Chapter 1. Observational Astronomy and Stellar Atmospheres

1. The star τ Sco has an apparent visual magnitude of $V = +2.8$ and a spectral type of approximately B0V. Parallax measurements indicate a distance of 470 ly. (a) Calculate the absolute visual magnitude. (b) Search in the literature the corresponding bolometric correction and calculate the luminosity L. (c) Adopt a typical T_{eff} value for its spectral class and calculate the radius. (d) Estimate the mass. (e) Calculate the acceleration of gravity at the stellar surface and the escape velocity. (f) Calculate the mean density of the star. (g) Compare these values with those of the Sun. L2.3

2. Consider a model of a star as a black body with a surface temperature 28000 K and a radius of 5.16×10^{11} cm at a distance of 180 pc. Determine: Luminosity, absolute and apparent bolometric magnitudes, distance modulus, flux at the star's surface, flux at the Earth's surface, wavelength of maximum emission. p.13

3. Show that $M_B - M_V = B - V = m_B - m_V$ and that this is valid for any combination of photometric index.

4. Consider a giant and a main-sequence star of the same spectral type. Usually a giant star has a lower atmospheric density and a slightly lower temperature than a main-sequence star. Use the Saha equation to explain this. (Note: this means that there is not a perfect correspondence between temperature and spectral type). p.46

5. The Gaia satellite measures parallaxes with an accuracy of 2×10^{-5} arcsec for stars with $V < 15$. (a) What is the distance d of an M5 I supergiant with $\log(L/L_\odot) = 5.50$ and $BC = -3.70$ that has $V = 12$, if the effect of interstellar extinction is ignored? (b) What is the relative distance accuracy $\sigma(d)/d$ of such a star? (c) Assuming that T_{eff} is known, what is the relative accuracy in radius $\sigma(R)/R$? L2.4

6. The average radial distance traveled by a photon in a random walk is $r \sim lN^{1/2}$, where N is the number of random steps and l is the step length (i.e., the mean-free path length). (a) Use $l \sim 1$ cm and $r = R_\odot$ to estimate the number of scatterings a photon will undergo before reaching the surface of the star. (b) Calculate the total path length L and the time it takes a photon created in the center to leave the Sun. (c) Is it still the same photon? (Same λ, ν?) L5.2

7. How far you could see trough the Earth's atmosphere if it had the opacity of the solar photosphere ($\kappa_{500\ nm} = 0.264$ cm^{2}g^{-1}). The air density is 1.2×10^{-3} gcm^{-3}. p.49

8. Assume that the intensity of the radiation field is a constant I_{out} in the $+z - direction$ and also constant I_{in} in the $-z - direction$. Calculate the mean intensity, flux and radiation pressure. p.51

9. Verify that if the source function is $S_\lambda = a_\lambda + b_\lambda \tau_\lambda$, the emergent intensity is $I_\lambda(0) = a_\lambda + b_\lambda cos\theta$. p.54

10. Calculate the Doppler width of a FeI line at $\lambda \sim 4700$ Å, a) in the photosphere of the Sun, b) in the photosphere of a red giant, c) in the atmosphere of a white dwarf. m.

11. Assume a star of radius R at a distance d which emitting a specific intensity I constant at the surface. Estimate the radiative flux at the surface of the star and that measured at d. m.

12. Very hot stars of spectral types B or earlier have unstable atmospheres and loss mass at a considerable rate due to the radiation pressure. For these stars the main opacity source in the atmosphere is given by the Thomson's scattering of electrons. Calculate the maximum luminosity for these stars to be in hydrostatic equilibrium. This luminosity is named *Eddington's luminosity).*

(Hint: use the equation of hydrostatic equilibrium.) m.

