ANEXOS

ANEXO 1. UNIDAD TEMÁTICA COMPLETA DEL ESTUDIO 1

COMPETENCIA PARA EL ANÁLISIS ONTOSEMIÓTICO DE TAREAS ESCOLARES

OBJETIVOS GENERALES:

- Reflexionar sobre diversidad de objetos y significados implicados en tareas matemáticas propias de educación secundaria.
- Reflexionar sobre las características de la visualización y el razonamiento diagramático (VRD) y su papel en la enseñanza y el aprendizaje de las matemáticas.
- Reconocer la diversidad de objetos y procesos implicados en tareas matemáticas propias de educación secundaria realizadas mediante la aplicación de visualizaciones y razonamiento diagramático.
- Conocer y aplicar herramientas teóricas específicas e innovadoras en el ámbito de la educación matemática para realizar análisis didácticos de tareas escolares.

CONTENIDO:

- Conceptos de visualización, diagramas y razonamiento diagramático.
- Uso de diagramas, visualización y recursos manipulativos en la enseñanza y aprendizaje de las matemáticas.
- Conocimientos implicados en la visualización y el razonamiento diagramático.

DURACIÓN:

- 3 sesiones de clases presenciales

REFLEXIÓN INICIAL SOBRE LOS OBJETOS QUE INTERVIENEN EN LA PRÁCTICA MATEMÁTICA

Apellido y nombre: ---

METODOLOGÍA:

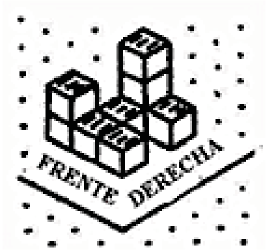
- Resolución de la Tarea 1, incluida a continuación, de manera individual.
- Presentación y discusión de resultados

DURACIÓN:

- 1 sesión de clases

Tarea 1. Exploración inicial

La figura adjunta muestra un edificio dibujado desde el ángulo frente-derecha.



1) Dibuja la vista del edificio desde atrás. Justifica la respuesta.
2) ¿Qué es para ti un concepto matemático? Identifica los conceptos matemáticos que intervienen en la resolución de la tarea.
3) ¿Qué es para ti una proposición matemática? Identifica las proposiciones matemáticas en la resolución de la tarea.
4) ¿Qué es para ti un procedimiento matemático? Describe el procedimiento matemático en la resolución de la tarea.
5) ¿Qué es para ti una demostración matemática? Elabora una justificación matemática para la respuesta dada en la tarea.
6) Uno de los conceptos que intervienen es el de cubo, usado para indicar cada una de las piezas que componen el 'edificio'.
    a) Elabora al menos dos definiciones diferentes para el cubo como concepto geométrico.
    b) Indica otros usos o significados que puede tener la palabra 'cubo'.
7) Indica qué papel desempeñan las proposiciones que has identificado en la justificación de la respuesta.
8) Describe otros posibles procedimientos que se podrían aplicar para resolver la tarea.
9) Describe una posible justificación de la respuesta que podría dar un estudiante usando algún tipo de material, secuencia de representaciones u otras explicaciones.
10) La figura geométrica dada se representa como una composición de piezas de forma cúbica.
    a) Identifica propiedades del cubo, como figura geométrica, que no se pueden representar de manera empírica.
    b) Enuncia la tarea utilizando lenguaje natural u ordinario.

ANÁLISIS ONTOSEMIÓTICO DE TAREAS MATEMÁTICAS

METODOLOGÍA:

- Lectura y elaboración de una reflexión sobre el siguiente artículo:

  Godino, J. D., Giacomone, B., Wilhelmi, M. R., Blanco, T. y Contreras, A. (2015). Configuraciones de prácticas, objetos y procesos imbricadas en la visualización espacial y el razonamiento diagramático.

- Presentación y discusión del artículo.
- Resolución de la Tarea 2, Tarea 3 y Tarea 4, incluidas a continuación, a partir de las siguientes consignas ontosemióticas, trabajando en equipos de 3 o 4 estudiantes:

  1. Resuelve el problema matemático.
  2. Describe el procedimiento seguido indicando la secuencia de prácticas elementales que has realizado para resolver la tarea; añade las explicaciones necesarias para justificar las respuestas.
  3. Completa la tabla incluida a continuación en la que se identifican los conocimientos que se ponen en juego en el enunciado y en cada una de las prácticas elementales, (añade las filas necesarias):

Uso e intencionalidad de las prácticas	Enunciado y secuencia de prácticas elementales para resolver la tarea	Objetos referidos en las prácticas *(Conceptos, proposiciones, procedimientos, argumentos)*
…	…	…

- Presentación y discusión de resultados

DURACIÓN:

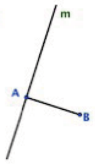
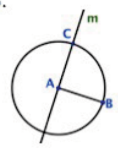
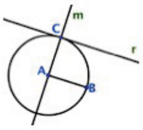
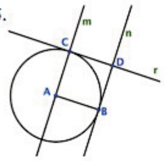
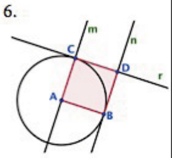
- 2 sesiones de clases

GRUPO DE TRABAJO:

Apellido y nombre: ---

Tarea 2. Construcción de un cuadrado con GeoGebra

La secuencia de pasos indicados a continuación es el procedimiento seguido por un alumno para construir un cuadrado con GeoGebra.

1. 	2. 	3. 	4. 	5. 	6. 
a) Represento un segmento AB.	b) Trazo una recta *m* perpendicular al segmento AB por el punto A.	c) Trazo una circunferencia de centro A y radio AB. d) Llamo C al punto de intersección entre la circunferencia trazada y la recta *m*.	e) Trazo una recta *r* paralela al segmento AB haciendo que pase por el punto C.	f) Trazo la recta *n* perpendicular al segmento AB por el punto B. g) Llamo D al punto de intersección de la recta *n* y la recta *r*.	h) El cuadrilátero ABCD es un cuadrado.

Justifica que, en efecto, el cuadrilátero ABCD es un cuadrado.

GRUPO DE TRABAJO:

Apellido y nombre: ---

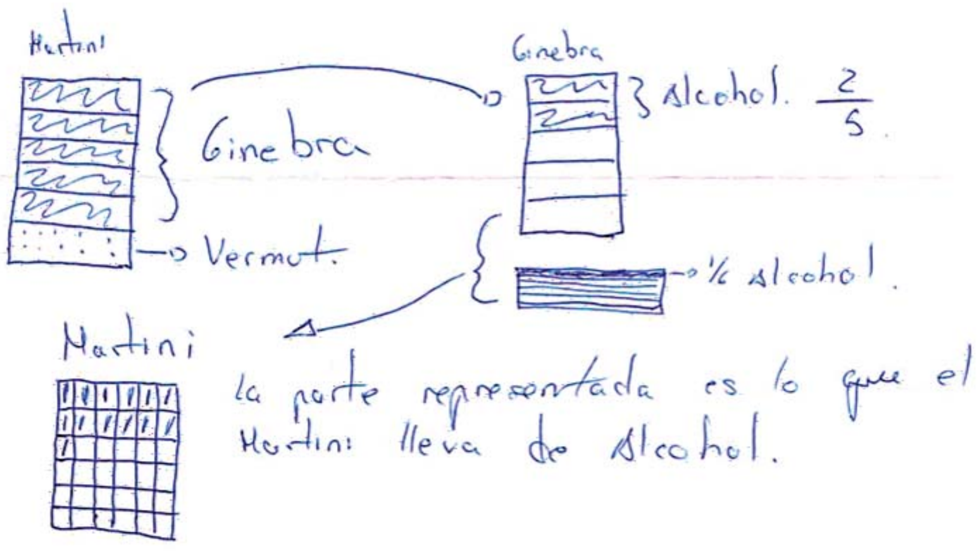
Tarea 3. Fracciones y diagrama de áreas

Un estudiante para maestro resuelve el siguiente problema:

Problema:

> *Un Martini es un cóctel que se hace con 5 partes de ginebra y 1 parte de vermut. Supongamos que 2/5 de la ginebra es alcohol y que 1/6 del vermut es alcohol. ¿Qué fracción de alcohol lleva un Martini? Resuelve el problema usando un diagrama de áreas.*

Solución:



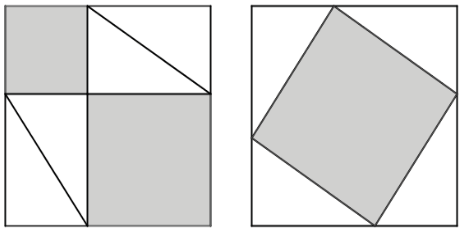
Responde: ¿Es correcta la solución dada por el estudiante? Justifica la respuesta.

