ASSESSING AND DEVELOPING PEDAGOGICAL CONTENT AND STATISTICAL KNOWLEDGE OF PRIMARY SCHOOL TEACHERS THROUGH PROJECT WORK

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In this paper we describe a model of pedagogical content knowledge with a formative cycle directed to simultaneously increase the teachers' statistical and pedagogical knowledge. In this cycle, teachers are first given a statistical project to work with and then carry out a didactical analysis of the project. An analysis guide, based on the notion of didactical suitability, helps increase the teachers' competence related to the different components of pedagogical content knowledge and their ability to carry out didactical analyses. At the same time it provides the teacher educator with information regarding the future teachers' previous knowledge and learning. Results of experimenting with this formative cycle for a particular project in a group of 55 prospective teachers indicated a need for better statistics preparation of these teachers and illustrated the usefulness of the formative cycle and analysis guide proposed.

INTRODUCTION

Prospective primary school teachers in many countries enter the Faculties of Education with a very limited statistical competence, and the time available for educating them in statistics and the related pedagogical knowledge is very limited. It is then important to find activities that serve to teach statistics to these teachers, while at the same time helping to bridge conceptualization and pedagogy (as suggested by Ball, 2000). Moreover we should present future teachers with activities based on a constructivist and social approach to teaching if we want them to use this same approach with their future students (Jaworski, 2001).

Pedagogical Content Knowledge and Didactic Suitability

In Batanero, Godino & Roa (2004), taking into account conceptions of pedagogical content knowledge (Schulman, 1986; Aichele & Coxford, 1994) and in particular those related to the specific case of statistics (Biehler, 1990; Steinbring, 1990; Watson, 2001), we described the following components of pedagogical content knowledge: a) *Epistemology:* Epistemological reflection on the meaning of concepts to be taught (e.g., reflection on the different meaning of randomness); b) *Cognition:* Prediction of students' learning difficulties, errors, obstacles and strategies; c) *Teaching resources and techniques:* Experience with good examples of teaching situations, didactic tools; critical capacity to analyse textbooks, curricular documents and to adapt statistics knowledge to different teaching levels; d) *Affect:* Ability to engage students' interest and take into account the students' attitudes and beliefs; e) *Interaction:* Ability to create good communication in the classroom and to use assessment as a way to guide instruction.

In order to introduce this pedagogical knowledge in an active way, we developed a "Guide to Analyse and Evaluate the Didactical Suitability" (GAEDS) where we suggest the following six dimensions for didactic analysis (Godino, Batanero & Font, 2007):

1. *Epistemic suitability* (extent to which the statistical content is representative of the curricular content for a specific teaching level and whether its inclusion in the teaching is justified).
2. *Cognitive suitability* (whether the content is adequate for the students' previous knowledge and the extent to which the instructional goals can be achieved).
3. *Resource suitability* (sound use of technical tools, resources and time).
4. *Emotional suitability* (whether the teaching/learning process takes into account the students' motivations, attitudes, affects and beliefs).
5. *Interactional suitability* (whether the interactions between the teacher and the students and among the students themselves favour overcoming learning difficulties).
6. *Ecological suitability* (degree to which the teaching/learning process is adapted to social environment; possibility of establishing interdisciplinary connections).

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Based on this previous research and taking into account the time and prospective teachers' previous knowledge restrictions, we started a new research project to prepare didactical materials that would simultaneously increase prospective primary school teachers' statistical and pedagogical knowledge and competence. Next we describe and carry out a didactical analysis of one of the units following the GAEDS guidelines. Then we analyse the statistical and pedagogical content knowledge of 55 prospective teachers, as reflected in their written work produced within the unit, and discuss the implication for training teachers.

EDUCATIONAL SETTING, METHODOLOGY AND DIDACTICAL ANALYSIS

The experience has been carried out within a 45 hour course of "Mathematical Curriculum in Primary Education" (second year of University) at the Faculty of Education. This course is mainly practical; prospective teachers are briefly introduced to some didactic materials that summarize the pedagogical content knowledge in the different mathematical areas. Different practical activities serve to contextualise and apply this knowledge. Along with the themes introduced previously in statistics, the prospective teachers were introduced to didactical analysis, and the GAEDS guide to assess the suitability of a teaching process. Because these teachers had previously taken a 90 hour course in elementary mathematics covering the curricular content for primary school in Spain, which included 20 hours of descriptive statistics and probability, we assumed they had mastered the basic statistical content. Moreover this content forms part of the primary and secondary school mathematics curriculum.