Chapter 2. Stellar Structure and Evolution

1. Assume that 10 eV could be released by every atom (H) in the Sun through chemical reactions. Estimate how long the Sun could shine at its current rate through chemical processes alone. p.59

2. What temperature would be required for two protons to collide if quantum mechanical tunnelling is neglected? Assuming that the Sun is pure hydrogen, estimate the number of H nuclei in the Sun. Could therebe enough protons moving with a speed 10 times the thermal velocity to account for the Sun's luminosity? p.59

3. Estimate the temperature of star of radius R, mass M and constant density using the Virial theorem. Does the Virial theorem apply to massive main-sequence stars, where hydrostatic equilibrium is partly supported by radiation pressure? Why? Does the Virial theorem apply to degenerate stars? m, LQ3.3

4. Show that the temperature dependence of the triple alpha process can be written as $\epsilon_{3\alpha} \sim \epsilon_o \rho^2 Y^3 f_{3\alpha} T_8^{41}$, where Y is the ^4He mass fraction,$f_{3\alpha}$ the screening factor of the triple alpha process and T_8 is the temperature measured in 10^8 K units. p.61

5. Compare the expected radius of a non-relativistic complete electron degenerate star with that of a non-relativistic complete neutron degenerate star of the same mass. Comment on the result. L4.6

6. Assume that through the star $\beta = P_{gas}/P_{total}$ and $1 - \beta = P_{rad}/P_{total}$ being $0 < \beta < 1$ constant. Show that this system admits a polytropic EOS. m.

7. At what rate is the Sun's mass decreasing due to nuclear reactions? p.73

8. The pressure at the base of the photosphere in the Sun is 5×10^4 dynes cm^{-2} and density $\rho = 2.5 \times 10^{-7}$ g cm^{-3}. Estimate the pressure scale at the base of the photosphere. Assuming the a mixing length parameter $\alpha = 2.2$, estimate the time required for a convective bubble to travel one mixing length in the Sun. (Hint, the turbulent velocity of solar granulation is 0.5 km s^{-1}.) p.75

9. Use the equation of hydrostatic equilibrium and the assumption of constant density to compute the central pressures for each of the following stars: a) a K0 V star, b) a K0 III star, c) a K0 I star. (Search in the literature the differences between the stars of different luminosity class.) p2.87

10. When the HR diagram is constructed from observed data, part of the width of the main-sequence is due to errors in distance measurements. If a typical uncertainty in parallax is 10%, by how much (in magnitudes) will stars typically be displaced from their true positions on the HR diagram? p2.88

11. Consider the Sun to be a sphere of uniform density that derives its luminosity from steady contraction. What fractional decrease in the Sun's radius would be required over the last 6000 years to account for the Sun's constant luminosity over that period of time? p2.92

12. Suppose the density of a star as $\rho(r) = \rho_o(1 - (r/R)^2)$, where R is the radius of the star. Calculate the mass (M) of the star, the mean density in terms of ρ_o and the central pressure in terms of M and R. p2.94

13. A protostellar cloud starts as a sphere of size 4000 AU and temperature 15 K. If it emits blackbody radiation, what is its total luminosity? What is the wavelength at which emits the most radiation? p2.103

14. Consider two clouds in the ISM, one H_2 cloud with T= 10 K and $n = 10^{12}$ m^{-3} and one HI with T= $120K$ and $n = 10^7$ m^{-3}. Calculate the Jeans mass for each cloud, the minimum radius each cloud must have to collapse and the timescale for the gravitational collapse of each cloud? p2.104

15. For a star of 1 $M_\odot$ compute the energy released in the triple alpha reaction. Then, assume that at the beginning of the central He burning (HB phase), 10% of the original mass of the star is in the form of ^{4}He in the core. Estimate the total energy released by fusing this amount of helium into carbon. Assume that during the HB phase, $L = 100L_\odot$. If all this luminosity is provided by fusion of helium, how long will the HB phase last? p2.105

16. Estimate the rotation period of a white dwarf rotating at break-up speed. Assume a mean density of ρ 2×10^9 kgm^{-3} p2.108

17. Photons leaving the surface of a compact stellar remnant are gravitationally redshifted by an amount

$$\Delta\nu/\nu_o \sim -r_{Sch}/2r$$

where r_{Sch} is the Schwarzchild radius and r is its actual radius. Calculate the gravitational redshift for the hydrogen line $H_\beta\lambda 4861$ Å for

a 1 $M_\odot$ white dwarf. It is possible to distinguish this redshift from a possible Doppler shift due to the motion of the white dwarf? p2.109

