LINKING LEARNING AND INSTRUCTIONAL THEORIES IN MATHEMATICS EDUCATION

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There is a dilemma between knowledge enquiry and transmission in general instructional theories, which are usually supported by constructivist, objectivist or a combination of constructivist and objectivist learning theories. Considering the epistemological, ontological, and semiotic assumptions about mathematical knowledge of the Onto-semiotic Approach, in this paper we describe a theoretical model of mathematical instruction that articulates the constructivist and objectivist approaches to learning in order to optimise the didactic suitability of the teaching and learning processes.

INTRODUCTION

The problem-situations designed to contextualise and provide meaning to curricular contents, the ways of interaction in the classroom and the resources used are, among other, factors that determine students' learning opportunities. The complexity of instructional design explains the existence of different instructional theories supported by differing psychological and pedagogical assumptions (Reigeluth et al., 2016). The family of instructional theories known as "Inquiry-Based Education" (IBE), "Inquiry-Based Learning" (IBL), "Problem-Based Learning" (PBL), advocate for inquiry-based learning, with little guidance from the teacher (Artigue & Blomhøj, 2013).

Contrary positions, such as those of Mayer (2004) and Kirschner et al., 2006) point to a wide range of research that concludes the greater effectiveness of instructional models in which the teacher and the transmission of knowledge are given a greater role. The problem about teaching models focused on the student or on the teacher can be related to the inquiry and transmissive instructional models, as well as to the debate between constructivism and objectivism (Jonassen, 1991), respectively. Consequently, a dilemma arises as to which paradigm is more effective in promoting the learning of mathematics: objectivism or constructivism? The objectivist model is based on the assumption that there is a real world and that the purpose of education is to map the entities of that world in the learner's mind. The constructivist paradigm relies on the premise that reality is constructed during interaction with the environment and classmates and that knowledge is both individual and collective.

> Radicals of each camp argue that is impossible to mix the two paradigms. [...] However, dominant paradigms, in both the physical and social sciences, [...] rarely

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> replace each other by falsification. Instead they tend to co-exist and are used whenever they are appropriate" (Vrasidas, 2000, p.12).

In this paper we first analyse the tension between student-centred didactic models, which are based on constructivist theories of learning, and teacher-centred models, which are supported by objectivist theories; in other words, we pose the dilemma between student enquiry and teacher transmission of knowledge. We then discuss a mixed didactic model, based on the epistemological, ontological, and semiotic assumptions from the Onto-Semiotic Approach (OSA) to mathematical knowledge (Godino & Batanero, 1994; Godino et al., 2007), through which the moments of transmission and enquiry are articulated to optimise the didactic suitability of the mathematics teaching and learning processes.

LEARNING AND INSTRUCTIONAL THEORIES

Theories of learning are often distributed along the objectivism - constructivism continuum (Jonassen, 1991), two extremes that are hardly ever proposed in isolation as the psychological underpinning of educational methods. Whether constructivist or otherwise, learning theories, in themselves, do not entail a theory of teaching; their implications for guiding educational processes are not prescriptive, but merely indicative. Ernest (2010) sees them as philosophies or generic orientations on how people manage to understand and appropriate knowledge, but without the characteristic of theories, which is the requirement of experimental contrast for their possible falsifiability. The implications for mathematics teaching of the four different versions of constructivism described by Ernest (2010) may legitimately be addressed by teachers who base their pedagogical practice on any of the learning theories "As a grand theory, or perhaps a paradigmatic theory, constructivism is too general to reach to the classroom directly" (Confrey & Kazak, 2006, p. 320).

The debate between direct teaching, linked to objectivist positions about mathematical and scientific knowledge, which supports the teacher's central role in guiding learning, and the minimally guided teaching, usually related to the constructivist model of teaching, is not clearly settled in the research literature. For Zhang (2016) the tissue between these two instructional positions is not whether one or the other is in favour of providing more or less guidance or support to students, but between explicitly discussing solutions with learners or letting learners discover them. "For the advocates of direct instruction, explicitly presenting solutions and demonstrating the process to achieve solutions are essential guidance" (Zhang, 2016, p. 908). In constructivist positions, although a certain amount of transmission of information from the teacher to the learner is accepted, it is still essential to hide some of the content.

