



The problem with rotation

Massive stars ($M > 2M_{\odot}$) are usually fast rotators \rightarrow structure distorted by centrifugal forces, gravity darkening making T_{eff} , $\log g$ dependent on inclination, Coriolis force impact mode propagation

\rightarrow need for 2D models and 2D complete mode calculations \rightarrow ESTER and TOP codes.

Machine learning for rotating-star seismology

Separating subsets of modes and identifying regular patterns

Oscillation modes in solar-like stars are known to follow regular patterns \rightarrow in massive, fast-rotating stars, modes can be split in subsets that exhibit such patterns.

Mirouh et al. (2019) used a **convolutional neural network** to separate modes into subsets based on their geometry. Among those, **island modes** are the most visible, and follow the scaling relation $\langle \rho \rangle \propto \Delta\nu^2$

Reaching a complete description of the island-mode spectrum

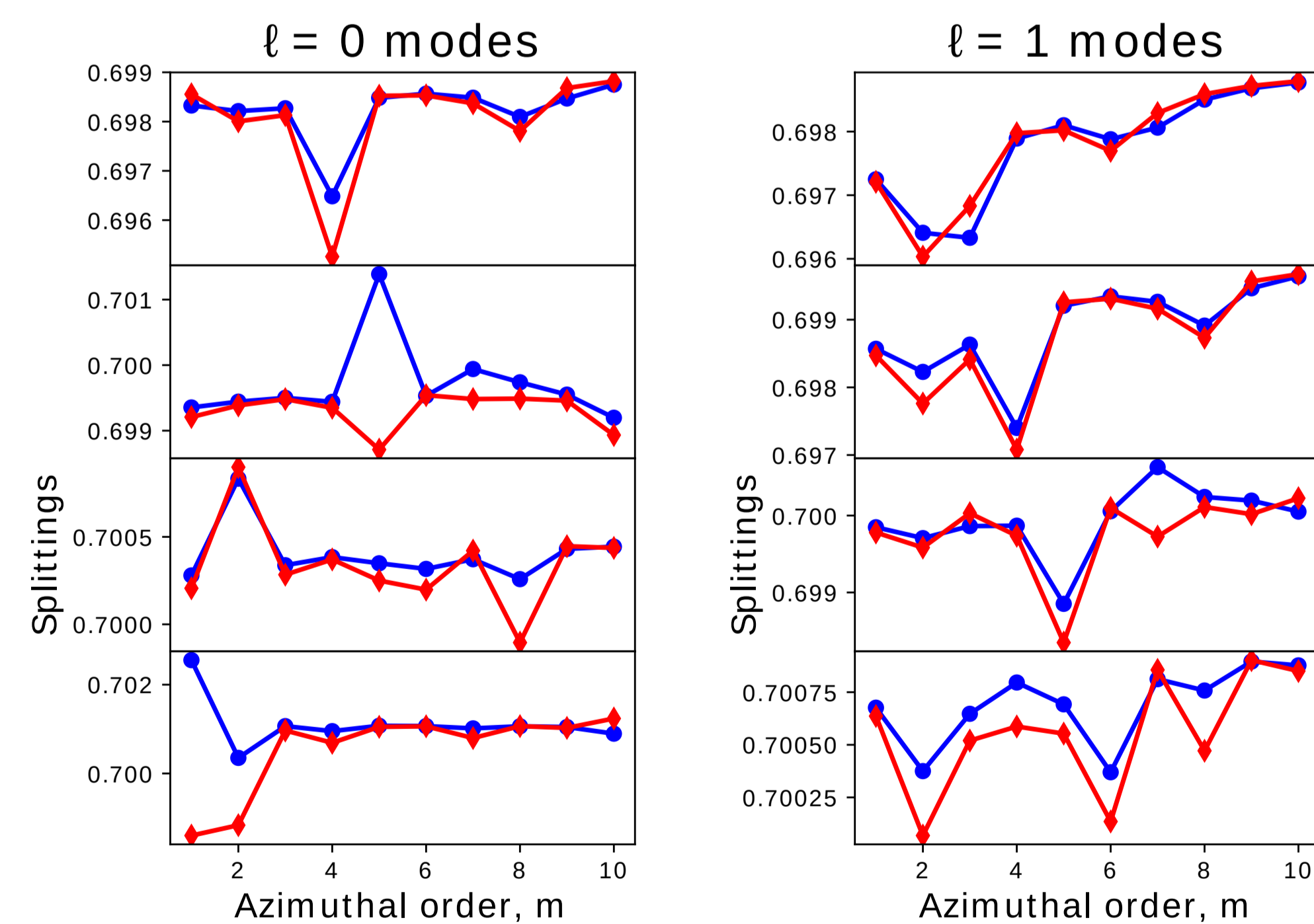
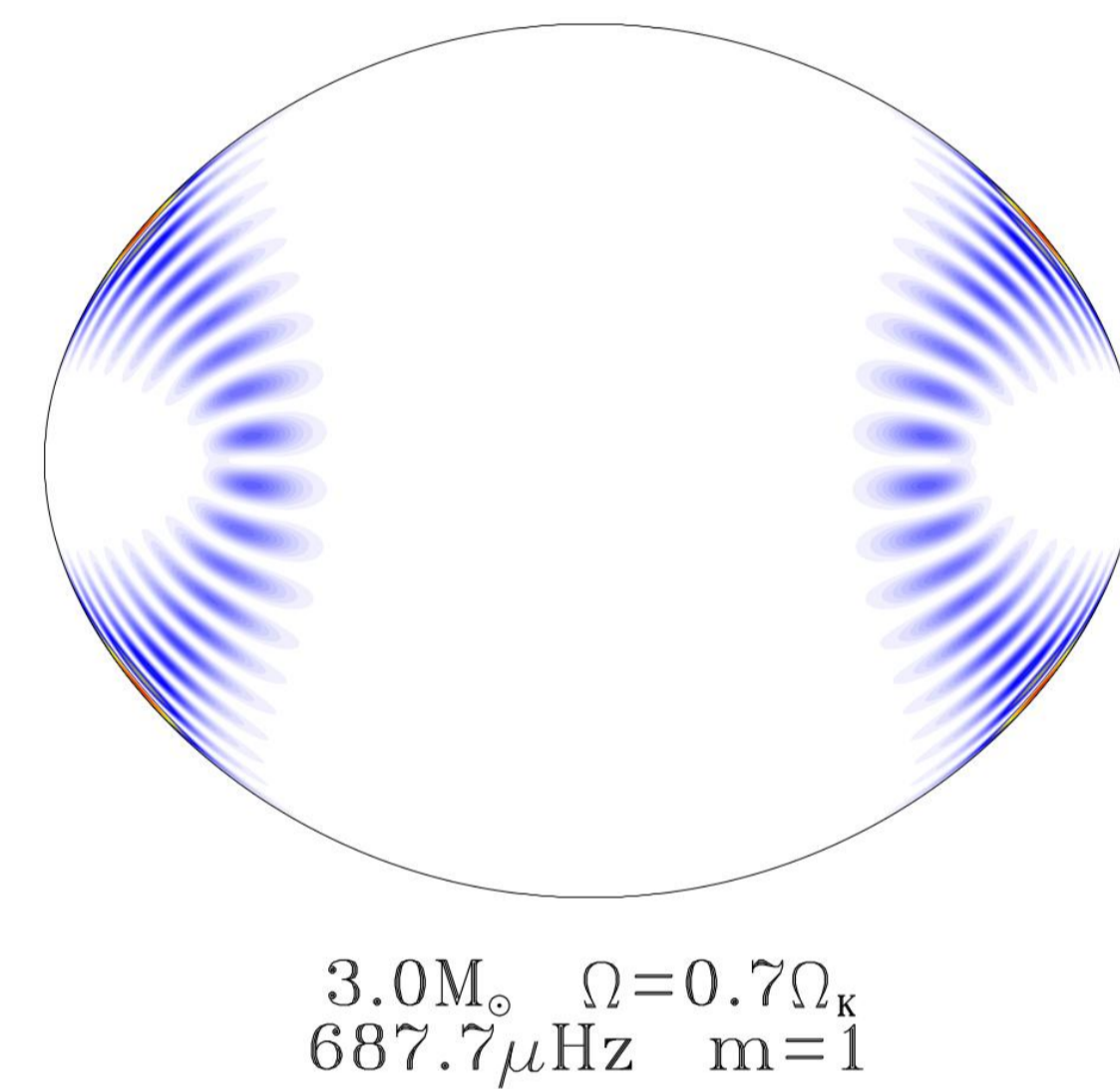
(Reese, Mirouh et al. submitted to A&A)

