Jornada de Geometría

Universidad de Granada

Grupo de investigación
Problemas variacionales en geometría

9–10 de diciembre de 2010
Sala de Conferencias
Planta baja, Sección de Matemáticas.
Facultad de Ciencias.

Probamos por medio de la lógica,
pero descubrimos por medio de la intuición.

—Henri Poincaré

Programa

Todas las conferencias tendrán lugar en la *Sala de Conferencias*, situada en la planta baja de la sección de Matemáticas, Facultad de Ciencias.

JUEVES 9

17:00 Ahmad El Soufi (Université François-Rabelais, Tours)

On the geometry of manifolds whose Laplace-Beltrami operator admits large eigenvalues

18:00 Joaquim Martín (Universidad Autónoma de Barcelona)

Pointwise symmetrization inequalities for Sobolev functions on Probability Metric Spaces

VIERNES 10

10:00 Miguel Sánchez (Universidad de Granada)

Lorentzian manifolds isometrically embeddible in Lorentz-Minkowski

11:00 Café.

11:30 Mark Haskins (Imperial College London)

Exceptional holonomy and calibrated submanifolds.

14:00 Comida.

Resúmenes

On the geometry of manifolds whose Laplace-Beltrami operator admits large eigenvalues

Ahmad El Soufi

The sequence of eigenvalues of the Dirichlet Laplacian on a bounded Euclidean domain satisfies several restrictive conditions such as : Faber-Krahn isoperimetric inequality, that is the principal eigenvalue is bounded above in terms of the volume of the domain, Payne-Pólya-Weinberger type universal inequalities, that is the k-th eigenvalue is controlled in terms of the $k-1$ previous ones, etc.

The situation changes completely as soon as Euclidean domains are replaced by compact manifolds. For example, according to results by Colin de Verdière and Lohkamp, given any compact manifold M of dimension $n \geq 3$, it is possible to prescribe arbitrarily and simultaneously, through the choice of a suitable Riemannian metric on M, a finite part of the spectrum of the Laplacian, the volume and the integral of the scalar curvature. Hence, Faber-Krahn and Payne-Pólya-Weinberger inequalities have no analogue in this context.

In this talk, we will discuss the effect of the geometry on the eigenvalues. We will try to understand what kind of geometric situations lead to large eigenvalues for the Laplacian on manifolds of fixed volume, and what does such a Riemannian manifold look like once realized as a submanifold of a Euclidean space.

On the other hand, we show that when the Laplacian is penalized by the squared norm of the mean curvature, then we obtain a Schrödinger type operator whose spectral behavior is similar to that of the Dirichlet Laplacian on Euclidean domains.

Pointwise symmetrization inequalities for Sobolev functions on Probability Metric Spaces

Joaquim Martín

To formulate Sobolev inequalities one needs to answer questions like: what is the role of dimension? What norms are appropriate to measure the integrability gains? Just to name a few... For example, in contrast to the Euclidean case, the integrability gains in Gaussian measure are logarithmic but dimension free (log Sobolev inequalities). So it is easy to understand the difficulties to derive a general theory. I will discuss some new methods to prove general Sobolev inequalities that unify the Euclidean and the Gaussian cases, as well as several important model manifolds.

Lorentzian manifolds isometrically embeddible in Lorentz-Minkowski.

Miguel Sánchez

Our aim is to give a simple characterization of the class of Lorentzian manifolds which can be isometrically embedded in Lorentzian-Minkowski L^N for some large N (in the spirit of classical Nash theorem) and, then, to show that this class includes the most relevant type of relativistic spacetimes, i.e., the globally hyperbolic ones. This last result was claimed by CJS Clarke (1970), but his proof was affected by the so-called *folk problems of smoothability* in Lorentzian Geometry. These problems will be specially discussed.

The talk is based in a joint work with O. Müller (arXiv:0812.4439v4).

Exceptional holonomy and calibrated submanifolds.

Mark Haskins

We give an introduction to recent developments in the geometry of compact manifolds with exceptional holonomy, focusing on recent work with Corti, Nordstrom and Pacini; we prove the existence of many compact 7-manifolds with holonomy G2 that contain rigid associative submanifolds. The main ingredients in the proof are: an appropriate noncompact version of the Calabi conjecture, gluing methods and a certain class of complex projective 3-folds (weak Fano 3-folds).