TRABAJO INDIVIDUAL

Apellido y nombre: ---

Tarea 4. Relación entre áreas de figuras planas

Dadas las siguientes figuras:



A B

- ¿Qué relación piensas que existe entre las áreas de las figuras sombreadas de la parte A y B? Usa las hipótesis que creas necesario.

- ¿Cómo se puede usar esta relación para probar el teorema de Pitágoras?

ANÁLISIS ONTOSEMIÓTICO DE TAREAS MATEMÁTICAS

TAREA OPCIONAL INDIVIDUAL

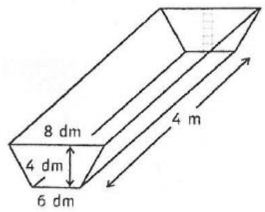
Tarea 5 optativa. Modelización matemática

Situación-problema:

En el campo, algunos bebederos para animales tienen una forma como la que se esquematiza en el dibujo. Se trata de un prisma recto de 4 m de largo, y dos de sus caras son trapecios isósceles congruentes de base menor 6dm, base mayor 8dm y altura 4dm.

Se necesita graduar una varilla colocada en forma vertical sobre uno de los trapecios para precisar el nivel de agua correspondiente a 100, 200, 300, … litros.

Encuentra la manera de preparar dicha varilla indicando las distancias a las cuales se deben trazar las marcas correspondientes.



Consignas de trabajo

a) Resuelve la situación-problema.

b) Describe el procedimiento seguido indicando la secuencia de prácticas elementales que has realizado para resolver la tarea; añade las explicaciones necesarias para justificar las respuestas.

c) Completa la tabla incluida a continuación en la que se identifican los conocimientos que se ponen en juego en el enunciado y en cada una de las prácticas elementales, (añade las filas necesarias):

Uso e intencionalidad de las prácticas	Enunciado y secuencia de prácticas elementales para resolver la tarea	Objetos referidos en las prácticas *(conceptos, proposiciones, procedimientos, argumentos)*
…	…	…

d) Identifica procesos matemáticos involucrados en la resolución de la tarea (particularización-generalización, materialización-idealización, ...).

e) Destaca entre las prácticas, objetos y procesos identificados cuáles consideras potencialmente conflictivos para los alumnos.

f) Enuncia variantes de la tarea e identifica los cambios que se producen en los conocimientos puestos en juego en cada variación.

ANEXO 2. UNIDAD TEMÁTICA COMPLETA DEL ESTUDIO 2

DESARROLLO DE LA COMPETENCIA DE ANÁLISIS DE LA IDONEIDAD DIDÁCTICA

OBJETIVOS GENERALES:

- Reflexionar sobre los factores que influyen en los procesos educativos.
- Conocer y aplicar herramientas teóricas específicas e innovadoras en el ámbito de la educación matemática para describir, explicar y valorar procesos de enseñanza y aprendizaje de las matemática.

DURACIÓN:

- 3 sesiones de clases presenciales

REFLEXIÓN SOBRE UNA CLASE DE MATEMÁTICAS

TRABAJO INDIVIDUAL

Apellido y nombre: ---

METODOLOGÍA:

- Resolución de la Tarea 1, incluida a continuación, de manera individual.
- Presentación y discusión de resultados

DURACIÓN:

- 1 sesión de clases

Tarea 1. Reflexión sobre una clase de matemáticas

A continuación, se presenta un texto que describe una clase de matemáticas imaginaria. Al final se incluye el texto descompuesto en párrafos numerados para que puedas referirte a ellos al responder a las siguientes cuestiones.

1) Lee el texto con atención. Subraya los puntos que consideres especialmente atractivos en la descripción.

2) Indica las características de las matemáticas que se consideran valiosas en el texto.

2.1. Explica por qué se consideran valiosas y si compartes esa opinión.

2.2. ¿Qué otros rasgos de las matemáticas consideras valiosos desde el punto de vista educativo?

3). Indica las características del aprendizaje matemático que se consideran valiosas en el texto.

3.1. Explica por qué se consideran valiosas y si compartes esa opinión.

3.2. ¿Qué otros rasgos del aprendizaje consideras valiosos desde el punto de vista educativo?

4). Indica qué características se mencionan en el texto relacionadas con los aspectos afectivos en el estudio de las matemáticas.

4.1. Explica por qué se consideran valiosos dichos aspectos y si compartes esa opinión.

4.2. ¿Qué otros rasgos de la enseñanza y aprendizaje de las matemáticas consideras valiosos desde el punto de vista de la afectividad?

5) Indica los modos de interacción entre profesor y estudiantes que se consideran valiosos en el texto.

5.1. Explica por qué se consideran valiosos dichos modos de interacción y si compartes esa opinión.

5.2. ¿Qué otros modos de interacción en el aula consideras valiosos para optimizar el aprendizaje matemático?

6) Indica qué características de la clase imaginaria de matemáticas se consideran valiosas relativas al uso de recursos tecnológicos.

6.1. Explica por qué se consideran valiosas dichas características y si compartes esa opinión.

6.2. ¿Qué otros aspectos del uso de recursos consideras valiosos para favorecer el aprendizaje matemático?

7) Identifica los factores externos a la clase que se mencionan en el texto como condicionantes de la enseñanza y el aprendizaje de las matemáticas.

7.1. Explica por qué se consideran factores condicionantes y si compartes esa opinión.

7.2. ¿Qué otros factores consideras que condicionan el logro de una clase ideal de matemáticas?

Lectura individual

Una Visión de las Matemáticas Escolares (NCTM 2000, p. 3):

> "Imagine una clase, una escuela, o un distrito escolar donde todos los estudiantes tienen acceso a una instrucción matemática atractiva y de alta calidad. Se proponen unas expectativas ambiciosas para todos, con adaptación para aquellos que lo necesitan. Los profesores están bien formados, tienen recursos adecuados que apoyan su trabajo y están estimulados en su desarrollo profesional. El currículo es matemáticamente rico y ofrece oportunidades a los estudiantes de aprender conceptos y procedimientos matemáticos con comprensión. La tecnología es un componente esencial del entorno. Los estudiantes, de manera confiada, se comprometen con tareas matemáticas complejas elegidas cuidadosamente por los profesores. Se apoyan en conocimientos de una amplia variedad de contenidos matemáticos, a veces enfocando el mismo problema desde diferentes perspectivas matemáticas o representando las matemáticas de maneras diferentes hasta que encuentran métodos que les permiten progresar. Los profesores ayudan a los estudiantes a hacer, refinar y explorar conjeturas sobre la base de la evidencia y usan una variedad de razonamientos y técnicas de prueba para confirmar o rechazar las conjeturas. Los estudiantes son resolutores flexibles de problemas y tienen recursos variados. Solos o en grupos y con acceso a la tecnología, los estudiantes trabajan de manera productiva y reflexiva, con la guía experimentada de sus profesores. Los estudiantes son capaces de comunicar sus ideas y resultados oralmente o por escrito de manera efectiva. Valoran las matemáticas y se comprometen activamente en su aprendizaje."