The project unit analysed here is a formative cycle consisting of two stages. In a first lesson (90 minutes long session), the future teachers were given the statistical project "Check your intuitions about chance", which is described below. After collecting the data and working in small groups they were asked (as an assignment) to complete the analysis at home and write a report with a complete discussion of the project, including all the statistical graphs and procedures they used and their conclusions regarding people's intuitions about chance and their ability to simulate (invent) a random sequence. Teachers were free to work in teams and use any statistical software available to them, but each future teacher had to write an individual report. These reports were collected the following week, when a second 90 minute long session took place. After the different solutions to the project given by the prospective teachers were collectively discussed in the classroom, a didactical analysis, following the GAEDS instrument was carried out in small groups and again an individual report was produced. Finally these reports were used by the lecturer to discuss in the following days the pedagogical content knowledge involved in teaching statistics in primary school, the statistical and didactical features of this project, the teaching of statistics through project work and the extent to which the project was useful to teach statistics in the upper level of primary school.

The statistical project: "Check your intuitions about chance"

This project is part of a didactical unit designed to introduce the "information handling, chance and probability" content included in the upper level of primary education. Some aims are: a) showing the usefulness of statistics to check conjectures and analyse experimental data; b) checking intuitions about randomness and realising these intuitions are sometimes misleading. The sequence of activities in the project was as follows.

1. *Presenting the problem, initial instructions and collective discussion.* We started a discussion about intuitions and proposed that the future teachers carry out an experiment to decide whether they have good intuitions or not. The experiment consists of trying to write down apparent random results of flipping a coin 20 times (without really throwing the coin, just inventing the results) in such a way that other people would think the coin was flipped at random.
2. *Individual experiments and collecting data.* The future teachers tried the experiment themselves and invented an apparently random sequence (simulated throwing). They recorded their sequences (Figure 1), using H for head and T for tail. Afterwards the future teachers were asked to flip a fair coin 20 times and write the results on the same recording sheet (real throwing).

3. *Classroom discussion, new questions and activities.* After the experiments were performed we started a discussion of possible strategies to compare the simulated and real random sequences. A first suggestion was to compare the number of heads and tails in the two sequences since we expect the average number of heads in a random sequence of 20 tosses to be about 10. The lecturer posed questions like: If the sequence is random, should we get exactly 10 heads and 10 tails? What if we get 11 heads and 9 tails? Do you think in this case the sequence is not random? These questions introduced the idea of producing a graph for the number of tails and heads in the real and simulated experiments for the whole class and then comparing the similarities and differences. The session continued by collecting data about the number of runs and length of the longest run, as students suggested these variables might be compared.
4. At the end of the sessions the future teachers were given a copy of the data set for the whole class, which contained six statistical variables: number of heads, number of runs and length of the longest run for each of real and simulated sequences from each student (55 cases with these 6 variables each). They were asked to complete the analysis at home and produce a report.

Simulated throwing																			
Real throwing																			

Figure 1. Recording sheet

Didactical Analysis of the Project: The Expert's View

An a-priori analysis of a potential teaching/learning process based on this project suggest a high *epistemic suitability*, since all the statistical content in the upper primary school level (collecting and recording data, experiments, statistical variables, frequency tables, graphs, averages, range, chance and probability) can be introduced and justified through a problem understood by the students. Moreover statistics is used to answer a meaningful question, and statistics and probability are related in the project. Regarding cognitive suitability, students are able to carry out the data analysis tasks with the help of the teacher. Since the concept of median is hard for primary school students, comparison of averages could be restricted to mean and mode. Children may also need help to use the same scale in producing the graphs to compare two distributions (e.g., number of heads in real and simulated sequences). *Emotional suitability* would seem to be high, since we expect children will be interested in checking their intuitions.

