Homogeneización y Micromecánica en materiales compuestos

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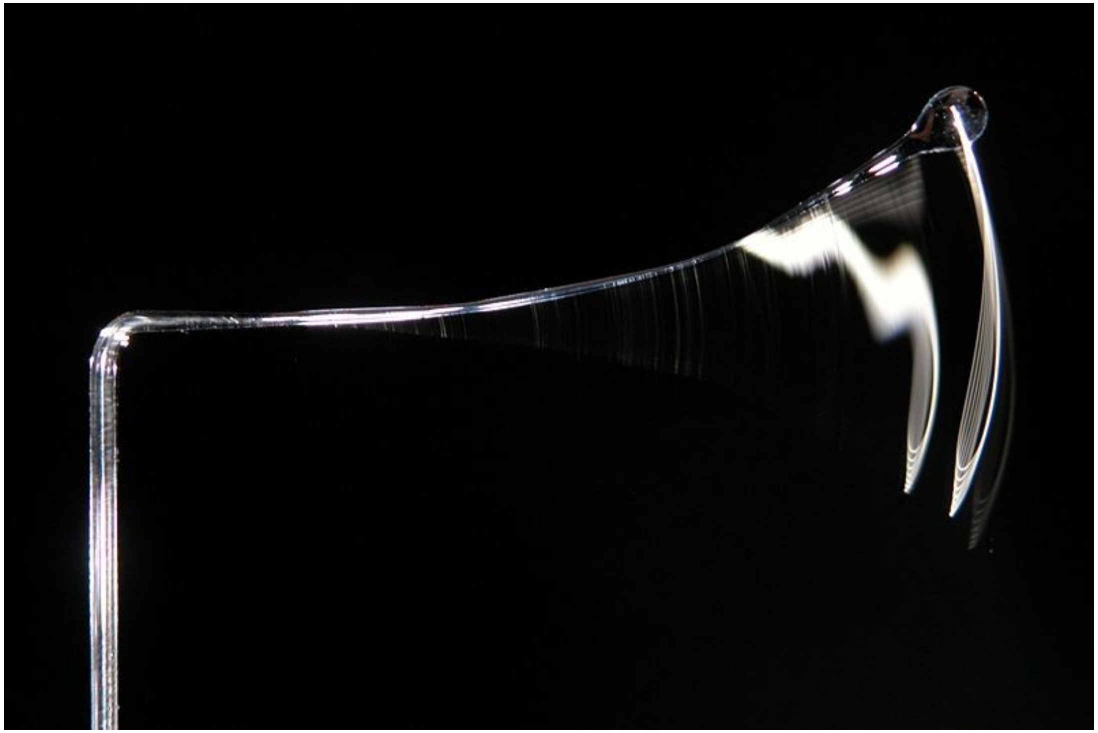
Índice

Elasticidad Lineal

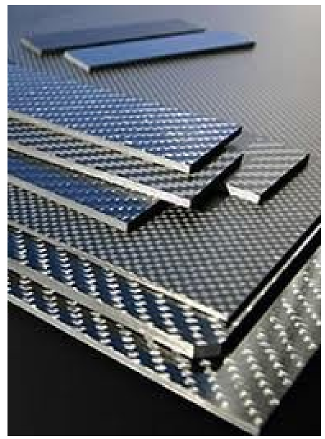
- **Elasticidad:** Propiedad de ciertos materiales de sufrir deformaciones reversibles cuando se encuentran sujetos a la acción de fuerzas exteriores y de recuperar la forma original si estas fuerzas exteriores desaparecen.

- **Elasticidad Lineal:** Un caso particular de la elasticidad en el que las tensiones aplicadas y las deformaciones sufridas por el material están relacionadas por una lineal como la que sigue:



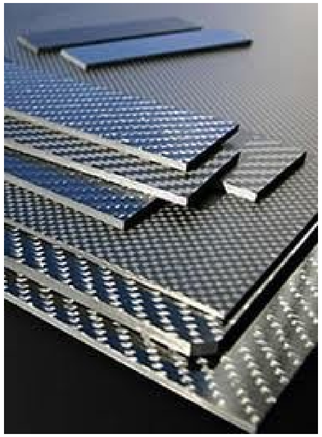
$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

