and pairwise relatively prime minimal generators. For all possible genders, Frobenius numbers or pseudo-Frobenius numbers we give a procedure to construct a numerical semigroup having the given gender, Frobenius or pseudo-Frobenius numbers.

Scott T. Chapman *Sam Houston State University*

DELTA SETS OF NUMERICAL MONOIDS: A PROGRESS REPORT

Let M be an atomic commutative cancellative monoid with set $\mathscr{A}(M)$ of irreducible elements. If x is a nonunit of M, then set

$$\mathscr{L}(x) = \{n \mid n \in \mathbb{N} \text{ such that } \exists\, x_1, \ldots, x_n \in \mathscr{A}(M) \text{ with } x = x_1 \cdots x_n\}.$$

$\mathscr{L}(x)$ is called the *set of lengths* of x in M. If we write $\mathscr{L}(x) = \{n_1, n_2, \ldots, n_t\}$ with $n_1 < n_2 < \cdots < n_t$, then set

$$\Delta(x) = \{n_2 - n_1, n_3 - n_2, \ldots, n_t - n_{t-1}\}$$

and

$$\Delta(M) = \bigcup_{x \in M} \Delta(x).$$

Several papers have recently appeared in the literature which analyze the structure of $\Delta(M)$ when M is a numerical monoid. I will review these papers as well as some recent work involving the case where M has embedding dimension 3.

Eric Emtander *Stockholms Universitet*

HIGHER DIMENSIONAL PROPERTIES OF NUMERICAL SEMIGROUPS

There are many properties of a numerical semigroup that are so natural that we take them for granted. For example one usually assumes that the greatest common divisor of the minimal generators of a numerical semigroups S are relatively prime. This immediately implies that $\mathbb{N} \setminus S$ (the set of gaps os S) is finite, and also that every integer larger than the Frobenius number $g(S)$ belongs to S. It is very natural to ask what these properties look like if considered in a higher dimensional affine setting instead.

In the talk I will show, based on considerations as above, that there is a natural higher dimensional affine version of a numerical semigroup. Some known result will be seen in a more general setting, and some new ideas will be presented.

Leonid Fel *Technion - Israel Institute of Technology*

NEW IDENTITIES FOR DEGREES OF SYZYGIES IN NUMERICAL SEMIGROUP

We derive a set of polynomial and quasipolynomial identities for degrees of syzygies in the Hilbert series $H\left(\mathbf{d}^m; z\right)$ of nonsymmetric numerical semigroups $\mathsf{S}(\mathbf{d}^m)$ of arbitrary generating set of positive integers $\mathbf{d}^m = \{d_1, \ldots, d_m\}$, $m \geq 3$. These identities were obtained by studying together the rational representation of the Hilbert series $H(\mathbf{d}^m; z)$ and the quasipolynomial representation of the Sylvester waves in the restricted partition function $W(s, \mathbf{d}^m)$. In the cases of symmetric semigroups and complete intersections these identities become more compact.

Vítor H. Fernandes *Universidade Nova de Lisboa, CAUL*

QUOTIENT NUMERICAL SEMIGROUPS

In this talk we introduce the notion of a quotient-numerical semigroup and explore its connection with the class of finite nilpotent commutative semigroups.

Ralf Froberg *Stockholms Universitet*

CURVES AND SEMIGROUPS

This is a survey talk about the use of semigroups in the study of curves (algebraic, complex, algebroid). I will describe what kind of problems in commutative algebra, connected to semigroups, that are investigated. Besides results, I will mention some open problems which I think are good.

Alfred Geroldinger *Karl-Franzens-Universität Graz*

THE CATENARY DEGREE OF NUMERICAL MONOIDS AND KRULL MONOIDS

Let H be an atomic monoid, $C \in \mathbb{N}_0$ a non-negative integer, $a \in H$ a non-unit and z, z' two factorizations of a (in other words, two product decompositions of a into irreducible elements of H). A finite sequence $(z_0, \ldots, z_s)$ of factorizations of a is called a C-chain of factorizations from z to z', if $z_0 = z$ and $z' = z_s$ and for every $i \in [1, s]$, z_i arises from z_{i-1} by replacing at most C atoms of z_{i-1} by at most C new atoms. The catenary degree $\mathsf{c}(H)$ of H is the smallest possible $C \in \mathbb{N}_0$ such that for all $a \in H$ each two factorizations of a can be concatenated by a C-chain. By definition, the monoid is factorial if and only if its catenary degree equals zero.