18. Estimate the temperature of a dust grain located 100 AU from a newly formed F0 main-sequence star. Assume that the grain is spherically symmetric and behaves as a blackbody. p.77

19. Three stars make up the 40 Eridani system: object A is a 4th-magnitude star similar to the Sun; object B is a 10th-magnitude white dwarf and object C is a 11th-magnitude red M5 star. The period of objects B and C system is 247.9 yr. The parallax of the system is $0.201''$ and the angular extend of the semi-major axis is $6.89''$. The ratio of the distances B to C from the centre of mass is $a_B/a_C = 0.37$. a) Find the mass of objects B and C. b) The absolute bolometric magnitude of object B is 9.6. Determine its luminosity. c) The effective temperature of object B is 16900 K; calculate its radius. d) Calculate the average density of object B. e) Calculate the product of the mass and volume of both 40 Eri B and Sirius B. Is there a departure from the mass-volume relation? p.95

20. Estimate the ideal gas pressure and the radiation pressure at the centre of Sirius B which has a central temperature 3×10^7 K. p.96

21. In the *liquid-drop-model* of an atomic nucleus, a nucleus with mass A has a radius of $r_o A^{1/3}$, where $r_o = 1.2 \times 10^{-15}$ m. Find the density of this nuclear model. p.97

22. If our Moon were as dense as a neutron star, what would its diameter be? p.98

23. Estimate tha maximum and the minimum mass for a star. m.

24. In the pp chain estimate the equilibrium abundance of deuterium (as the number of atoms relative to H). Assume that the amount of H atoms does not change significantly. m.

25. Calculate the mean molecular mass μ of a fully ionised gas as a function of X and Y. Assume that $Z << Y << X$. m.

26. Obtain the general relationship between the constant K, the mass M and the radius R of a generic polytropic system as seen in the class room. m.

27. Using dimensional analysis, estimate the relationship between the luminosity and the radius with the mass of a star belonging to the lower and upper main-sequence. m.

28. Sketch the chemical profile of He in a massive star in which the mass of the convective core decreases during the H-fusion phase. Sketch it at three epochs: at the beginning of the main sequence phase (ZAMS), halfway through the main sequence phase and at the end of the main sequence phase (TAMS). L7.4.

29. (a) Calculate the mass defect fractions of the following fusion reactions:

$4\mathbf{H}+2e \rightarrow {}^{4}\mathbf{He}$

$3\,{}^{4}\mathbf{He}\rightarrow{}^{12}\mathbf{C}$

${}^{12}\mathbf{C}+{}^{4}\mathbf{He}\rightarrow{}^{16}\mathbf{O}$

$2\,{}^{16}\mathbf{O}\rightarrow{}^{28}\mathbf{Si}+{}^{4}\mathbf{He}$

$2\,{}^{28}\mathbf{Si}\rightarrow{}^{56}\mathbf{Fe}$

(b) Describe the trend and discuss what this trend implies for stellar evolution.L8.1

Chapters three and four: Galactic Morfology and Dynamics

1. A globular cluster has a integrated apparent visual magnitude of +13 and an integrated absolute visual magnitude of -4.15. It is located 9.0 kpc from Earth. Estimate the amount of interstellar extinction between the cluster and the Earth in magnitude per kpc. p.153

2. A typical temperature in the ISM is 15 K, estimate the average thermal energy density of hydrogen gas in the disk of the Galaxy. Assume that the disk has a radius of 8 kpc and a height of 160 pc. Compare the result with the typical energy density of the interstellar magnetic field 6×10^{-14} **J m**$^{-3}$**.** p.155

3. Assuming that the high-velocity stars are near the escape velocity from the Galaxy, estimate the mass of the Milky Way. Repeat the calculating using the extremely high-velocity stars ($v_{esc} \sim 500$ **kms**$^{-1}$**). What could account for the extra mass compared with your answer in the first case?** p.156

4. Assuming that the dark matter mass density is given by $\rho(r) = \rho_o/(1+(r/a)^2)$ **for any arbitrary distance from the centre of the Galaxy, calculate the amount of dark matter interior to a radius r.** p.159