UNIDADES DE ANÁLISIS:

1. Imagine una clase, una escuela, o un distrito escolar donde todos los estudiantes tienen acceso a una instrucción matemática atractiva y de alta calidad.
2. Se proponen unas expectativas ambiciosas para todos, con adaptación para aquellos que lo necesitan.
3. Los profesores están bien formados, tienen recursos adecuados que apoyan su trabajo y están estimulados en su desarrollo profesional.
4. El currículo es matemáticamente rico y ofrece oportunidades a los estudiantes de aprender conceptos y procedimientos matemáticos con comprensión.
5. La tecnología es un componente esencial del entorno.
6. Los estudiantes, de manera confiada, se comprometen con tareas matemáticas complejas elegidas cuidadosamente por los profesores.
7. Se apoyan en conocimientos de una amplia variedad de contenidos matemáticos, a veces enfocando el mismo problema desde diferentes perspectivas matemáticas o representando las matemáticas de maneras diferentes hasta que encuentran métodos que les permiten progresar.
8. Los profesores ayudan a los estudiantes a hacer, refinar y explorar conjeturas sobre la base de la evidencia y usan una variedad de razonamientos y técnicas de prueba para confirmar o rechazar las conjeturas.
9. Los estudiantes son resolutores flexibles de problemas y tienen recursos variados.
10. Solos o en grupos y con acceso a la tecnología, los estudiantes trabajan de manera productiva y reflexiva, con la guía experimentada de sus profesores.
11. Los estudiantes son capaces de comunicar sus ideas y resultados oralmente o por escrito de manera efectiva.
12. Valoran las matemáticas y se comprometen activamente en su aprendizaje.

TAREA DE REFLEXIÓN DIDÁCTICA

TRABAJO GRUPAL

Apellido y nombre: ---

METODOLOGÍA:

- Lectura y elaboración de una reflexión sobre el siguiente artículo:

  Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos de Investigación y Formación en Educación Matemática, 11,* 111-132.

- Presentación y discusión del artículo.
- Resolución de la Tarea 2, incluida a continuación, trabajando en equipos de 3 o 4 estudiantes.
- Presentación y discusión de resultados

DURACIÓN:

- 2 sesiones de clases

Tarea 2. Reflexión didáctica

En el siguiente link encontramos un video de una clase de matemáticas: http://www.youtube.com/watch?v=60s_0Ya2-d8.

Después de visionado el vídeo, y trabajando en equipos, elaborar un informe respondiendo a las siguientes cuestiones:

1) Descripción: *¿Qué sucede?*

a. ¿Qué contenido matemático se estudia?
b. ¿Qué significados caracterizan el contenido estudiado?
c. ¿Cuál es el contexto y nivel educativo en que tiene lugar la clase?
d. ¿Qué hace el profesor?
e. ¿Qué hace el alumno?
f. ¿Qué recursos se utilizan?
g. ¿Qué conocimientos previos deben tener los alumnos para poder abordar la tarea?
h. ¿Qué dificultades/conflictos de aprendizaje se manifiestan?
i. ¿Qué normas (regulaciones, hábitos, costumbres) hacen posible y condicionan el desarrollo de la clase?

2) Explicación: *¿Por qué sucede?*

e. ¿Por qué se estudia ese contenido?
f. ¿Por qué se usa un problema realista para estudiar el contenido?
g. ¿Por qué actúa el docente de la manera en que lo hace?
h. ¿Por qué actúa los alumnos de la manera en que lo hacen?

3) Valoración: *¿qué se podría mejorar?*

Emitir un juicio razonado sobre la enseñanza observada en las siguientes facetas, indicando algunos cambios que se podrían introducir para mejorarla:

j. Epistémica (contenido matemático estudiado)
k. Ecológica (relaciones con otros temas, currículo)
l. Cognitiva (conocimientos previos, aprendizaje, …)
m. Afectiva (interés, motivación, …)
n. Interaccional (modos de interacción entre profesor y estudiantes)
o. Mediacional (recursos usados)

4) *Limitaciones de la información disponible:*

Para discutir en clase: ¿qué información adicional sería necesario tener para que el análisis realizado fuera más preciso y fundamentado?

EXTENDED SUMMARY

INTRODUCTION

A major problem in mathematics education, which has recently been gaining more attention, is to clarify the kind of didactic-mathematical knowledge and professional competences that mathematics teachers should have in order to carry out their teaching appropriately (Chapman, 2016; English, 2008; Sowder, 2007). Having mathematical knowledge is not guaranty of professional performance, "It is not only important what mathematics teachers know but also how they know it and what they are able to mobilize for teaching" (Chapman, 2014, p. 295). Certainly, characterizing this knowledge necessary for mathematics teaching is a relevant research topic, among other reasons, because "there is limited understanding of what it is, how one might recognize it, and how it might develop in the minds of teachers" (Silverman & Thompson, 2008, p. 499). These researches have been gestated in the light of several theoretical approaches; "however, there is neither a consensus nor a common perspective regarding the nature of this knowledge" (Chapman, 2014, p. 296).

Several authors develop tools and strategies to promote teacher's analysis and reflection about the processes of mathematics teaching and learning; in addition, they provide tools that allow the teacher to be competent to describe, explain, and assess systematically, their own practice (Llinares & Krainer, 2006; Pino-Fan, Assis, & Castro, 2015). In these works, it is recognized that the teacher should have mathematical and didactic knowledge, but the teacher should be also competent in the use of such knowledge to address the profession performance.

Within the Onto-Semiotic Approach (OSA) of mathematical knowledge and instruction (Godino, Batanero, & Font, 2007), a theoretical model of the mathematics teacher's knowledge, known as the Didactic-Mathematical Knowledge model (DMK model), has been developed (Godino, 2009; Pino-Fan, Assis, & Castro, 2015). As stated by these authors, one of the aspects that the aforementioned model considers is the interconnection of the notion of the teacher's knowledge with that of her/she competence. Further, from the OSA, significant research has been carried out on the competences of the mathematics teacher (Giménez, Font, & Vanegas, 2013; Nogueira, 2015; Rubio, 2012; Seckel, 2016; Pochulu, Font & Rodríguez, 2016), which has also brought to light the need for such a model on teachers' knowledge in order to evaluate and develop their competences. Both of these research topics have converged to create the model known as *Didactic-Mathematical Knowledge and Competences* of the mathematics teacher (DMKC model) (Godino, Giacomone, Batanero & Font, 2017). This theoretical model is addressed in Chapter 2.

From DMKC model is assumed that mathematics teachers should develop *the specific competence of didactical analysis and intervention*; whose fundamental nucleus (Font, 2011; Pino-Fan, Assis & Castro, 2015) consists of designing, applying and assessing mathematical study processes through of didactic analysis techniques and criteria of quality, in order to establish cycles of planning, implementation, evaluation and to put forward proposals for improvement. This didactical analysis competence can be split into sub-competencies, which can be identified linked to the use of specific theoretical tools, allowing to approach teaching problems:

1) C*ompetence for global meanings analysis*, linked to the knowledge and competent use of the *system of practises* tool (Godino y Batanero, 1994).

2) *Onto-semiotic analysis competence,* linked to the knowledge and competent use of the *onto-semiotic configuration of practices, objects, and processes tool* (Godino, Font, Wilhelmi y Lurduy, 2011).

3) *Competence for the management of Interactions and conflicts analysis*, linked to the knowledge and competent use of the *didactic trajectory* tool (Godino, Contreras et al., 2006).

4) *Norms and meta-norms analysis competence* linked to the knowledge and competent use of the *normative dimension* tool (D'Amore et al., 2007; Godino et al., 2009).

5) *Didactical suitability analysis competence,* linked to the knowledge and competent use of the *didactic suitability* tool (Godino, 2013a).

Continuing with this research line, this thesis deals with the development of an educational cycle, that is, its design, implementation and retrospective analysis, aimed at prospective secondary school mathematics teachers. The aim is to initiate them in the development of their competence for the analysis and didactic intervention, and didactic knowledge linked to mentioned competence. We focus our attention on two aspects: firstly, on developing the onto-semiotic analysis competence, understanding it as the competence to identify the variety of objects and meanings involved in solving mathematical tasks; secondly, on developing the didactical suitability analysis competence or professional reflection.