Rotational splittings = degeneracy lift for modes at $m \neq 0$, can be predicted

Considering a doublet at $+m$ and $-m \rightarrow$ models give us frequencies

We compute integrals over the propagation domain \rightarrow **integration kernels** \blacktriangleright

\rightarrow we need a theoretical link with internal rotation



Theoretical prediction from the variational principle

\rightarrow effective rotation probed $\Omega_{\pm}^{\text{eff}}$ for modes at $\pm m$

$$\frac{\nu_+ - \nu_-}{2|m|} \approx \frac{\Omega_+^{\text{eff}} - \Omega_-^{\text{eff}}}{4\pi} + \frac{-C_+ + C_-}{4\pi|m|}$$

$$\text{with } \Omega_{\pm}^{\text{eff}} = \frac{\int_V \Omega \rho_0 |\xi_{\pm}|^2 dV}{\int_V \rho_0 |\xi_{\pm}|^2 dV}$$

$$\text{and } C_{\pm} = \frac{\int_V \rho_0 \Omega \cdot (\xi_{\pm}^* \times \xi_{\pm}) dV}{\int_V \rho_0 |\xi_{\pm}|^2 dV}$$

\blacktriangleleft Good match between **model** and **theory**, usually \rightarrow discrepancies can be attributed to **avoided crossings**

Spectroscopy of fast-rotating stars

Two-dimensional models to fit observations

We want to find the best $(M, X_{\text{core}}, \Omega, i)$ combination for a given set of observables $(T_{\text{eff}}, \log g, v \sin i, L, \Delta\nu)$.

From a grid of two-dimensional models for various masses, ages, rotation rates and inclinations

\rightarrow 1D atmospheres computed at each point + limb-darkening + flux-weighted average on the visible surface

\rightarrow estimate of **surface properties** to compare with observations.

Finding the best-fit models

We use both a full 4D Bayesian analysis and a Monte-Carlo Markov Chain (MCMC) algorithm to find the best-fit model quickly and reliably. These methods allow to include observational errorbars and provide uncertainties on the model.