5. From the information given in the classroom etc, determine the approximate mass-to-light ratio of the Galaxy interior to a radius of 25 kpc from the centre. Repeat the calculation for a radius 100 kpc. What can you conclude about the effect that dark matter might have on the average mass-to-light ratio of the universe? p.160

6. By using Newtonian gravity, estimate the amount of energy required to move 10^7 **M**$_\odot$ **from a position just above the event horizon of the super-massive black hole at the centre of the Galaxy to 3 kpc. Compare your answer with the energy liberated in a typical Type II supernova.** (Search in the literature the mass estimated for the galactic central black hole). p.162

7. Assume that a black hole (point) of mass 3.7×10^6 **M**$_\odot$ **is located at the centre of the Galaxy and that the remainder of the mass has a density distribution that varies as** r^{-2}**. Show that if the mass distribution is spherically symmetric, the mass interior to a radius is given by** $M(r) = kr + M_o$**, where** k **is a constant. Assuming perfectly circular motion and Newtonian gravity, show that the orbital velocity curve is given by** $v = (G(k + M_o/r))^{1/2}$**. Plot** v **as a function of log** r **over the**

range $0.01 < r < 1$ **kpc. At what radius does the contribution of the central point mass begin to become significant?** p.162

8. Estimate the M_B **for our Galaxy. Using the Tully-Fisher relation, calculate the maximum rotation speed of the Galactic disk.** p.164

9. Transform the de Vaucouleurs profile into units of mag. arc sec^{-2}. p.164

10. NGC 2639 is a Sa galaxy with a maximum rotational velocity of 324 kms^{-1} and an apparent magnitude B= 12.22. **a) Estimate its absolute magnitude in B from the Tully-Fisher relation. b) Determine the distance of NGC 2639.** p.165

11. The rotational velocity of M32 at $1''$ **from the centre is about 50 kms^{-1}. Estimate the mass within this radius.** (a parallax of $1''$ is d 3.73 pc) p.165

12. Show that if the surface brightness of an elliptical galaxy follows the $r^{1/4}$ **law, then the average surface brightness over the area of a circular disk of radius** r_e **is given by** $< I >= 3.607 I_e$. p.167

13. According to the virial theorem, the central radial-velocity dispersion for a typical elliptical galaxy is related to the mass and size of the galaxy by $\sigma_r^2 \propto M/R$. **Show that in that case** $L \sim \sigma_r^4$, **which is call the Faber-Jackson relation.** p.168

14. M31 has approximately 350 globular cluster. If its absolute visual magnitude is -21.7, estimate the specific frequency for its clusters. b) Do the same for NGC 3311 which is a cD galaxy with about 17000 globular cluster and absolute visual magnitude -22.4. Discuss the results in terms that cD galaxies are due to mergers of already formed spiral galaxies. p.168

15. Suppose that the average luminosity L of a quasar with a redshift z **has the form** $L = L_o(1+z)^{3.3}$, **where** L_o **is the luminosity today. How much more luminous is an average quasar at** $z = 2$ **than today?** p.183

16. A spiral galaxy displays the rotation curve in Figure 1. Use this curve to set a lower limit to the mass of the galaxy.EG.1

17. An edge-on spiral galaxy is observed to have the HI line profile depicted in Figure 2 and a H-ban flux of m$_H$ $= 15.1$. **Estimate the distance of this galaxy.**

