Identifying torsion in the tensor product...

Micah Leamer

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Notations and Definitions

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- Throughout this talk Γ will denote a numerical semigroup;
- A and B will denote relative ideals of Γ; and
- The dual of A is denoted by $A^* = \Gamma - A = \{z \in \mathbb{Z} | \ z + A \subseteq \Gamma\}$.

Definition

A splitting of A is a pair of relative ideals P and Q such that $P \cup Q = A$

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A is said to be Huneke-Wiegand if either it is principal, or there exists a splitting $P \cup Q = A$ such that

$$(P \cap Q) + A^* \neq (P + A^*) \cap (Q + A^*)$$

$$\subseteq \quad \leftarrow \text{This inclusion is automatic}$$

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The Huneke-Wiegand Conjecture for Numerical Semigroups

Conjecture

All relative ideals are Huneke-Wiegand.

Recall: A is Huneke-Wiegand provided there exists a splitting $P \cup Q = A$ such that $(P \cap Q) + A^* \neq (P + A^*) \cap (Q + A^*)$

Question

Why would we make this conjecture and where does it come from?

Answer

It is equivalent to a special case of the Huneke-Wiegand Conjecture, which is a well known conjecture in commutative algebra related to torsion and tensor products.

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Introducing torsion and tensor products

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- R will denote a commutative Noetherian domain
- M and N will be R-modules

Definition

The torsion submodule of M is
$T(M) := \{m \in M | \; rm = 0 \text{ for some } r \in R \setminus \{0\}\}$

It is often the case that $T(M \otimes_R N) \neq 0$

Example

Suppose $R = k[\Gamma]$ is a numerical semigroup ring with monomial ideals I and J. Then $T(I \otimes_R J)$ is the k-linear span of elements of the form

$$t^a \otimes t^b - t^c \otimes t^d \in I \otimes_R J \quad \text{where } a + b = c + d$$

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$I = t^3R + t^4R$ and $J = t^4R + t^5R$
Then $t^3 \otimes t^5 - t^4 \otimes t^4 \in T(M \otimes_R N) \neq 0$

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Let $R = k[t^4, t^5, t^6]$
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Conjectures

The Huneke-Wiegand Conjecture (HWC)

Let R be a one-dimensional Gorenstein domain. Let M be a finitely generated R module such that $T(M) = T(M \otimes_R \operatorname{Hom}_R(M, R)) = 0$, then M is projective.

- HWC is around 30 years old and is well known.
- Proving HWC would imply the Auslander-Reiten Conjecture is true for Gorenstein domains of any dimension.
- The Auslander-Reiten Conjecture is one of the most sought after results in commutative algebra.
- HWC is known to be true when R is a hyper-surface
- HWC is open when R is complete intersection with $codim(R) \geq 2$
- HWC is open when M is a 2-generated monomial ideal in a NSGR.

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Theorem

Let I and J be mononomial ideals of $k[\Gamma]$ and $A := \deg(I)$, $B := \deg(J)$.
Then $T(I \otimes_{k[\Gamma]} J) = 0 \iff$
$(P \cap Q) + B = (P + B) \cap (Q + B)$ for every splitting $P \cup Q = A$.

Let K be the total quotient ring of R.
Then $Hom_R(I, R) \simeq (R :_K I)$ and $\deg((R :_K I)) = \Gamma - \deg(I) = A^*$.

Corollary

Let I be a monomial ideal in $k[\Gamma]$ and $\deg(I) = A$.
Then $\mathrm{T}(I \otimes_R \mathrm{Hom}(I, R)) \neq 0 \iff$
$\exists$ a splitting $P \cup Q = A$ such that $(P \cap Q) + A^* \neq (P + A^*) \cap (Q + A^*)$.

Theorem (G-S,L)

Let $R = k[\Gamma]$ be a complete intersection numerical semigroup ring and I a 2-generated monomial ideal of R.
Then $T(I \otimes_R \mathrm{Hom}_R(I, R) \neq 0$.

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Remark

If $A = (a_1, a_2)$ is two generated. Then there is only one non-trivial splitting $P = a_1 + \Gamma$ and $Q = a_2 + \Gamma$.
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Monoids of arithmetic sets

Definition

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Identifying torsion in the tensor product...

Micah Leamer

Thank You!

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