Materiales Compuestos

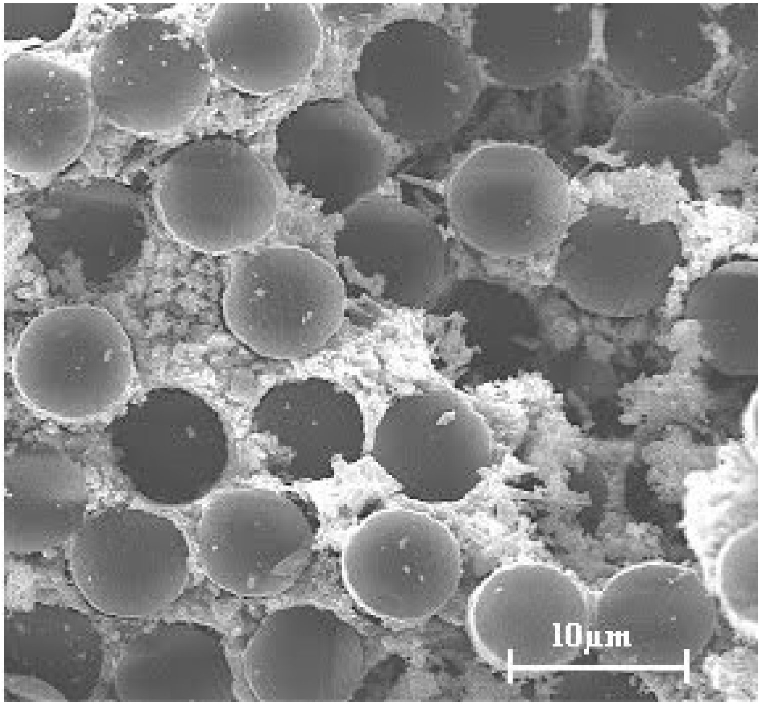
- **Materiales Compuestos:** En esta primera aproximación, nos vale con tener en cuenta que son materiales formados por varios componentes, que se denominan fases, de manera que cada uno de ellos tiene unas propiedades físicas y mecánicas (que pueden ser lineales o no) y cuya unión da como resultado un material nuevo cuyas propiedades no son las de sus componentes ni superposiciones de ellos tampoco, sino un material nuevo con características que habrá que definir.


Materiales Compuestos



- **Fibra de Carbono:** CFRP es un material compuesto o composite formado por dos fases: un polímero denominado matriz, y un refuerzo formado por fibras de carbono. También puede considerarse un tercer implicado, la interfase entre ambos. A menudo el nombre del refuerzo da nombre a todo el compuesto, como ocurre con el material frecuentemente denominado fibra de vidrio.

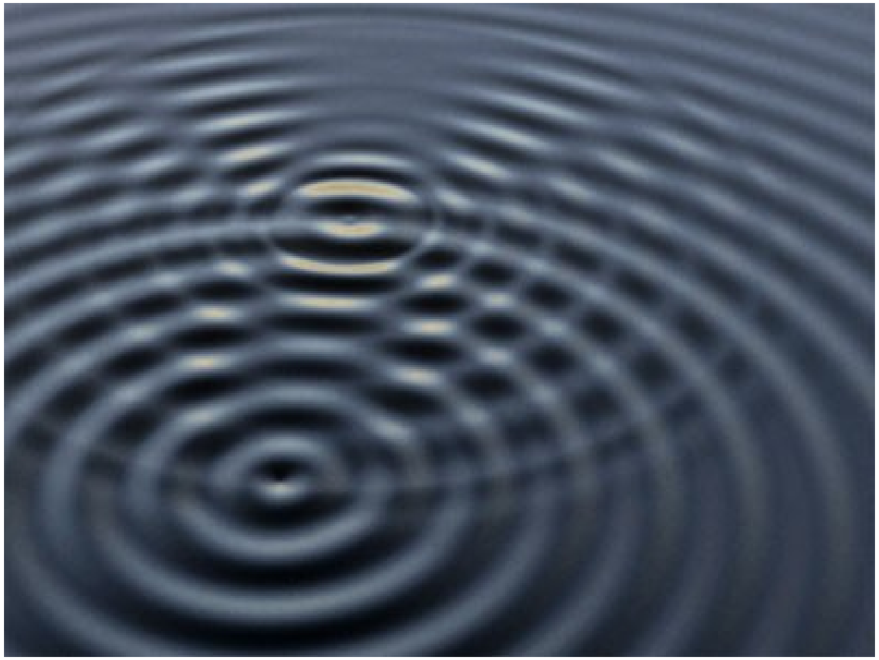
Micromecánica



- **Micromecánica:** Es el análisis de los materiales compuestos heterogéneos a nivel de sus componentes individuales. Uno de los principales objetivos de la micromecánica es la homogeneización del material compuesto.