In relation to resource suitability, the project can be adapted to different media. It is possible to work with only paper and pencil, provided the class is divided in small groups, and each group takes responsibility for analysing one of the variables. Spreadsheets (recommended for the last year in primary school level in Spain), when available, can reduce computation and give more time for interpretative activities. Working in small groups and whole class discussions can contribute to identifying and overcoming students' difficulties (interactional suitability). Finally, the experiment serves to check the student's intuitions about chance (ecological suitability) and may help in realising some probabilistic biases. It might interest students in social problems e.g., compulsive gambling, so there is interaction with social and psychological areas. When we use this project with prospective teachers, we give them an example of working with statistical projects that introduces an exploratory and participating dynamic in the classroom, in agreement with recent recommendations for teaching statistics

ANALYSIS OF PROSPECTIVE TEACHERS' REPORTS

The written reports of future teachers were analysed to produce an overall view of their knowledge and learning. Below we first analyse the graphs, statistical summaries and interpretations included in the solutions to the project and then the contents of the preservice students' didactical analyses reports.

Solutions to the Project and Teachers' Statistical Knowledge

Statistical graphs and summaries correctly or incorrectly used by the future teachers are presented in Table 1 (n = 55), where it is visible that these future teachers, in general, were able to use elementary statistical concepts in their analysis. The main errors were the identification of statistical variables (comparing individual values of only their own sequences, instead of using the distributions in the whole group) and inadequate graphs (e.g., plotting in the same graph all of the values for each individual). The number of prospective teachers who were able to reach a conclusion about the research question was very small, so mostly teachers showed some statistical knowledge but not statistical literacy.

Table 1. Frequency (and percentage) of students using different statistical knowledge

Statistical graphs and summaries	Correct (percent)	Incorrect (percent)
V1. Statistical variables and values	44 (80.0)	11 (20)
V2. Mean	16 (29.1)	1 (1.8)
V3. Median		1 (1.8)
V4. Range or standard deviation	16 (29.1)	
V5. Bar charts or line diagrams		13 (23.6)
V6. Matched bar charts or line diagrams	10 (18.2)	
V7. Frequency tables	22 (40.0)	3 (5.5)
V8. Comparing averages	16 (29.1)	
V9. Comparing spread	10 (18.2)	
V10.Conclusions on probabilistic intuitions	3 (5.5)	

A cluster analysis of subjects (variables were V1 to V10 as in Table 1) served to identify four levels of statistical knowledge and competence in these teachers:

- Level 1 (33 teachers, 60%): These prospective teachers did not recognise the individual experiments carried out in the class as part of a global sample that should be analysed to draw conclusions about people's intuitions. Some of them only analysed their own data and expected a coincidence in the two sequences, so they showed an incorrect conception of randomness. They built frequency tables, but they either produced an incorrect graphical representation or no graph at all; they usually computed the mean but reached no conclusion about the differences in the distribution or about the probabilistic intuitions in the group of students.
- Level 2 (10 teachers, 18.2%): These students analysed the three pairs of variables (number of heads, number of runs, length of the longest run, for the simulated and real sequences in the whole sample). They produced tables, graphs (bar charts or line graphs), and statistical summaries independently for each variable, but they did not compare the simulated and real distributions; they only described separately some of the data features, e.g., the averages.
- Level 3 (9 teachers 16.3%): These prospective teachers plotted each pair of variables on the same graph (for example, representing the number of heads in the real and simulated sequences in a paired bar chart). This facilitated the comparison of each pair of distributions, although, usually the comparison was limited to the averages, without taking spread into account. Other students focused on spread, and did not use the averages. They reached no conclusion about the probabilistic intuitions in the group.
- Level 4 (3 teachers, 5.5%): Similar to Level 3, but additionally they compared both averages and spread. They concluded that the group had a good intuition about the average number of heads and tails in a random sequence but not about the dispersion. They also concluded that the group had poor intuition about runs.