Suppose that H is a Krull monoid with finite class group G (e.g., the ring of integers of an algebraic number field). Then the catenary degree of H equals the catenary degree of the associated monoid of zero-sum sequences over G. We study the relationship of the catenary degree with the set of distances, and we compare the arithmetical behavior of Krull monoids with that of numerical monoids.

This is joint work with David J. Grynkiewicz and Wolfgang A. Schmid.

Kurt Herzinger *United States Air Force Academy*

BRICKS AND PERFECT BRICKS OF ALL SIZES

We examine numerical semigroups that have a non-principal relative ideal such that $\mu_S(I)\mu_S(S-I) = \mu_S(I+(S-I))$. If $\mu_S(I) = k$ and $\mu_S(S-I) = n$, then we call the pair (S, I) a $k \times n$ brick. In the case $I + (S-I) = S \setminus \{0\}$, we say (S, I) is a perfect brick. There exists a classification for 2×2 perfect bricks in terms of the minimal generating set for S. In this talk we will examine families of $k \times n$ perfect bricks for $k, n \geq 3$. We will compare and contrast characteristics of these families with those of 2×2 perfect bricks as well as discuss progress toward a classification of $k \times n$ perfect bricks in general.

Ulrich Krause *Universität Bremen*

NUMERICAL SEMIGROUPS AND THE CALE PROPERTY

For a numerical semigroup S the semigroup ring $k[S]$ as well as the toric ideal I_S are analyzed with respect to the Cale property. The latter roughly means (for a domain or a multiplicative monoid) that for each atom some power can be uniquely represented as a product of elements taken from some special subset. Conclusions will be drawn concerning the elasticity of $k[S]$ as well as a minimal description of I_S or the variety induced by simple binomials.

Vincenzo Micale *Università di Catania*

APERY SET OF A NUMERICAL SEMIGROUP AND PROPERTIES OF THE ASSOCIATED GRADED RING OF A NUMERICAL SEMIGROUP RING

We study the associated graded ring of a numerical semigroup ring using the properties of the Apery set of the semigroup. In particular, we analyze two different orders on the semigroup and we use them in order to study the property for the associated graded ring to be Cohen-Macaulay or Buchsbaum. Finally we show how these properties are connected to the properties of the Apery set of the semigroup.

Ignacio Ojeda *Universidad de Extremadura*

UNIQUELY PRESENTED FINITELY GENERATED COMMUTATIVE MONOIDS

A finitely generated commutative monoid is uniquely presented if it has only a minimal presentation. We give necessary and sufficient conditions for finitely generated, combinatorially finite, cancellative, commutative monoids to be uniquely presented. We use the concept of gluing to construct commutative monoids with this property. Finally for some relevant families of numerical semigroups we describe the elements that are uniquely presented. This is a joint work with P. A. García-Sánchez.

Jean-Eric Pin *CNRS - Université Paris 7*

LANGUAGES ON A ONE-LETTER ALPHABET AND THEIR CONNECTIONS WITH ORDERED SEMIGROUPS

In this lecture, I will try to explain how some questions on formal languages reduce to some questions on subsets of the additive semigroup $\mathbb{N}$.

A language on a one-letter alphabet a can be identified with a subset of $\mathbb{N}$. A recognizable language is then a finite union of arithmetic progressions. The marked product of two languages L and K is the language LaK. We are interested in families of recognizable languages that are closed under finite union, finite intersection and quotients (this latter condition corresponds to the operation $x \to x - k$ for a fix k): these families are called "quotienting algebras". A general result states that quotienting algebras can be characterized by a set of profinite equations, but these equations are simpler in the one letter case. For instance, we shall explain why the quotienting algebra generated by the language $a + a^6 a^*$ is defined by the equations $x^5 \leq 1$, $x^3 \leq x^2$ and $x^6 = x^7$ and why the closure under marked product of this quotienting algebra is defined by the equations $x^5 \leq 1$, $x^6 \leq 1$, $x^7 \leq 1$, $x^8 \leq 1$ and $x^9 \leq 1$. This will lead us to some recent results obtained jointly with Z. Esik.