- **Homogeneización:** La homogeneización consiste en la predicción de la respuesta de los materiales heterogéneos en base a la geometría y propiedades de los materiales que los componen, denominados fases. Una de las principales ventajas de la homogeneización es que se puede predecir el comportamiento del material heterogéneo compuesto sin haberlo probado a él, lo que puede resultar útil cuando dichas pruebas sean muy costosas. No obstante, para confiar en la micromecánica, hay que comparar los datos experimentales con aquellos predichos por nuestro modelo teórico en un proceso conocido como validación del modelo.

Teoría de Ondas



- **Onda:** Una onda es una perturbación que viaja en el espacio y en el tiempo, generalmente acompañada de una cierta transferencia de energía.

Ondas Unidimensionales – Ondas Bidimensionales
Ondas Longitudinales – Ondas Transversales
Ondas Mecánicas – Ondas Electromagnéticas

- **Magnitudes de la Onda:**

Período (T)

Frecuencia (f)

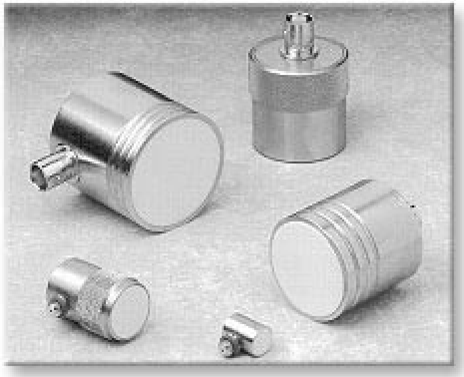
Longitud de Onda (λ)

Velocidad de Propagación (v)

$$T = 1/f = 2\pi/\omega;$$
$$v = \lambda/f;$$

Ultrasonidos

- **Ultrasonidos:** Onda sonora (por tanto elástica) cuya frecuencia es elevada (mayor de 10 kHz) y no puede ser percibida por el oído humano. Es posible determinar las propiedades mecánicas de un determinado material mediante el uso de ultrasonidos. Conocidas algunas propiedades del material tales como sus dimensiones, densidad y coeficiente de Poisson, se pueden obtener algunas constantes elásticas tales como los módulos de Young (E) y de Cizalla (G).



Relación entre velocidades obtenidas experimentalmente y los datos conocidos del material:

Velocidad onda longitudinal

$$V_L = \sqrt{\frac{E(1-\mu)}{\rho(1+\mu)(1-2\mu)}}$$

V_L = velocidad onda longitudinal
E = módulo Elástico
r = densidad
m = coef. Poisson

Velocidad onda transversal

$$V_S = \sqrt{\frac{E}{2\rho(1+\mu)}} \text{ or } \sqrt{\frac{G}{\rho}}$$

V_S = velocidad onda transversal
E = módulo Elástico
r = densidad
m = coef. Poisson
G = módulo cizalla

John Douglas Eshelby

John Douglas Eshelby (21 December 1916 - 10 December 1981) was a scientist in micromechanics. His work has shaped the fields of defect mechanics and micromechanics of inhomogeneous solids for fifty years and provided the basis for the quantitative analysis of the controlling mechanisms of plastic deformation and fracture.


Scanned at the American Institute of Physics

- The continuum theory of lattice defects (1956)

- The determination of the elastic field of an ellipsoidal inclusion, and related problems (1957)

- Elastic inclusion and inhomogeneities (1961)

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- Elastic inclusion and inhomogeneities (1961)