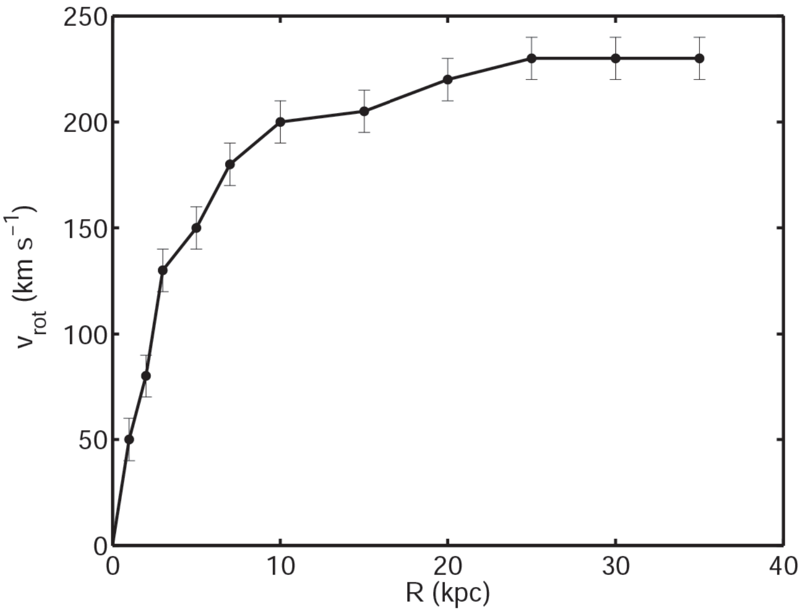

Figure 1: The inclination-corrected rotation curve of a spiral galaxy.
EG.5

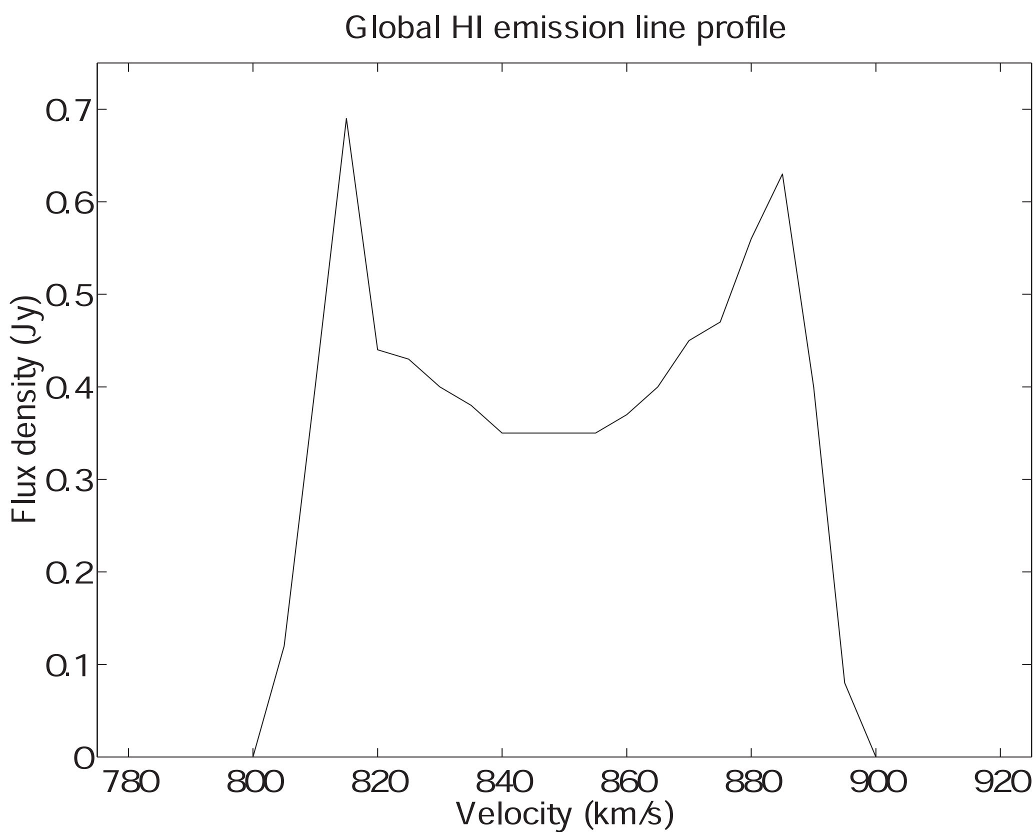

Figure 2: The global HI profile of an edge-on disk galaxy.