> *OG-1. Design, implement, and evaluate an educational experience with prospective secondary school mathematics teachers aimed at promoting the development of their competence for the* onto-semiotic analysis.

> OG-2. *Design, implement, and evaluate an educational experience with prospective secondary school mathematics teachers aimed at promoting the development of their competence for the* analysis of didactical suitability.

The research work is framed in a qualitative approach (McMillan & Schumacher, 2001) that collects and analyses data throughout a design cycle composed of two descriptive and exploratory studies, which respond to the objectives outlined above. The educational cycle is developing as part of a master course of mathematics education for secondary teachers (academic year 2015-2016) in a real classroom setting. According to the design based research methodology (Kelly, Lesh, & Baek, 2008) it is carry out from the following four phases supported by the OSA tools: preliminary study, design of tasks, implementation, and retrospective analysis.

The sample consists of 52 students for teacher—prospective mathematics teachers, separated into two groups (group A: 27, group B: 25), with no teaching experience and consolidated mathematical knowledge. In the first study, all the students participated; in the second study, only group A participated for administrative reasons.

For both studies, the teaching techniques used combine: reading and discussion of documents; presentations by the teacher; participation in problem solving workshops; didactic analysis; extracurricular support for students. The instruments for collecting data are: notes of the researchers on the different instances of work in class audio recording of all sessions of the course written responses to the group activities final written material, delivered by the students individually, with two weeks of deadline.

The research group consists of the teacher of the course—teacher educator and director of the Doctoral Thesis, and the doctoral candidate who plays the role of participant observer.

The thesis is organized in 5 chapters. In Chapter 1, previous researches in the field of teacher education are described in order to adequately support research. The contributions allow us to approach the research problem. In Chapter 2, we present the research problem—research questions, objectives, and hypotheses, the theoretical framework, and the methodology. In Chapter 3, we present the study 1, as part of a design research, on developing the *onto-semiotic analysis competence*, that is, knowledge and competence to identify and describe objects and processes involved in school mathematical tasks. This competence will allow the prospective teachers "to anticipate potential and effective learning conflicts, evaluate the mathematical competences of the students, and identify objects (concepts, propositions, procedures, arguments) that should be remembered and institutionalized at the appropriate moments of the study processes" (Godino, 2017, p. 94). The conclusions of the first study are also exposed. In Chapter 4, we present the study 2, as part of a design research, on developing the *didactical suitability analysis competence* understood as the competence for global reflection on a mathematical study process, its assessment and progressive improvement. The conclusions of the second study are also exposed. Finally, in Chapter 5, the research objectives are retaken, and the final conclusions are presented highlighting the limitations of the work and future continuation lines.

FIRST RESEARCH STUDY

This study describes an experience with prospective mathematics teachers on developing the *onto-semiotic analysis competence*, that is, knowledge and ability to identify and describe practices, objects, and processes involved in school mathematical tasks.

From onto-semiotic perspective, various type of mathematical objects (problems, languages, concepts/definitions, propositions, procedures, and arguments), intervene and emerge from mathematical practices. These types of objects are interconnected to each other through referential and operational semiotic functions building configuration of knowledge. These configurations can be contemplated from five duals points of view

(Godino et al., 2007): expression content; personal (cognitive)-institutional (epistemic); intensive (general)-extensive (particular); ostensive-non-ostensive; unitary-systemic. On the other hand, the dualities lead to the following processes: institutionalization-personalization; generalization-particularization; analysis (splitting)-synthesis (reification); materialization-idealization (abstraction); expression (representation)-signification. In the Tables 3.1. to 3.8. showed in Chapter 3 we discuss the role that some of these processes play in the emergence of the primary objects involved in a priori analysis of the tasks; for this extensive summary we will use Table 6.1 to exemplify the analysis.

The formative action includes the development of the following four phases. The first phase corresponds to the recognition of students' initial personal meanings on the nature of mathematical objects and their ability to recognize these objects in mathematical practices. Prospective teacher worked individually with the Task 1 (see Appendix 3) answering a series of questions from an isometric perspective drawing; following, students' answers were presented and discussed in class.

In the second phase, the required reading and discussion of a specific document was proposed (Godino et al., 2015a). The proposed article is an introduction to ontosemiotic analysis using as a context the reflection on the role of diagrams, visualization, and manipulative materials in the mathematics teaching and learning processes. After the article discussion, it begins the third phase: put into practice. Working in teams of 3 or 4 students, two tasks were implemented followed by presentation and discussion of the answers with the whole class. In Task 2 (see Appendix 3), students had to justify a procedure given by a student to build a square with GeoGebra. In Task 3 (see Appendix 3), a problem on fractions together with its solution based on a sequence diagram areas, were proposed; the prospective teachers had to justify if such solution was correct. Finally, phase 4 is related to the final evaluation process. Students worked individually with a Task 4 (see Appendix 3), based on a demonstration of the Pythagorean theorem. The resolution was not addressed in class and was regarded as a final assessment instrument. In addition, prospective teachers are proposed 1 optional task, in order to consolidate the achievement of the intended competition and provide researchers with relevant data sources. Finally, the development of all the tasks is presented at the end of the subject, considering a period of two weeks.

Following, the teaching methodology used for the four tasks is described. It should be taken into account that each one of them was analysed by the research group, that is, an a priori analysis of each problem was carried out; however, in order to exemplify this type of analysis, in this summary only the *a priori* analysis of the task 1 is shown. The results of the entire implementation are then discussed.

Teaching methodology

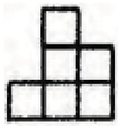
Task 1. Initial exploration

The designed questions to investigate the students' personal meanings on the nature of mathematical objects and their initial level of onto-semiotic analysis competence is included in Appendix 3. Down below an a priori analysis (epistemic analysis) of Task 1, which was used to support the sharing of individual students' responses, is presented. The Table 6.1. summarizes the configuration of objects and meanings involved in the resolution of the task; it can be seen that both the statement and the task resolution are broken down into units of analysis that we have listed from 1) to 7).

Table 6.1.

Onto-semiotic analysis of the initial task

Use and purpose of the practices	*Statement and sequence of practices to solve the task*	*Objects referenced in the practices (concepts, propositions, procedures, arguments)*
Presenting the problem; interpretation of an isometric view of a 3-D object.	1) The attached figure shows the drawing of a building from the front-right angle:  Draw the view of the building from behind.	Concepts: isometric perspective of a 3-D object, viewpoint (or focus), opposing viewpoints, orthogonal projection, projection plane, straight lines of projection, visual ray, cube, composition of cubes, square, 3-D reference system, front, above, right, visible object, hidden object.
To induce the development of a	2) Justify your answer	Concept of justification of a

justification of the required response.		geometric proposition
Answer to the task.	3) The view from behind should be the following figure: 	Concept: elevation view (rear view). Procedure: counting of cubes by rows and columns. Proposition 1: the view from behind is the attached figure
To establish a fundamental hypothesis to give a rational response to the task, and a property of orthogonal projections.	4) If the building pieces are drawn of cubic form, the orthogonal projections of their faces are squares.	Concept: cube, orthogonal projection, square. Proposition 2: orthogonal projections of a cube are squares.
To evoke the properties of orthogonal projections necessary to justify deductively the answer to the task.	5) Orthogonal projections preserve the shape, size and relative position of the projected objects.	Argument: justification of the proposition 2. Concepts: form, size, and relative position
To describe the relative positions of the components of the "building" to justify the shape of the flat projection from behind.	6) If I get behind the building, I would see 1 cube on my left, 3 cubes stacked up on the center, and 2 cubes stacked up on my right, because in the isometric perspective given, there are 1 cube on the right-back, 3 cubes on middle-back, and 2 cubes on the left - front.	Concepts: behind, left, center, and right. Proposition and its argumentation based in the task data
To evoke a previously established property to justify the final answer.	7) Orthogonal projections of a cube are square, then the view of the object must be the shown in practice 3)	Argument: justification of the proposition 1.