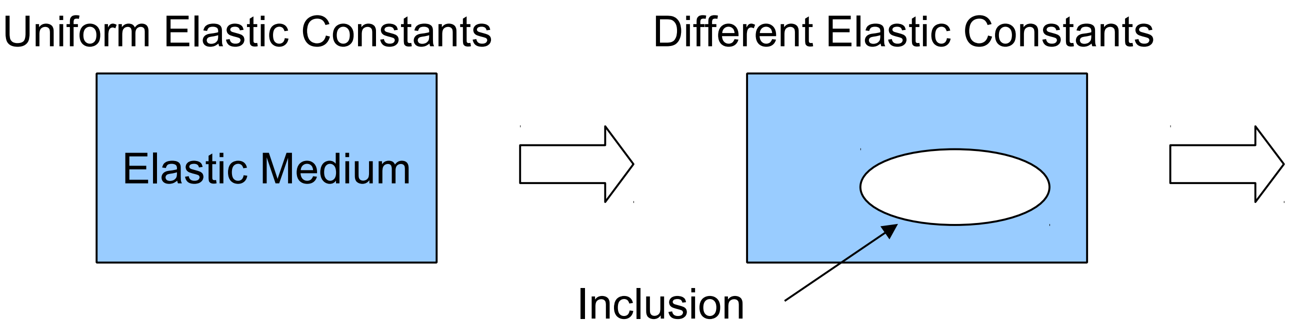
Eshelby 1957

"The determination of the elastic field of an ellipsoidal inclusion, and related problems"

- Original Problem:

Region within Isotropic elastic solid + Spontaneous change of form → Stresses matrix & inclusion

Region ≡ Ellipsoid → Strain Uniform, expressed by elliptic integrals



- Simple Solving Method
- Ellipsoid solves wide variety of particular cases

Eshelby 1957

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- Original Problem:

Region within Isotropic elastic solid + Spontaneous change of form → Stresses matrix & inclusion

Region ≡ Ellipsoid → Strain Uniform, expressed by elliptic integrals

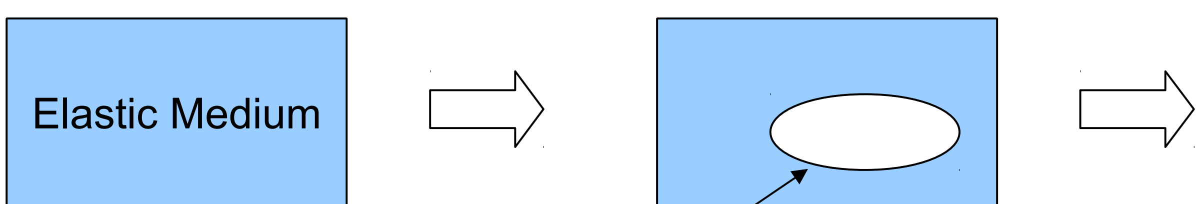
- Further problem:

$\left.\begin{array}{l}\text{Infinite medium, elastic constants K1} \\ \text{Ellipsoidal region, elastic constants K2}\end{array}\right\}$ → How is the applied stress-field?

Physical & Engineering questions are function of the Elastic Field inside the ellipsoid

