Las abejas, en virtud de una cierta
intuición geométrica,
saben que el hexágono es mayor
que el cuadrado y que el triángulo,
y que podrá contener más miel
con el mismo gasto de material.

—Pappus de Alejandría, s. IV

**Universidad de Granada**

# Jornada de Geometría


Grupo de investigación
*Problemas variacionales en geometría*

Proyecto de excelencia
*Análisis Geométrico y Aplicaciones*

3–4 de diciembre de 2009
Sala de Conferencias.
Facultad de Ciencias.

## Programa

Todas las conferencias tendrán lugar en la *Sala de conferencias*, situada en la planta baja de la sección de Matemáticas, Facultad de Ciencias.

JUEVES 3

**17:00** Antonio Ros (Universidad de Granada)

*Area minimizing surfaces in flat tori.*

**18:00** Antonio Alarcón (Universidad de Murcia)

*Minimal surfaces in $\mathbb{R}^3$ properly projecting into $\mathbb{R}^2$.*

VIERNES 4

**10:00** Graham Smith (Centre de Recerca Matemàtica)

*Non-linear Dirichlet problems in Hadamard manifolds.*

**11:00** Café.

**11:30** Isabel Fernández (Universidad de Sevilla)

*Complete minimal surfaces with a prescribed coordinate.*

**12:30** Giuseppe Tinaglia (University of Warwick)

*The geometry of constant mean curvature surfaces embedded in $\mathbb{R}^3$.*

**14:00** Comida.

## Resúmenes

**Area minimizing surfaces in flat tori**

*Antonio Ros*

A surface $S$ in a complete 3-manifold $M$ is area-minimizing mod 2 if it has least area among all surfaces, orientable or nonorientable in the same homology class. These surfaces present a rich and interesting geometry, even in flat or positively curved 3-manifolds. For instance, if $M$ is flat, Fischer-Colbrie and Schoen, Do Carmo and Peng, and Pogorelov proved that complete two-sided stable minimal surfaces are flat, but Ross proved that some nonorientable quotient of the classic Schwarz P and D surfaces are estable, and we proved that an area minimizing surface in $\mathbb{R}^2 \times S^1$ is either planar or a quotient of the Helicoid. We will review some results about this problem and we will prove that area minimizing surfaces in flat 3-tori are planar.

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**Minimal surfaces in $\mathbb{R}^3$ properly projecting into $\mathbb{R}^2$**

*Antonio Alarcón*

On the one hand, given an open Riemann surface $\mathcal{N}$ and a real number $\theta \in ]0, \pi/4[$, we construct a conformal minimal immersion

$$X = (X_1, X_2, X_3) : \mathcal{N} \to \mathbb{R}^3$$

such that $X_3 + \tan\theta |X_1| : \mathcal{N} \to \mathbb{R}$ is positive and proper. Furthermore, $X$ can be chosen with arbitrarily prescribed flux map. This construction is related with a problem posed by Shoen and Yau.

On the other hand, we produce properly immersed hyperbolic minimal surfaces with empty boundary in $\mathbb{R}^3$ lying above a negative sublinear graph. This construction can be linked to a conjecture by Meeks.

The main tool used in the construction of the above examples is an approximation theorem by minimal surfaces. We will also remark some other applications of this result.

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**Non-linear Dirichlet problems in Hadamard manifolds**

*Graham Smith*

We prove the existence of constant curvature hypersurfaces in Hadamard manifolds subject satisfying topological/boundary conditions.

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**Complete minimal surfaces with a prescribed coordinate**

*Isabel Fernández*

We will show that any (non-constant) harmonic map on an arbitrary open Riemannian $N$ surface can be realized as the third coordinate of a complete minimal immersion of $N$ in $\mathbb{R}^3$. As a consequence we will prove that any open Riemann surface admits a complete conformal minimal immersion whose Gauss map misses two points.

This is a joint work with F.J. López and A. Alarcón.

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**The geometry of constant mean curvature surfaces embedded in $\mathbb{R}^3$.**

*Giuseppe Tinaglia*

In this talk I will discuss recent results on the geometry of constant mean curvature ($H \neq 0$) surfaces embedded in $\mathbb{R}^3$. Among other things I will prove a radius and curvature estimates for constant mean curvature disks embedded in $\mathbb{R}^3$. It follows from the radius estimate that the only complete constant mean curvature disk embedded in $\mathbb{R}^3$ is the round sphere. This is joint work with Bill Meeks.