Despite the fact that Table 6.1. shows the analysis of objects and meanings at stake, it is necessary to complement this analysis with the recognition of the processes involved in solving the task. Elaborating a full epistemological analysis of semiotic functions plot involved in the practices, both referential (an object refers to another object) and operational (pragmatic use of objects) type is not the aim of this article. But it should be noted that the processes of generalization-particularization, and materialization-idealization are always present. For example, the task shows the material representation in the paper sheet of a real object (the building) but ideal (imagined). This representation in isometric perspective refers to the view that a hypothetical ideal observer would build it. This kind of perspective has the advantage of allowing representation to scale, and the disadvantage of not reflecting the apparent decrease in size perceived by the human eye. The drawing of the building is then an embodiment of an ideal object: the view of a building that would have a hypothetical observer. The drawings (isometric and orthogonal projections) can be interpreted as materialization of ideal objects (cubes compositions) that facilitate the realization of the "mathematical actions" done on them (recognize the views).

Complementary methodology

The four tasks (situations-problems) were selected with the purpose of they put into play visualizations and reasoning with diagrams in order to provoke reflection on the dialectic between ostensive and non-ostensive objects involved in mathematical practices.

The onto-semiotic methodology for Task 2, 3, 4 and optative are included below:

a) Solve the mathematical task (Task 2, Task3, and Task 4)

b) Describe the procedure followed, indicating the actions to be performed and the necessary explanations to justify the answers.

c) Identify mathematical knowledge put at stake in the statement and each of the elementary practices, completing the table below (add the necessary rows)

Use and purpose of the practices	Statement and sequence of practices to solve the task	Objects referenced in the practices (Concepts, propositions, procedures, and arguments)
…	…	…
…	…	…

d) In addition to the signifying processes indicated in the above table, identify other mathematical processes involved in solving the task.

An a priori analysis is performed for each task as shown in the previous Table 6.1.

Observations during the educational process and the analysis of students' responses have allowed drawing some conclusions about the difficulties of understanding the instructions, achievements, and the possibilities offered by the didactical design.

Discussion

Discussion of the Task 1: Initial exploration of personal meanings

During the first phase of individual work, it was observed that students were not clear what was the nature of the primary mathematical objects and their meanings. Due to the visualization process involving the statement (front-right building perspective) and its solution (drawing seen from behind), recognition of the students on mathematical objects has focused on perceptive visual objects. For instance, they recognize as intervening and emerging concepts in the resolution: cube, square, volume, height, rotation, reference system, (...), but none of them refers, for example, to the orthogonal projections. The notion of proposition is also controversial for them; for example, a student arguments:

> Propositions are applied to prove theorems. This task does not involve demonstrations, is simply to draw what you see (...). It is a problem of technical drawing, not a mathematical problem.

The conflicts identified were deal with in the classroom sharing, aiming to discuss and share the understanding of the entities put at stake and their role in mathematical practice. The aim was that students share the pragmatic and anthropological vision of mathematical knowledge that OSA postulates, according to which a concept is conceived as a functional entity (i.e., it has a role in mathematical practices), whose meaning has been socially reified as a rule or definition, and a proposition is a statement that is either true or false.

On the question six of this task (Appendix 3) students have to develop at least two different definitions for the cube as geometric concept. This part of the task generated some confusion among some students. Then in the next part, students must identify other uses or meanings of the word *cube* that do not relate to the geometric concept. This was used as a reagent to explain the diversity of meanings that might have a concept or proposition depending on the context in which they participate, and to discuss some aspects of language, such as polysemy and homonymy.

Another important aspect is the complex dialectic between the ostensive objects (material representations) and non-ostensive objects (immaterial, mental or ideal objects), which is manifested in different dialogues recorded. Thus, for instance, to the question ten: What properties cannot be represented empirically? A prospective teacher comments:

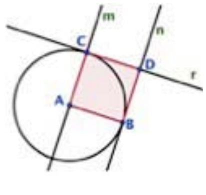
> All properties of a cube can be represented empirically, except the faces that are behind. For example (pointing with a finger to the picture): this is a cube, and I am representing it empirically; these are its edges, these are its faces, ...

The ostensive/non-ostensive duality has an essential role within the OSA framework, since the activity of mathematical production and communication cannot be performed without the synergistic relationship between the two types of objects, which are interwoven in mathematics practices. This reflection is necessary, because it allows prospective teachers become aware that such objects are understood as the rules of use of the visual and analytical languages that represent them. So, dialogue and interaction took a key role in the didactical action

Discussion of the Task 2: Building a square with GeoGebra

While working in teams to deal with the Task 2, it has been observed that students were able to identify all the concepts and procedures involved in solving the task; however, cases where the notion of proposition remains unclear is evident. For instance, a student identified as a proposition the definition of square; it is clear that the definition of square is neither true nor false.

In the collective discussion, in order that students make a real mathematical activity, it is necessary to ask for the justification of the procedure based on the use of the software, since this way they have to think about the mathematical knowledge involved in the resolution. Geogebra does not require the explicit use of definitions, propositions or properties of geometric objects to carry out the mathematics practices, being masked their features of figural concepts (Fischbein, 1993). It is necessary that the teacher asks the student explicit justification of procedures to ensure the validity of the statements. For example, in the figure below:



the segment AC is congruent to AB, not because "they are seen on the screen of the same length" but because they are the radii of the same circle with center A, then by definition they are congruent. Necessarily the quadrilateral ABCD is a square because the conditions of the definition are met: all four angles are right and the four sides are congruent. "A square is not an constructed image. It is a shape controlled by its definition (though it may be inspired by a real object" (Fischbein, 1993, p. 141). Finally, using software as part of the task is a positive aspect because it provides opportunities for students to engage in mathematical processes and reflect about particularization (materialization of concepts to particular figures) and generalization processes (the particular figures are representative of a family of similar figures).

Discussion of the Task 3: Fractions and area diagrams

Developing Task 3 aims that prospective teachers use the knowledge gained through the previous activities and discussions to analyse and assess a possible solution given by a subject to a problem on fractions. The answers that students have given to this task indicate some progress in the recognition and identification of the different objects involved in the task, that is, their onto-semiotic analysis competence.

In general, the prospective teachers solve the Martini problem using other diagrammatic procedures. They are not able to develop a justification based on the area diagrams, since the addition and multiplication of fractions represented with these kind of representations, requires an unusual mathematical work. In this way, students need to resort to other types of languages. The problem is that from different procedures for the same task involve that the mathematical objects mobilized in each of them are different. This generates a major consequence if the aim is to analyze the mathematical activity involved in a given response. This fact is a fundamental problem, which was discussed in the whole class discussion.

Discussion of the Task 4: Pythagoras Theorem

The analysis of the answers allows observing that students have been able to identify the concepts and procedures involved in mathematical practices, making detailed and deep analyses. However, difficulties persist mainly for the argument and proposition notions, resulting complex their identification and their onto-semiotic analysis.

The type of analysis that has been implemented, that is, recognition and management of knowledge put at stake in tasks, allows the prospective teacher analyze the intervening and emerging objects in the resolution, and become aware of the diversity of meanings attributed to them in the specific context.

Retrospective analysis and conclusions

The notion of didactical suitability and the system of suitability criteria (Godino 2013a) are used to reflect and evaluate the teaching experience described in the previous paragraphs. These criteria are classified into six facets that characterize teaching and learning processes: epistemic (mathematical institutional meanings), ecological (socio-professional and curricular context), cognitive (personal meanings), affective (emotional factors), interactional (personal interactions) and media (didactical resources). This retrospective analysis of an implemented teaching cycle allows the teacher/researcher reflecting on each of the six facets, and determine potential improvements for future cycles.

In order to collect additional useful information for this analysis, students were asked to respond an anonymous opinion survey on the following five aspects of the four proposed tasks: 1) clarity of the task and its instructions; 2) suitability of the teacher's explanations and interactions; 3) degree of motivation and interest generated; 4) learning level achieved; 5) degree of overall relevance of the activities for your education as a teacher of mathematics.