Uniform Elastic Constants

Different Elastic Constants

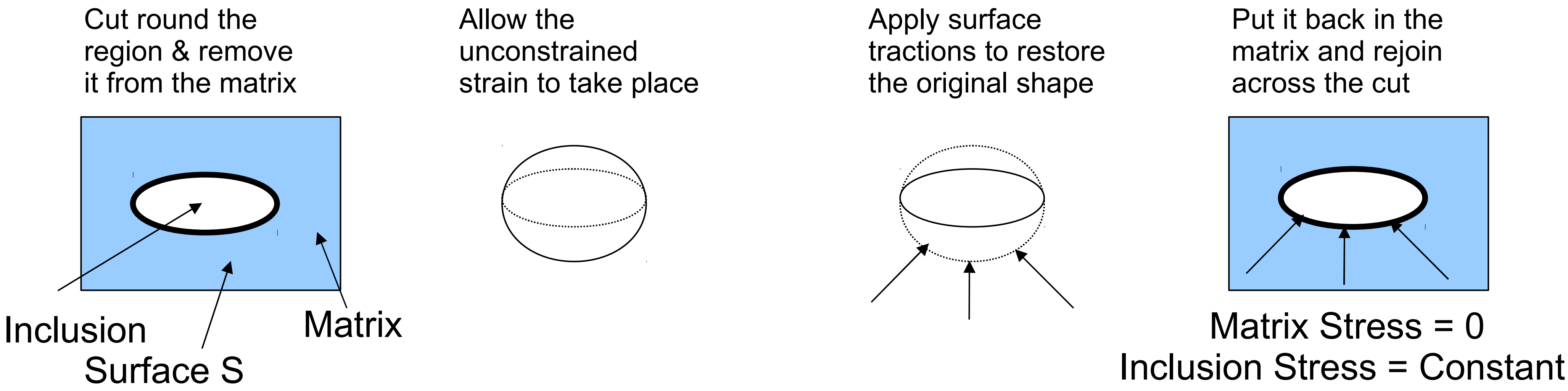


Inclusion

- Simple Solving Method
- Ellipsoid solves wide variety of particular cases

Eshelby 1957

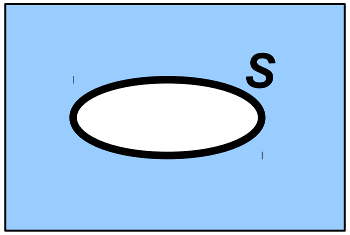
Transformation problem: What is the elastic state of inclusion and matrix?



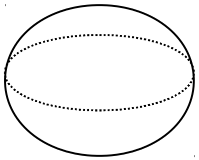
Eshelby 1957

Transformation problem: What is the elastic state of inclusion and matrix?

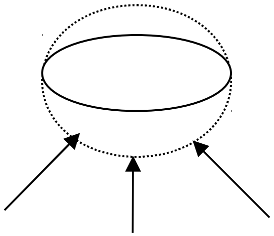
Cut round the region & remove it from the matrix



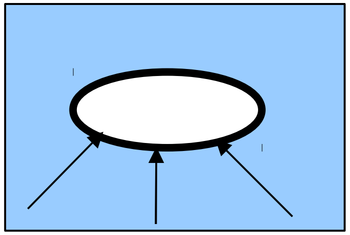
Allow the unconstrained strain to take place



Apply surface tractions to restore the original shape



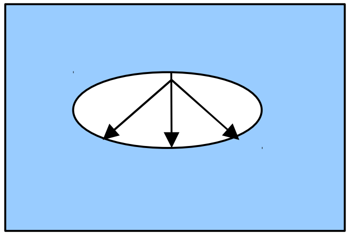
Put it back in the matrix and rejoin across the cut



Matrix Stress = 0
Inclusion Stress = Constant

If inclusion is an ellipsoid → Stress Uniform within the inclusion

Uniform stress to the solid annulling inclusion stress



Just remove the unstressed inclusion → Problem of the **ellipsoidal cavity solved**

Eshelby 1957

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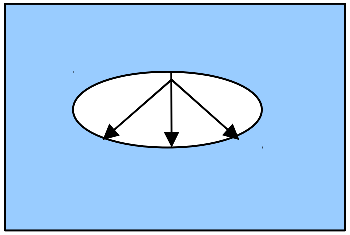


Just remove the unstressed inclusion → Problem of the **ellipsoidal cavity solved**

If the applied uniform stress does not annul the stress in the inclusion

Hooke Law of the material

$$\sigma_{ij} \neq C_{ijkl}\varepsilon_{kl}$$

Relationship between strain and stress $\equiv \sigma_{ij} = C'_{ijkl}\varepsilon_{kl}$,

where C'_{ijkl} is some hypothetical material constants

<u>Statement:</u>

Transformed ellipsoid may be replaced by an ellipsoid of the hypothetical material which has suffered the same total strain, but purely elastically →

<u>We have thus solved the Inhomogeneity Problem</u>

Eshelby 1957

The general inclusion

Stress-free strain, ε^T_{ij} : Uniform strain which the inclusion would undergo in the abscence of the matrix

Contrained strain, ε^C_{ij} : Strain occuring in the inclusion when it transforms while embedded in the matrix

I. Allow the stress-free strain: $p_{ij}^T = \lambda \varepsilon^T \delta_{ij} + 2\mu \varepsilon_{ij}^T$

II. Surface tractions to the inclusion: $-p_{ij}^T n_j$

III. Apply body forces over S: $+p_{ij}^T n_j$

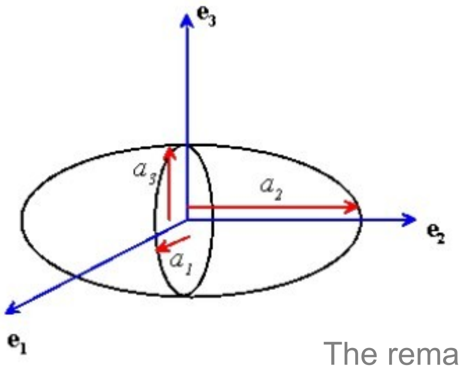
The ellipsoidal inclusion

Relationship between Stress-free (T) and Contrained (C) Strain:

$$\boxed{\varepsilon_{il}^C = S_{ilmn} \varepsilon_{mn}^T} \quad \rightarrow \quad S_{ilmn} \equiv \text{Eshelby's Tensor}$$