Chapter five: Evolution of Galaxies and Large Structure of the Universe

1. The mass density of stars in the solar neighbourhood of the Sun is around 0.05 $M_\odot$ pc^{-3}. Assuming that it is constant and that all stars are main-sequence M stars, estimate the fraction of the Galactic disk's volume that is occupied by stars. b) Suppose that a main-sequence M star travels perpendicularly through the Galactic disk. What are the odds of this star colliding with another star during its passage through the disk? (Search in the literature the typical mass and radius for a M star in the main-sequence.) p.170

2.By using dimensional analysis write a general expression for the force of dynamical friction as $f_d \sim C(GM)^\alpha v_M^b \rho^c$ where C is dimensionless and $\alpha,\ b$ and c are constants. p.170

3. Use the age of the Milky Way's oldest globular clusters and the orbital velocity of the local standard rest to estimate the greatest distance from which globular clusters could have spiralled into the nucleus due to dynamical friction. Use 5×10^6 $M_\odot$ for the cluster's mass (take C=76). p.171

4. The LMC has a mass of about 2×10^{10} $M_\odot$ and orbits our Galaxy at about 51 kpc of distance. Assuming that the Galaxy's flat rotational curve (including the dark-matter halo) extend out the LMC, estimate how much time it will take for the LMC to spiral into the Galaxy (take $C = 23$). p.171

5. Suppose that the original density distribution of the proto-Galactic nebulae had a radial functional dependence similar to the dark-matter halo ($\rho(r) \sim C/r^2$), show that the free-fall time would be proportional to radius. (Hint: start with the radial equation of motion.) p.172

6. Assume an IMF Salpter-like with $x = 1.8$. Calculate the ratio of the number of stars that are formed in the mass range between 2 and 3 $M_\odot$ to those formed with masses 10 and 11 $M_\odot$. Assuming the mass-luminosity relation for main-sequence stars, derive an expression for the number of main-sequence stars formed per unit luminosity interval, dN/dL. Calculate the ratio of the number of stars formed with main-sequence luminosities 2 and 3 $L_\odot$ to the number formed with 10 and 11 $L_\odot$. Explain the results in terms of the physical properties of main-sequence stars. p.173

7. Estimate the number of stars currently formed per year in the Milky Way (as an average). p.174

8. Using the Jeans mass estimate the upper mass limit of galaxies that are in virial equilibrium. Assuming that the least massive galaxies could have formed out of gas with an initial temperature near the ionisation temperature of hydrogen, estimate the lower mass cutoff. By using the Jeans length, estimate the radius of the largest galaxies that are in virial equilibrium. p.174

9. Calculate the maximum mass of a protogalactic nebulae that can undergo a free-fall collapse if $R = 60$ **kpcand** $\Lambda(T) \sim 10^{-37}$ **$\mathbf{Wm^3}$.** p.174

10. M87 is a giant galaxy with mass 3×10^{13} **$\mathbf{M_\odot}$ within a radius of 300 kpc. How long would it take for a star near the outer edge of the galaxy to orbit the centre once? Assuming that M87 has been capturing smaller satellite galaxies up until the present time, would you expect the outer portions of M87 to be in virial equilibrium?** p.174

11. Estimate how much longer the Galaxy can effectively create new stars without an appreciable infall of new material. (Search into the literature the estimated current star formation rate in the Galaxy). p.175

12. Estimate the escape speed from a dwarf elliptical galaxy and compare the result with the speed of the ejecta from a typical Type II supernova, and with thermal velocity of hydrogen ions in a gas having a temperature of $\mathbf{10^6}$ K. How might you explain the evolution of ancient, gas-poor systems such as dE's and dSph's? p.175

13. Estimate how long a galaxy in the Coma cluster would take to travel from one side of the cluster to the other. Assume that the galaxy moves with a constant speed equal to the cluster's radial dispersion. How this compare with the Hubble time? Then...are the galaxies in the Coma cluster gravitationally bound? p.180

14. The dispersion in the radial velocity of the galaxies in the Virgo cluster is 666 $\mathbf{kms^{-1}}$. Use the virial theorem to estimate the mass of the Virgo cluster. p.180

15. The brightest galaxy in the A1060 cluster has an apparent magnitude $V = 10.99$**. Estimate the distance of the cluster.** p.181