The retrospective analysis reveals a high epistemic-ecological suitability, showing itself as a design consistent with the objectives of the institutional context in which is carried out, and with preparation of prospective secondary school teachers. The medial-interactional suitability can be considered average, bringing to light aspects that should be taken into account in the future, for example, it should increase the time used for the development of this competence, incorporating more fundamental moments of group discussion. In the class sharing, the teacher facilitates the inclusion of students in the class dynamics; students are encouraged to explain, justify, disagree, question, and reflect on different alternatives, so most of them were involved in the discussion of the answers. Although awareness of the existence of conflicts in some students is taken, there is no evidence that these conflicts have been resolved. The time factor is perhaps the most important resource that must be considered for proper management of the teaching and learning process; but certainly, given the complexity of mastering the onto-semiotic analysis competence, the time spent was insufficient. The use of manipulative and technological resources is an aspect that should be improved in this intervention; the following comment retrieved from a student's opinion survey, regarding Task 1, allows progress in this direction:

> "Incorporate manipulative material that may help solve the task and analyze other possible methods".

The cognitive-affective suitability can be considered average. In the analysis of the evaluation results, common confusions were recorded in the students' responses, which also took place during the study process, indicating that the *a posteriori* cognitive suitability has not been adequate.

In this research, a didactical design cycle for teacher education focused on developing the so-called onto-semiotic analysis competence has been designed, implemented, and evaluated a education cycle, leading to the achievement of objective OG-1., reflecting a contribution that can be taken into account in the initial teacher mathematics education programs.

SECOND RESEARCH STUDY

In this second study we describe, analyse, and evaluate the implementation of an educational design to develop the prospective mathematics teacher's didactical analysis and reflection competence, addressing the general objective OG-2.

The Didactic-Mathematical Knowledge and Competence model (DMKC) proposed by Godino, Giacomone et al. (2017) focuses, among others, on the competence of didactical suitability analysis, as the competence for global reflection on the teaching practice, its assessment and its progressive improvement. Also, these authors suggest the importance of designing and implementing training resources that promote the realization of this type of macro-analysis by teachers.

The notion of Didactical Suitability (DS) is part of the Onto-Semiotic Approach (OSA) to mathematical knowledge, a theoretical framework introduced by Godino, Batanero, and Font (2007) within mathematics education field. This notion, its components and indicators, allow the systematic analysis and assessment of mathematics teaching and learning processes. It is understood as the degree to which an educational process (or a part of it) combines certain characteristics in order to be classified optimal or appropriate for the adaptation between the personal meanings achieved by students (learning), and the intended or implemented institutional meanings (teaching), taking into consideration the circumstances and the available resources (environment). This assumes the coherent and systemic articulation among the six following facets or dimensions:

- Epistemic suitability: it refers to the degree of representativeness and interconnection of institutional meanings implemented (or intended) regarding to a reference meaning. The tasks/situations-problems are an important component in this facet, and they should include various types of mathematical objects and processes.
- Ecological suitability: the extent to which the process of study is adapted to the educational/curricular project, scholar norms, and social environment.
- Cognitive suitability: the extent to which intended and implemented meanings are within the students' zone of proximal development, as well as the correlation between students' achieved meaning and the intended and implemented meanings.
- Affective suitability: it refers to the degree of the students' involvement (interest, emotions, motivation, attitudes, and beliefs) in the study process.
- Interactional suitability: it is the degree to which the didactic configurations and classroom discourse served to identify and solve semiotic conflicts that appeared throughout the instructional process.
- Media suitability: is the extent to which the teaching process fit the school and society educational process, and took into account other factors influencing the setting in which it was developed.

For each of these six facets, Godino (2013) identifies a system of associated components and general empirical indicators that constitute a guide for the analysis and systematic reflection; thus, this theoretical model provides criteria for the progressive improvement of the teaching and learning processes.

Starting from considering the teacher as a reflective professional (Schön, 1983; Elliot, 1993), with this design we intend that prospective teachers know the criteria and use them competently to reflect systematically and professionally. The instructional device uses the possibilities offered by episodes of video-recorded lessons. The participants in these types of experiences have:

> [...] the opportunity to develop a different kind of knowledge for teaching — knowledge not of "what to do next", but rather, knowledge of how to interpret and reflect on classroom practices. (Sherin, 2004, p. 14)

However, in this work, video recording of classes remains in the background (Stockero, 2008). These should be a mere resource that may facilitate access to the teacher educator and future teachers *fragments of educational reality* in all its complexity, and develop in the training students specific teaching skills through the systematic didactic analysis of the various facets, components and conditioning factors.

The implementation is organized in four phases, which include different didactic resources as well as moments of autonomous and group work, and final evaluation. First phase, called *Initial exploration phase,* includes reading and discussion of a highly ambitious document about the characteristics of an ideal mathematics class, taken from the curriculum orientations of the NCTM —National Council of Teachers of Mathematics (2000, p. 3): *A Vision for School Mathematics.* The goal is that students develop a first reflection on possible ideal characteristics of a math class. The students worked individually upon a reflection guide, which played a key role in motivating the discussion about previous ideas, beliefs and conceptions that prospective teachers may have about mathematics and the complex processes of their teaching and learning. The discussion made the epistemic, cognitive, affective, interactional, media, and ecological components to emerge and how they articulate each other and how they affect the development of a study process. In addition, the prospective teachers' meanings of possible suitability criteria of teaching and learning processes were highlighted. This phase ends with a reflection on the need to know about specific tools that allow the teacher to assess the teaching practice in a systematic way and to be competent using them. It's not just about describing and explaining what is happening in that ideal class, but also to reflect on what aspects could be improved.

In the second phase, *Introduction of a tool for reflection,* the required reading and discussion of a specific document was proposed (Godino, 2013a). In the second-class session the article, previously read by students, is jointly discussed. The notion of didactic suitability is presented in this article, in addition to a system of didactic suitability indicators for each of the different facets involved, and the concordance between this system and other proposals from various authors.

The third phase, *Put into practice,* begins after discussing the article. It is proposed that students watch a fragment of a high school mathematics class. This episode was selected from the internet, being free open-access material, in which it is possible to observe 10 minutes of a class taught in Mexico. After watching the class episode, a second activity, *Didactic reflection task*, was delivered, and the prospective teachers worked on it in teams of two or three peoples. Peer discussion took place during the development of the whole task. The sharing inside of the classroom of the sections 1 (description) and 2 (explanation) was done during the second session. From the information gathered in these two sections, students worked in teams during the last class session, in section 3 (evaluation) and then the final sharing was done. The a priori analysis carried out by the research team allowed to support the sharing done within the class, as well as to prevent possible learning conflicts. Finally, the forth phase, *Final evaluation process, was carried out; that is* the development of all the tasks/activities was presented by the students at the end of the subject, considering a period of two weeks.

Discussion

When students watch the video for the first time, they focus on specific elements, which are known to them as good practices. In this way, positively valued issues appear, such as the use of problems with context, collaborative work, classroom arrangement, the use of technological resources, sharing, respect, class dynamics and fieldwork. However, the first analysis they perform is based on superficial characteristics and without connections between the information collected by the items in the reflection guide.

The group discussion aims "[...] to help the prospective teachers to acquire professional teaching competencies" (Llinares, 2012, p.24). In this case, these competencies are focused on implementing the system of indicators and components previously studied. In this way, it is possible to find more elaborate and organized analysis in the answers (portfolios), where the students seek to establish key connections between those elements that seemed important to them. It should be highlighted that of 25 portfolios delivered in the final stage, 20 of them presented an analysis where possible improvements are proposed to increase the suitability of the observed study process. However, not every participant wanted to give a low, medium or high assessment of each facet considering that a lot of additional information is required in order to assess them.

To following, we synthesize significant responses collected from the experience, considering the participants' reflection on each of the six facets. In this sense, we have considered how their answers contributed in the first and second part of the task (description and explanation of the teaching situation) to make the assessment of the third part. The aim is to confront the analysis of the participants with the a priori analysis of the researchers. The latter allows opening a range of possible expert responses and thus to highlight the importance of knowing and being competent in the use of the didactic suitability tool and training as reflective professionals capable of assessing and improving their own practice.