Eshelby 1957

Eshelby's Tensor



$$\varepsilon_{ij}^{C} = S_{ijkl}\,\varepsilon_{kl}^{T}$$

S_{ijkl} *isafunctionof*
theelasticpropertiesofthesolid
semi − axisoftheellipsoid

The remaining components may be computed by the cyclic permutation. The others are zero

$$S^{*}_{1111} = \frac{3}{8\pi(1-\nu)} a_1^2 I_{11} - \frac{1-2\nu}{8\pi(1-\nu)} I_1$$

$$S^{*}_{1122} = \frac{3}{8\pi(1-\nu)} a_2^2 I_{12} - \frac{1-2\nu}{8\pi(1-\nu)} I_1$$

$$S^{*}_{1133} = \frac{3}{8\pi(1-\nu)} a_3^2 I_{13} - \frac{1-2\nu}{8\pi(1-\nu)} I_1$$

$$S^{*}_{1212} = \frac{a_1^2 + a_2^2}{16\pi(1-\nu)} I_{12} - \frac{1-2\nu}{16\pi(1-\nu)} (I_1 + I_2)$$

Ellipsoidal Inhomogeneities

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} \le 1,$$

Elasticconstantsinside theregion C'_{ijkl}
Elasticconstantsoutside theregion C_{ijkl}

$\longrightarrow$

Substituting , we find the transformation strain can be computed by solving

$$\boxed{\left(C_{ijkl} - C'_{ijkl}\right) u^{\infty}_{k,l} = \left(C_{ijpq} - \left(C_{ijkl} - C'_{ijkl}\right) S_{klpq}\right) \varepsilon^{T}_{pq}}$$

where

$$u^{\infty}_{k,l} = \textit{displacementfieldby}\,\sigma^{\infty}_{ij}$$

$$\varepsilon^{T}_{pq} = \textit{stress} - \textit{freestrain}$$

$$S_{klpq} = \textit{Eshelby'sTensor}$$

Mori – Tanaka 1973

“Average Stress in Matrix and Average Elastic Energy of materials with misfitting inclusions”

T. Mori & K. Tanaka, 1973.

Huge contribution to micromechanics. ***2052 mentions, according to official sources.***

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Material → Calculating the average internal stress in matrix

Matrix
&
Inclusions

<u>Mori Tanaka Statements:</u>

- Average internal stress is →
  - Uniform throughout the material
  - Independent → $\sigma_A \neq f(x_D)$

  where x_D = Position of the domain of the average treatment

- $\sigma_A = \sigma_{AV} + \sigma_{LF}$ →
  - σ_A is the actual stress
  - σ_{AV} is the average stress
  - σ_{LF} is the locally fluctuating stress

Average Internal Stress in Matrix (I)

- Case C_{ijkl} uniform throughout specimen V_0.
- N inclusions, uniformly distributed in matrix

e^T_{ij} in inclusion V → e'_{ij} in specimen V_0.

e'_{ij} → e^∞_{ij} constrained strain – elastic outside inclusion
e^{im}_{ij} image strain in the actual specimen by free boundary – elastic in nature
$(e^\infty_{ij} - e^T_{ij})$ – elastic inside the inclusion

All inclusions identical transformation strain e^T_{ij} → Average strain in the specimen $\langle \varepsilon_{ij}{}^F \rangle_{V_0} = f \varepsilon_{ij}{}^T$
Where f is a volume fraction of the inclusions.

Assume (1): The average strains, in domain Vr, from all the inclusions is also equal to f e^T_{ij} if Vr is a representative domain for the specimen.

$\langle \varepsilon_{ij} \rangle_{V_R} = -f(S_{ijmn} e_{mn}{}^T - e_{ij}{}^T).$ → Thus, the average elastic strain defined above is independent of the position and the size of V_R.