Chapter six: Cosmology

1. Suppose that all the matter in the universe were energy if the form of black-body radiation. Take the average density of matter to be the WMAP value for the density of the baryonic matter. Find the temperature of the universe in that situation. At what wavelength would the blackbody spectrum peak? p.191

2. Check by substitution in the equation that gives $(dR/dt)^2$ **the solutions for an open universe (**$k < 0$**).** p.192

3. Show that $\Omega(t) = 1 + kc^2/(dR/dt)^2$ **in one-component universe of pressure-less dust. What does this have to say about the nature of the early universe?** p.194

4. In the same type of universe than above show $1/\Omega - 1 = (1/\Omega_o - 1)(1 + z)^{-1}$**.** p.195

5. Derive the acceleration equation $d^2R/dt^2 = -4/3\pi G(\rho + 3P/c^2)R$ p.195

6. Consider a model of universe consisting of neutral hydrogen atoms for which the average speed of the atoms is 600 km^{-1}**. Show that** $\rho >> P/c^2$ **for the gas. For an adiabatically expanding universe, for what value of R and z will** $\rho = P/c^2$**?** p.196

7. Show that $R^{3(1+w)}\rho = constant = \rho_o$ **using the fluid equation where** ρ_o **is the present value of** ρ**.** p.196

8. Show that for a pressure-less universe, $q(t) = 1/2\Omega(t)$**.** p.197

9. When the light from a distant quasar passes trough an intergalactic cloud may produce absorption C lines. By studying the relative intensity of these lines one conclude that the temperature of a intergalactic cloud is 7.4 K. The lines show a redshift of $z = 1.776$**. How does the temperature of the cloud compare with the temperature of the CMB at that redshift?** p.197

10. Consider a comoving sphere whose surface expands with the universe. Let it be centred at the origin and filled with the CMB photons. Show that $R^4\rho_{rel} = \rho_{rel,0}$**, is consistent with the conservation of the energy within the sphere.** p.200

11. Use the scale factor $R(t)$**, to show that the characteristic time for the expansion of the universe is** $\tau_{exp}(t) = 1/H(t)$**. Assuming a flat universe containing only matter and radiation, find a expression for**

the characteristic expansion time τ_{exp} as a function of the scale factor R. p.200

12. Show that one-component universe of relativistic particles is flat in the limit $z \to \infty$. p.202

13. Assuming $\rho_{b,0} = 4.17 \times 10^{-31}$ gcm^{-3}, what was the density of the matter at the time of the Big Bang nucleosynthesis, when $T \sim 10^{10}$ K? p.202

14. Calculate the time of decoupling t_{dec} for an universe of matter and radiation using the WMAP values for $z_{dec} = 1089$ and other quantities. p.204

15. Find an expression for Λ in terms of the density ρ_m of a static model of a pressure-less dust universe with a cosmological constant. Find also an expression for the curvature k for this static model. Is this model universe closed, open, or flat? p.205

16. Evaluate Ω_m, Ω_{rel} and Ω_Λ at the time of decoupling ($z = 1089$) using WMAP values. (Search into the literature the necessary WMAP data) p.206

17. Derive a general expression of the deceleration parameter, $q(t) = 1/2\ \Sigma_i(1 + 3w_i)\Omega_i(t)$. p.207

18. Use the acceleration equation to show that the acceleration of the universe changed sign when the scale factor was $R_{accel} = (\Omega_{m,0}/2\Omega_{\Lambda,0})^{1/3}$. Evaluate R_{accel} and z_{accel} at this time with the WMAP values. p.207

19. The cosmological constant becomes dominant as the scale factor R becomes increasingly larger in the Λ era. a) Show that the Hubble parameter is a constant in a flat universe deep in the Λ era. b) Suppose that starting today, only the cosmological constant contributes to the Friedmann equation. Solve this equation and show that for $\Lambda > 0$, the scale factor will increase exponentially. c) Use WMAP values to evaluate the characteristic time for the exponential expansion. p.208

20. In a one-component universe pressure-less dust show that the angular diameter observed for an extended object of linear diameter D at redshift z is $\theta = (H_o D/2c)\frac{(1+z)^{3/2}}{\sqrt{(1+z)-1}}$. p.215