Didactical analysis reports and teachers' pedagogical conceptions

Once the didactical analysis reports produced by the prospective teachers were collected (38 reports), we scored the reports on a scale 0-5 for each of the six dimensions of the GAEDS guide, depending on the completeness and correctness of the analysis (total score in the report ranging from 0 to 30). The mean score was 11.63 with a standard deviation of 8.49, which shows that the didactical analysis was, in general. difficult for these prospective teachers. Average scores on the different components were Ecological (1.29), Interactional (1.95), Emotional (1.95), Epistemic (2.03) and Cognitive (2.16), which shows the relative difficulty of the different components. We defined different levels in the pedagogical content competence for these teachers according to the total score in the reports:

- Level 1: 18 students (47.4%: score between 0 - 10 points) did not apply the descriptors in each dimension or the application was inadequate "we needed to differentiate heads, tails and runs and we threw the coin" (emotional suitability). The epistemic, interactional and ecological dimensions were particularly hard for them. Regarding possible improvements, their suggestions were very general "there should be good communication", "the teacher should resolve all the conflicts".
- Level 2: 10 students (26.3%: score between 10 - 20 points) listed the descriptors in the GAEDS guideline for each dimension but were only able to apply some of them in a sometimes imprecise way: "the problem is about throwing a coin". They produced a global evaluation of didactical suitability, although they proposed some improvements in specific points: "there should be more time to consult and receive explanations from the teacher".
- Level 3: 10 students (26.3%: score higher than 20 points) were able to apply at least one indicator of suitability for each of the dimensions and showed personal criteria in interpreting and evaluating the project. They made a global judgment of didactical suitability and remarked on some of the positive features ("It is possible to relate different statistical ideas"; "Students were autonomous; they could explore by themselves") and even suggested some improvements.

We also identified some conceptions of future teachers regarding the teaching and learning that could be included in or complement Eichler's (2007) categorization: a) "Theory should precede practice": To face the solution of a problem, students first should know all the concepts and procedures needed in the solution. For example, students commented, "Along the project we did not explain what central tendency measures or types of graphs are and when you have to use them; these important questions were not explained, but you assumed they derive from the experiment"; b) "Overabundance of resources": Students should consider using all the possible technical tools in the same situation, even if they are not needed. Students remarked, "I would also include more resources, more technology, as computers"; c) "Decomposing": Splitting activities and problems in elementary tasks can avoid students' difficulties. Students commented, "I basically will propose many more activities of application: computing mean or modes, and making many statistical graphs"; d) "Time unawareness": Assuming learning time is reduced to time working in the classroom, the future teachers said, "I do not consider it correct to give an assignment to work at home. I would only use the classroom time".

DISCUSSION AND IMPLICATIONS FOR TRAINING TEACHERS

The formative cycle based on the project "Check your intuitions about chance" served to contextualise the elementary statistical notions and procedures included at primary school, to use them to solve a research problem and provoke didactical reflection on the pedagogical content knowledge in a second stage. The future teachers were given a model where the traditional "knowledge division" in textbooks (concepts versus procedures) was overcome and where statistical concepts and techniques are justified by a realistic situation, so that these concepts acquire a "situational meaning" for students. Organising an experiment to check our intuitions about the behaviour of random sequences led us to compare frequency distributions, and thus justifies the introduction of statistical tables, graphs and summaries. Another feature of statistical projects is the multivariate approach to data analysis. Decision making in random situations often requires taking into account, not just a variable, but a multiple approach: the average number of heads and tails in the simulated and real sequences were quite similar, but not the average of longest run, or of the number of runs, so the project also permitted future teachers to reflect on the multiple models present in a random sequence. People have a good intuition of equiprobability, but misperceive variation, assuming less variation than that present in a random process. Recognising these properties of randomness is crucial to overcome probabilistic biases such as the "gambler fallacy".

Regarding the didactical analysis, many students had difficulties in applying the GAEDS guide and in judging the suitability of the didactical process, which is reasonable given the scarce time devoted to preparing these teachers and the complexity of pedagogical content knowledge. However, GAEDS proved to be a useful tool to introduce systematic reflection on different facets affecting the teaching and learning of statistics, and, moreover, responses by even the most advanced future teachers showed some underlying conceptions about teaching and learning mathematics that should be made explicit and confronted. It also provided a multivariate approach to didactical analysis by including six different dimensions that interact with the teaching and learning processes of statistics, such as: Technical resources modify the type of problems and language used; previous knowledge affects the possible content to be included and the emotional dimension. To conclude, these analyses and results suggest the need to improve the statistical training of future primary school teachers that will only be possible if significant changes are introduced in the initial teachers' training syllabus assigning more time to statistics education.

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