Average Internal Stress in Matrix (II)

Next, let us consider the following sum,

$$\sigma_{ij}{}^{\infty}(1, M) = \sum_{P=1}^{M} \sigma_{ij}{}^{\infty}(x, x^P), \tag{8}$$

where $\sigma_{ij}{}^{\infty}(x, x^P)$ is internal stress due to the P-th inclusion at x^P within V_R when it is in an infinite body. This sum, $\sigma_{ij}{}^{\infty}(1, M)$ cannot be assumed nearly constant in V_R; instead, it fluctuates and the wave length of the fluctuation is apparently of the order of the inter-inclusion spacing. $\sigma_{ij}{}^{\infty}(1, M)$ is called locally fluctuating stress, to which nearby inclusions obviously contribute predominantly. However, the average of

In summary, the internal stress at any point in the matrix is the uniform average stress $\langle\sigma_{ij}\rangle_M$ plus the locally fluctuating stress from nearby inclusions. This locally fluctuating stress is averaged to be zero in the matrix. It is important to note that $\langle\sigma_{ij}\rangle_M$ is the average, within the matrix, of the sums of the stresses of the inclusions when they are present in an infinite body $(\Sigma\sigma_{ij}{}^{\infty})$ and of the image stresses of all the inclusions $(\Sigma\sigma_{ij}{}^{im})$. The relative contributions of $\Sigma\sigma_{ij}{}^{\infty}$ and of $\Sigma\sigma_{ij}{}^{im}$ are not generally determined. However, as shown in the appendix, the contribution from each term can be calculated, if the specimen is of ellipsoidal shape. Especially when the specimen is

Main Equations

APPENDIX: Average Stress and image stress in a uniform ellipsoidal body containing ellipsoidal inclusions with transformation strain

$$\int_{V_0} \sigma_{ij}{}^{im}\, dD = -\int_V \sigma_{ij}{}^{\infty}\, dD - \int_{V_0 - V} \sigma_{ij}{}^{\infty} dD. \tag{A1}$$

$$-\int_V \sigma_{ij}{}^{\infty}\, dD = -VC_{ijkl}[S_{klmn}(v)\varepsilon_{mn}{}^T - \varepsilon_{kl}], \tag{A2}$$

$$-\int_{V_0 - V} \sigma_{ij}{}^{\infty}\, dD = -VC_{ijkl}[S_{klmn}(V_0) - S_{klmn}(V)]\varepsilon_{mn}{}^T, \tag{A3}$$

$$\begin{aligned}\langle \sigma_{ij}{}^{\infty} \rangle_M &= \langle \sigma_{ij}{}^{\infty} \rangle_{V_0 - V} \\ &= \{V/(V_0 - V)\} C_{ijkl} \\ &\quad \times [S_{klmn}(V_0) - S_{klmn}(V)]\varepsilon_{mn}{}^T.\end{aligned} \tag{A4}$$

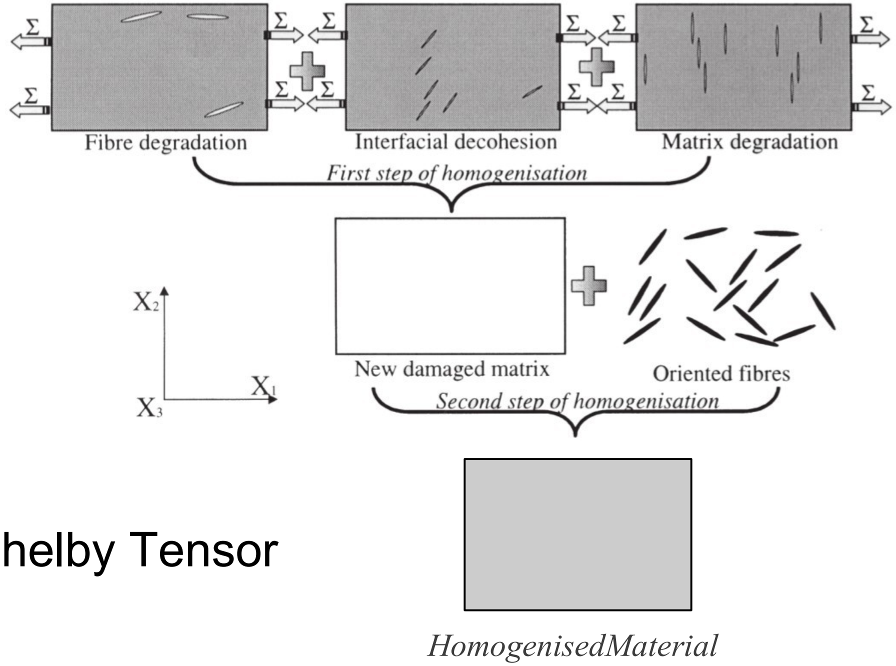
$$\int_{V_0} \sigma_{ij}{}^{im} dD = -VC_{ijkl}[S_{klmn}(V_0)\varepsilon_{mn}{}^T - \varepsilon_{kl}{}^T]. \tag{A5}$$