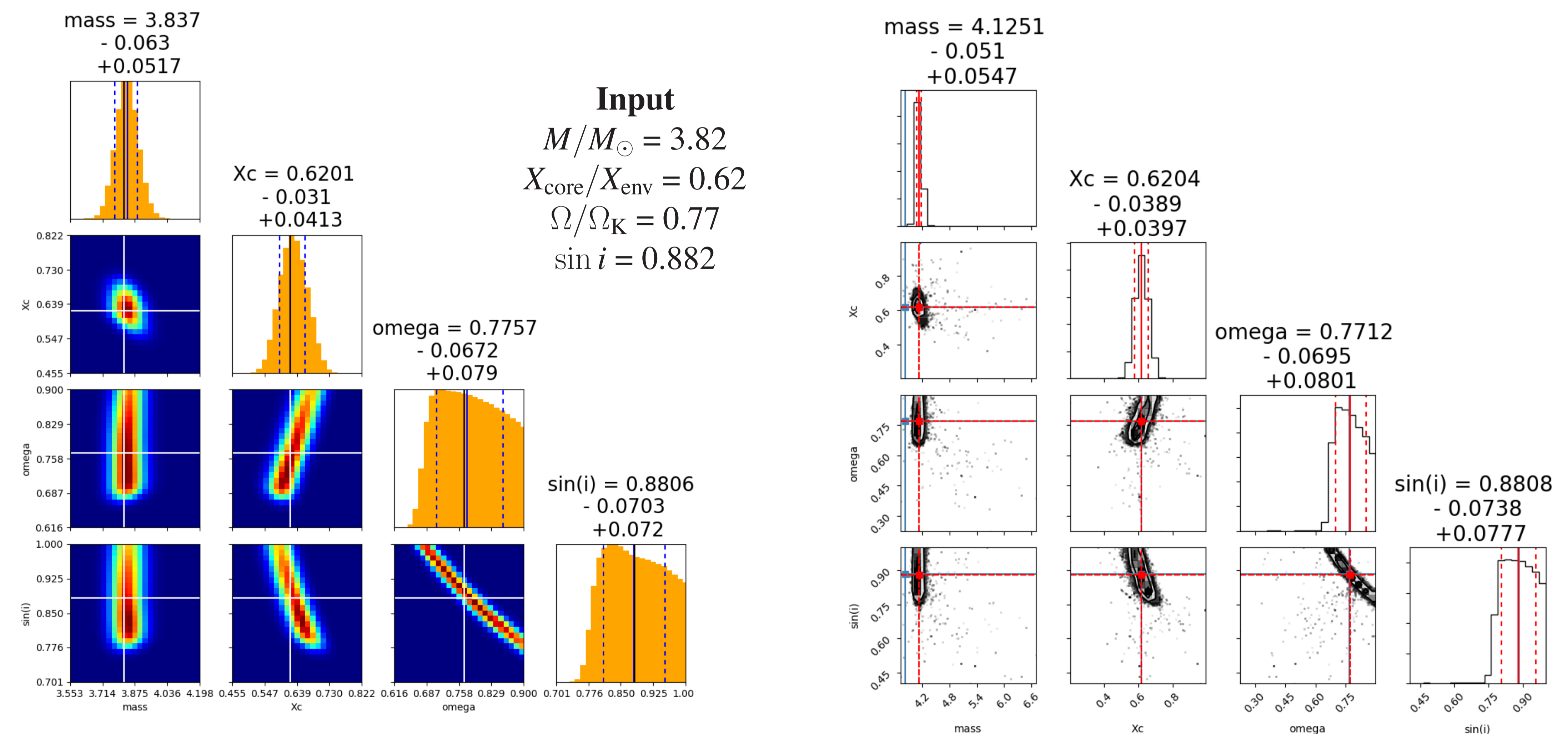
The likelihood of a model is computed as $L = \exp\left(-\frac{\chi^2}{2}\right)$ with $\chi^2 = \frac{(L_{\text{model}} - L_{\text{obs}})^2}{\sigma_L^2} + \dots$

Hare-and-hounds exercise to test the method

Create a random 2D model \rightarrow compute their surface properties $(T_{\text{eff}}, \log g, v \sin i, L) \rightarrow$ feed those values in the pipeline and retrieve the original model.

Including seismic diagnosis such as $\Delta\nu$ reduces the degeneracy of the problem

\rightarrow low uncertainties for fast rotators, both full Bayesian and MCMC show **very promising results** \blacktriangledown



Asteroseismology : summary

Island modes are the key to the seismology of fast-rotating stars

\rightarrow the convolutional neural network allows us to **identify modes**

\rightarrow separation-density scaling relation and workable rotational splittings

Future development = extension to **other subsets of modes** \rightarrow non-island pressure, gravity, and inertial modes

Spectroscopy : summary

Combining seismology and spectroscopy \rightarrow automatically constrain fast-rotating stars and **determine their inclination** \rightarrow builds on available observations, avoids expensive interferometry

Both **theoretical developments work on test cases** and must now be applied to actual observations.

Future development = spectral energy distributions \rightarrow chemical constraints, use on actual data