Discussion about the epistemic facet

A key point to assess the epistemic facet (institutional mathematical knowledge) is to reflect on the type of situations-problems implemented in the video-recorded class episode. Although the participants notice that it is not possible to observe a representative and articulated sample of contextualization, exercising and application tasks in only nine minutes, the presence of a problem guide on inaccessible height calculation stands out, as well as the field work in the school courtyard. From a mathematical point of view, the prospective teachers notice that the studied content allows us to put into practice significant and relevant mathematical practices (knowledge, comprehensions and competences): geometric proportionality, linear function, similarity of triangles, calculation of inaccessible heights and distances. They assess positively the type of problems that allow to explore this content, as well as the type of languages that they mobilize.

The prospective teacher highlights important aspects such as: *Lack of precision in the teacher's language and concepts referred* to; likewise, they identify aspects that should be improved, such as the lack of didactic situations to argue and generate definitions or propositions:

> Although the problematic situations that appear seems to enhance the connections between the different concepts, propositions and procedures, the

> absence of moments of argumentation or justification, make the task itself become a mere exercise of application of a rule. It does not mean that it is incorrect, but it would be appropriate to add statements as 'justify your answer', in this way students can establish relationships between previously studied concepts and the teacher can evaluate their knowledge on the subject. (Prospective teacher)

Discussion about the ecological facet

The participants were able to identify components and indicators that characterize this facet by articulating their responses with the previous analyses obtained from part 1 and 2 of the task. They evaluated the adequacy of the content and its implementation according to the curricular guidelines that mark the new reform of Mexico (2011), which conditions the development of the class. Some aspects to be improved are the implementation of problems emphasizing intra/interdisciplinary connections, as well as situations of innovation and reflective practice.

Discussion about the cognitive facet

The prospective teachers focused their attention on the prior knowledge needed to address the calculation of inaccessible height. If we focus on the section: *what prior knowledge students should have to approach the task?*, only three of twenty-seven prospective teachers do not answer 'the simple rule of three'. These participants are aware that 'the rule of three' is a procedure to solve a task and not the objective itself; their reflections are coherent and highlight the importance of justifying procedures. For instance, the following prospective teacher reflects:

> During fieldwork they [students] collect information and apply Thales' theorem when the conditions of the theorem are not met (e.g., parallelism). At least they should consider certain assumptions to solve them [tasks], or the teacher could take advantage to do it, and thus generate instances of institutionalization.

Twenty-one participants indicate that the simple rule of three is *a prior* knowledge necessary to solve the tasks. Among these answers, nineteen of them believe that the similarity of triangles and Thales's theorem are not prior knowledge. In addition, they value positively the cognitive facet, since they consider that the students are capable of satisfactorily applying the rule of three, or at least it is an accessible objective.

While this was discussed in class, it seems that the rule of three is a procedure that is deeply rooted in its formation, as well as the deliberate use of proportionality relations:

> Students use the rule of three because the segments are proportional; that is easily demonstrated from the measurement of the sides. (Prospective teacher)

The two remaining prospective teachers emphasize that the use of the simple rule of three is important as *a prior knowledge necessary* to solve the task. However, their later reflections are in contradiction since they value negatively the learning linked to the application of rules and mechanical procedures.

Discussion about the affective facet

The assessments that are identified as related to this facet are very superficial, such as: fieldwork generates motivation; it allows assessing mathematics in everyday life.

Discussion about the interactional facet

The participants' reflections are related to the information obtained from the previous questions. In general, competences about reflection on the different modes of interaction are observed: between students and teachers, between students and about the autonomous study. The participants identify rules established in class such as the classroom arrangement, raising a hand to call the teacher, the role of the teacher as an observer in the class. They also make reasonable judgments about these. Regarding the teacher's role, the prospective teachers classify him as the protagonist of the class. They admit that the purpose of sharing is to present the answer to the problem. Four answers show superficial analysis, such as: *There is a fluid dialogue between the students and the teacher, and between the students among themselves*. We consider it a false impression created by the collaborative work dynamics.

Discussion about the mediational facet

The prospective teachers referred to the use of different manipulative materials, as scarce and unproductive, valuing this facet as not very suitable; while recognizing the importance of problem guidance and the use of calculators, participants emphasize that computer resources are very valuable in this type of content but they are not present:

> It would be advisable to use dynamic software to show, for example, how the shadow of a tree varies as the sun passes through different points, thus generating moments where students should estimate, test hypothesis and search

> for relationships between height and shadow, without needing to calculate it. (Prospective teacher)

The results indicate the effectiveness of the theoretical model put into practice, as well as awareness of the importance of incorporating reflective learning in university teaching.

Retrospective analysis and conclusions

Firstly, an *a priori* analysis of the didactic situation reveals a high *epistemic-ecological suitability*. The implementation stages are articulated to each other and appropriate to the formative level involved. In each educational situation proposed, the prospective teachers are faced with moments in which they have to investigate, interpret, relate meanings, discuss, and argue. In addition, this didactic design shows openness to innovation based on research and reflective practice. The mediational-interactional suitability can be considered average; it is attributed mainly to the limitations of the time allotted. While this type of design research occurs in real class environments, where it is not possible to have a greater workload, the short period of time between the implemented tasks reveals a great limitation of this study. The discussion of the final answers, delivered in the portfolio, did not take place within the class. In this sense, we consider that an exchange of final answers would have provided a greater opportunity for the participants to develop ways of reflecting on the different facets and appropriating the theoretical framework offered by this design. According to Amador (2016), the inclusion of additional experiences, or thinking about continuous cycles in teacher training, would be beneficial for prospective teachers to acquire greater competence in reflecting on the practice. Regarding the quality of the interactions in the classroom, we consider that it has been high, highlighting the dialogue and discussions in the classroom, the inclusion of the prospective teachers in the class dynamics, the appropriate presentation of the topic using various resources. In addition, moments of autonomous study and continuous evaluation were contemplated.

On the other hand, the use of video recordings as a resource has been widely recognized in teacher training (Alsawaie & Alghazo, 2010), and has undoubtedly proved to be a suitable training strategy, as it allows prospective teachers "[...] to view a lesson from a perspective of an observer" (Sherin, 2004, p. 22).

In summary, this study proposes an example of design research in which *Didactical suitability* tool is made operational in the different stages of implementation, addressing the achievement of the general objective OG-2. Although the different factors affecting the educational processes are complex, the participants of this study have positively evaluated this type of didactic situations for their formation, pointing them out as necessary. Moreover, highlighting their usefulness for the next stage of his/her professional work, such as lesson planning and implementation of professional practices in a school institution. It is worth noting that three students have continued their master's thesis using the didactic suitability tool to reflect on their own teaching practice.

APPENDIX 3

Task 1. Initial exploration of personal meanings

The attached figure shows the drawing of a building from the front-right angle.

1) Draw the view of the building from behind. Justify your answer.
2) What is it for you a mathematical concept? Identify the mathematical concepts involved in solving the task.
3) What is it for you a mathematical proposition? Identify mathematical propositions in solving the task.
4) What is it for you a mathematical procedure? Describe mathematical procedure in solving the task.
5) What is for you a mathematical proof? Provide a mathematical justification for the answer given in the task.
6) One of the concepts involved is *cube* used to indicate each of the pieces that make up the *building*:
    a. Give two different definitions (at least) for *cube* as a geometric concept.
    b. Indicate other uses or meanings that the word *cube* can have.
7) Indicates the role the propositions, that you have identified, plays in the answer justification.
8) Describe other possible procedures that could be applied to solve the task.
9) Describe a possible justification for the answer that could give a student using some kind of manipulative material, sequence of representations or other explanations.
10) The figure given is represented as a composition of cubic form pieces.
    a. Identify properties of a cube, as a geometric figure, which cannot be empirically represented.
    b. State the task by using natural or ordinary language.