$$\langle \sigma_{ij}{}^{im} \rangle_{V_0} = -(V/V_0) C_{ijkl}[S_{klmn}(V_0)\varepsilon_{mn}{}^T - \varepsilon_{kl}{}^T]. \tag{A6}$$

Otros modelos micromecánicos

Desrumaux 2001

Generalised Mori-Tanaka Scheme to model anisotropic damage using numerical Eshelby Tensor.



- New micromechanical model:
  Two levels of homogenisation

- Statistical Failure Criterion:

- Mori-Tanaka theory for 3-phase system
  First, Overall Effective Stiffness
  Second, Stress in Matrix

- Developed Model with: Stiffness Tensor & Eshelby Tensor

<u>Applications</u>:

- Glass material reinforced polyester
- Comparison between model and experience

Otros modelos micromecánicos

Fahim Al Ameri 2009

Model-Based Damage Reconstruction in Composites from Ultrasound Transmission

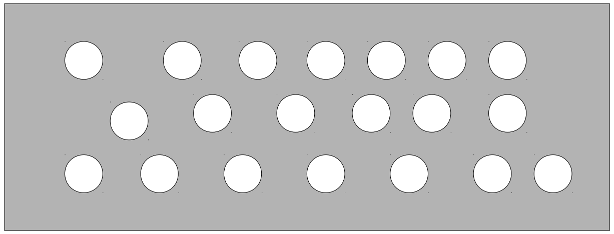
Mori-Tanaka method

Shear modulus G
Bulk modulus K

Damage variable $$D = \frac{2(1 - \frac{\overline{E}}{E})(5\nu - 7)}{2(5\nu - 7) + \frac{\overline{E}}{E}(15\nu^2 + 2\nu - 13)}$$

Graphics: Density of micropores ↔ Damage Energy

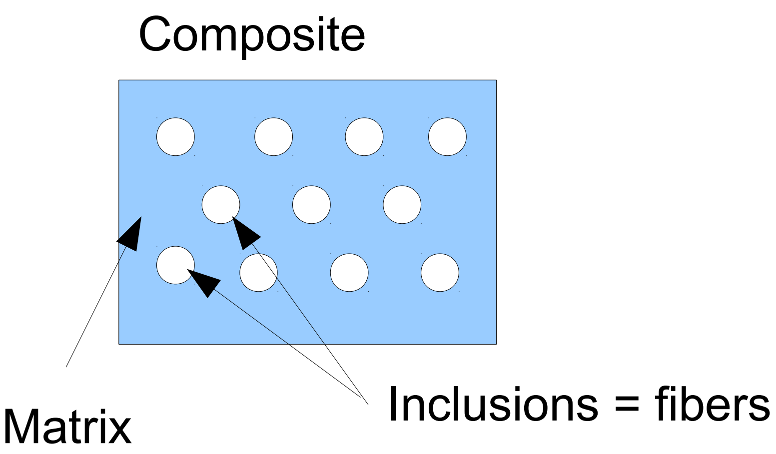
Increase of micropores → Reduction in the Young modulus



Otros modelos micromecánicos

Propuestas

Proponer un modelo micromecánico para el comportamiento de composite con despegue de fibras



<u>Pasos:</u>
- Confección del modelo micromecánico.
- Estudio de la solución analítica si existe.
- Propuestas de resolución numérica.
- Validación mediante muestras ensayadas por técnicas de ultrasonidos.

<u>Variaciones:</u>
- Propuesta estocástica de fibras despegadas
- Propuesta de estudio de porcentaje de fibra que sigue trabajando.

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