Vadim Ponomarenko *San Diego State University*

FACTORIZATIONS COMMON TO SOME SUBSEMIGROUPS OF $(\mathbb{N}, +)$ AND $(\mathbb{N}, \times)$

Let M be any subsemigroup of $(\mathbb{N}, \times)$, and let $n > 0$. We produce a subsemigroup N of $(\mathbb{N}, +)$, and a nontrivial natural map $f : M \to N$, such that f respects factorization on $[1, n]$. That is, $f(xy) = f(x) + f(y)$ for all $x, y \in [1, n]$. As an application, we count factorizations in a half-factorial domain.

Jorge Ramírez Alfonsín *Université Montpellier 2*

APÉRY SETS AND HILBERT SERIES FOR ALMOST ARITHMETIC SEMIGROUPS

Let $a_1 \ldots, a_n$ be relatively prime positive integers, and let S be the semigroup consisting of all non-negative integer linear combinations of $a_1, \ldots, a_n$. In this talk, we focus our attention on *almost arithmetic semigroups* (AA-semigroups, for short), i.e., semigroups generated by almost arithmetic progressions that is, all but one of the a_i form an ordinary arithmetic progression. After some general considerations, we present a characterization of the symmetric AA-semigroups. We also give an efficient method to determine an Apery set and the Hilbert series of an AA-semigroup.

Klara Stokes *Universitat Rovira i Virgili*

THE SEMIGROUP OF COMBINATORIAL CONFIGURATIONS

A $(v, b, k, r)-$configuration is an incidence structure with v points and b lines,such that there are k points on every line, r lines through every point, two different lines intersect each other at most once and two different points are connected by a line at most once. It is not trivial to decide whether, given a $4-$tuple (v, b, k, r), there exists a $(v, b, r, k)-$configuration. We define a way to uniquely assign an integer to each 4-tuple and we prove that the set of these integers has the structure of a numerical semigroup. As a result we deduce general results (including explicit bounds) on the existence of configurations. Also, we deduce an algorithm to construct a $(v, b, r, k)-$configuration.

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Grazia Tamone *Università di Genova*

ON THE ORDER BOUND OF ONE-POINT AG CODES

Let $S = \{s_i\}_{i\in\mathbb{N}} \subseteq \mathbb{N}$ be a numerical semigroup. For $s_i \in S$, let $\nu(s_i)$ denote the number of pairs $(s_i - s_j, s_j) \in S^2$. When S is the Weierstrass semigroup of a family $\{\mathscr{C}_i\}_{i\in\mathbb{N}}$ of one-point algebraic-geometric (AG) codes, a good bound for the minimum distance of the code $\mathscr{C}_i$ is the so called *order bound* introduced by Feng and Rao, $d_{ORD}(C_i) := min\{\nu(s_j) : \ j \geq i+1\}$. Since there is an integer m such that the sequence $\{\nu(s_i)\}_{i\in\mathbb{N}}$ is non-decreasing for $s_i \in S, \ \ s_i \geq s_m$, then $d_{ORD}(C_i) = \nu(s_{i+1})$ for $i \geq m$. By way of some suitable parameters related to the semigroup S, we can find upper bounds for s_m, we can evaluate s_m exactly in many cases, further we give a lower bound for several classes of semigroups.

Fernando Torres *Universidade estadual de Campinas*

ON THE WEIGHT OF NUMERICAL SEMIGROUPS

We investigate the weights of a family of numerical semigroups by means of eves gaps and the Weierstrass property of such a family.

Paulo Vasco *Universidade de Trás-os-Montes e Alto Douro*

THE FROBENIUS VARIETY OF THE SATURATED NUMERICAL SEMIGROUPS

A Frobenius variety is a nonempty family of numerical semigroups closed under finite intersections and under the adjoin of the Frobenius number. We see that the variety of the saturated numerical semigroups, is the least Frobenius variety satisfying that for any integers $2 \leq m < r$, with $r \not\equiv 0 \pmod{m}$, there exists an element of the variety with multiplicity m and smallest generator greater than the multiplicity equal to r. As a consequence we obtain that every saturated numerical semigroup admits a Toms decomposition. Finally, we give a characterization of the saturated numerical semigroups in terms of a certain type of Diophantine inequalities.

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