Instructions for the onto-semiotic analysis and mathematical tasks

For the following three mathematical tasks, perform the following activities:

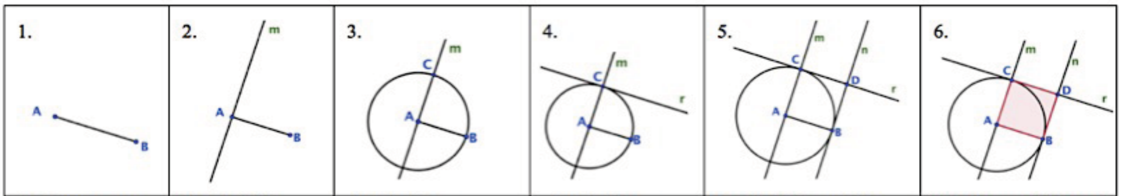
a) Solve the mathematical task
b) Describe the procedure followed, indicating the actions to be performed and the necessary explanations to justify the answers.
c) Identify knowledge at stake in the statement and each of the elementary practices, completing the table below (add the necessary rows)

Use and purpose of the practices	Statement and sequence of practices to solve the task	Objects referenced in the practices (Concepts, propositions, procedures, and arguments)
…	…	…
…	…	…

d) In addition to the processes of meaning indicated in the above table identify other mathematical processes involved in solving the task.

Task 2. Building a square with Geogebra

The procedure followed by a student to build a square using GeoGebra, is shown in the following sequence:



1.	2.	3.	4.	5.	6.
a) I represent a segment AB.	b) I build a straight line *m* perpendicular to segment AB through point A.	c) I build a circumference of centre A and radius AB. d) I call C to the point of intersection between the circle and the straight line *m*.	e) I build a straight line *r* parallel to the segment AB through point C.	f) I build a straight line *n* perpendicular to the segment AB through point B. g) I call D to the point of intersection between the straight lines *n* and *r*.	h) The quadrilateral ABCD is a square.

a) Justify why the quadrilateral ABCD is a square.

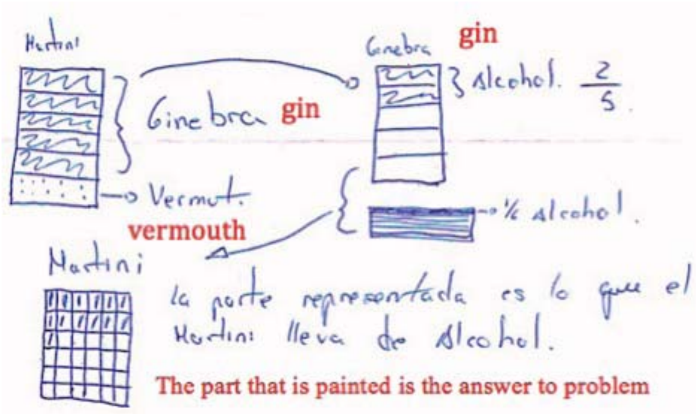
Task 3. Fractions and area diagrams

A student solves the following problem:

Martini cocktail problem:

> A Martini is a cocktail, which is made up of 5 parts gin and 1 part vermouth. Suppose that 2/5 of the gin is alcohol and 1/6 of the vermouth. What fraction of alcohol does a Martini have? Solve the problem by using an area diagrams.

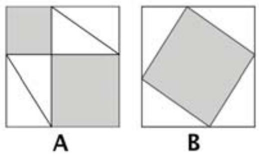
Solution:



a) Is it the solution given by the student correct? Justify

Task 4. Relationship between areas of plane figures

Observe the following figures:



a) What is the relationship between the areas of the figures shaded A and B?

b) How can you use this relationship to prove the Pythagorean theorem?

APPENDIX 4

Didactic reflection task (guide for prospective teachers)

At the following link we find a video of a math class: http://www.youtube.com/watch?v=60s_0Ya2-d8. After watching the video, work in teams and prepare a report answering the questions below:

1) Description: What is happening?

a. What mathematical content is studied?

b. Which meanings characterize the content studied?

c. What are the context and the educational level in which the class takes place?

d. What does the teacher do?

e. What does the student do?

f. What resources are used?

g. What prior knowledge should students have in order to tackle the task?

h. What learning difficulties/conflicts are manifested?

i. What norms (regulations, habits, customs) make possible and condition the development of the class?

2) Explanation: Why is it happening?

a. Why is that content studied?

b. Why is a realistic problem used to study the content?

c. Why does the teacher act the way he does?

d. Why do students act the way they do?

3) Evaluation: What could be improved?

Issue a reasoned judgment on the teaching observed in the following aspects, indicating some changes that could be introduced to improve it:

a. Epistemic (mathematical content studied)

b. Ecological (relations with other subjects, curriculum)

c. Cognitive (previous knowledge, learning, ...)

d. Affective (interest, motivation, ...)

e. Interactional (modes of interaction between teacher and students)

f. Media (resources used)

4) Limitations of the available information:

What additional information would be necessary to make the analysis carried out more accurate and reasoned?

Transcription of the video-episode focused on the participants' voices.

1T	Good afternoon, everyone
2Ss	Good afternoon
3T	Look, today we are going to work with a new task. From the curriculum content: shape, space and measurement, under the topic geometric shapes and under the sub-topic (emphasis) Similarity
4T	We will work normally, as always, as we have been doing
5T	Professor Martín Eduardo Martínez Morales is here and will take evidence of the classes, of what we do and how we do it. You all have to work in a normal way, as usual.
6T	We hope all of you solve this task
	DISTRIBUTION OF TASKS [minute 00:52]
7T	Now, you all can turn over the tasks sheet and start reading
	READING INSTRUCTIONS [01:07]
8T	Attention boys and girls. Have you all read the problem?
9T	Who can tell me, what does the task ask?
10T	Mr. Legarre
11S	Based on the drawing that is there, calculate the height
12T	Good. What do the others say? Do you agree?
13Ss	Yes!!!
	VERBALIZATION [01:49]
14T	You have to calculate the height of the tree that appears in a drawing.
15T	Okay?
16Ss	Yes!!!
17T	Go ahead. Calculate the height of the tree according to the information.
18T	Now. Now. Look here
	USE OF ICT [02:18]
19T	There on the blackboard, we can see the projected problem that we are solving
20T	Use the knowledge acquired in the previous problems, because there, you have calculated the value of the measurements of some triangles with their homologous sides
21T	You have also previously calculated the value of proportionality
	DIDACTIC SITUATIONS [02:52]
	STUDENTS SPEAKING SPANISH [03:18]
22T	Understood?
	STUDENTS SPEAKING NAHUATL DIALECT [03:40]

23T	Here you have two possibilities. To solve the problem, you can use one of the two methods, ok, but also you can verify the solution using the other method.
24T	The most correct thing is to be "like that" (the teacher points out the student's sheet).
	SHARING [03:44]
25S	The answer to the problem is 5.23 (She explains the procedure used and writes it on the board)
26S	Then we apply a rule of three, and X is 5.23
27T	You got the same results by both methods. Good
28T	So, the height of the tree is 5.23
29	The students go to study outside, into the schoolyard
30T	'This' times 'this' divided 'this other' is equal to the height of the post
31S	Ah!
32T	Now you have to do the same procedure. You are going to choose a small tree and measure its shadow with the measuring tape.
	ADDITIONAL ACTIVITIES [06:41]
33I	Teacher, we have to present to the school supervision evidence of the problems that are carried out according to the secondary reform. Could you briefly comment on what are you doing, the students educational level, the kind of instruction, and what mathematical knowledge you are studying in this moment?
34T	These students are grade 3 (course A)
35T	We are studying the similar triangles. So, the reform involves exercises applying similarity. So we are solving some problems about that.
36T	We are working now in the schoolyard; in this way, the students have practical experience to calculate the height of some trees/poles, which are difficult to measure.
37T	This problem is solved using similarity of triangles
38T	They measure the shadow of some objects, and based on that data, they calculate their heights
39I	Okay teacher. Thank you so much. These are the problems currently proposed by the reform. In this moment, are you developing any particular task?
39T	Of course, Similarity triangles