For supporters of direct instruction who assume the cognitive load theory with an emphasis on worked examples, it is essential to provide the solutions. Authors such as Mayer (2004) and Kirschner, et al. (2006) claim that empirical research over the last half century provides overwhelming and clear evidence that minimal guidance during instruction is significantly less effective and efficient than guidance specifically designed to support the cognitive processing necessary for learning. In a similar vein, Radford states: “Indeed, it does not seem reasonable to expect that the child (working alone or in collaborative groups) would be capable of reconstructing by him/herself the complex theories featured in the curriculum” (Radford, 2012, p.103).

ONTOSEMIOTIC APPROACH TO MATHEMATICAL KNOWLEDGE

The problem of what mathematics should be taught and how to teach it is being addressed from various theoretical perspectives. The OSA framework problematises the very nature of mathematical knowledge, as does the Theory of Didactic Situations (Brousseau, 1997) and the Anthropological Theory of the Didactics (Chevallard, 1992). In the OSA, a double dimension of mathematics is considered, as an activity of people involved in the resolution of some kind of problems, and as a system of historically and culturally shared objects. The OSA ontological postulates are in line with those formulated in Wittgenstein's philosophy of mathematics:

> Concepts/definitions and propositions are regarded as “grammatical” rules of a certain kind. From this point of view, mathematical statements are rules (of a grammatical kind) governing the use of certain types of signs, since that is precisely how they are used, as rules. They do not describe properties of mathematical objects with any kind of existence that is independent of the people who wish to know about them or the language through which they are known, even if this may appear to be the case (Font et al., 2013, p. 110).

The central notion of *semiotic function*, as a relationship between mathematical objects and systems of practices, together with the proposed typology of mathematical objects and processes allowing to articulate the operational-pragmatist and referential positions of meaning and to reveal the onto-semiotic complexity of mathematical knowledge (Font et al., 2013; Godino et al., 2021).

The theory of mathematical knowledge embodied in the OSA on anthropological (Wittgenstein, 1953), pragmatist (Peirce, 1931-58) and semiotic (Hjelmslev, 1969/1943) foundations entails crucial implications for educational-instructional processes, by providing articulation elements between the theories of learning and mathematical instruction. The theory of didactic suitability (Godino et al., 2016), as a module of OSA, recognises the complexity of educational processes by taking into account not only the cognitive - affective (learning) and the instructional (interactions and resources) facets, but also the epistemic (content) and ecological (context) dimensions, as well as the interactions between these facets.

Didactical suitability is defined as the degree to which an instructional process (or part of it) meets specific characteristics that qualify it as optimal or adequate to achieve the alignment between the personal meanings achieved by learners (learning) and the intended institutional meanings (teaching), while considering the available circumstances and resources (environment). This involves the coherent and systemic articulation of six criteria related to the facets involved in an instructional process (Godino et al., 2007, p. 133):

- *Epistemic suitability*, expressing the degree of representativeness of the institutional meanings implemented, with regard to a reference meaning.
- *Ecological suitability*, referring to the degree to which the instruction is in line with the school and society educational project and the environmental conditions in which it is developed.
- *Cognitive suitability*, describing the degree to which the meanings implemented correspond to the learners' zone of potential development, as well as the closeness of the personal meanings achieved to the intended meanings.
- *Affective suitability*, indicating the learners' degree of involvement (interest, motivation, etc.) in the instructional process.
- *Interactional suitability*, indicating the degree to which the types of didactic configurations implemented, and their articulation help to identify and settle the potential semiotic conflicts that arise during the teaching process.
- *Mediational suitability* is the degree of availability and adequacy of the material and temporal resources necessary for the development of the teaching-learning process.

Achieving educational-instructional processes with high didactic suitability involves a complex human activity that requires coherent articulation of teaching and learning activities with curriculum development and teacher education activities, all of which are carried out within an institutional and ecological background that supports and conditions them.

SUITABLE MATHEMATICAL INSTRUCTION

Within the OSA framework, a didactic model has been developed seeking to locally optimise mathematics teaching and learning processes by considering the triple dialectic between the teacher`s and students' work and the mathematical content (Godino et al., 2019; Godino & Burgos, 2020). In line with the principles of cultural/discursive psychology (Lerman, 2001), it is assumed that autonomy and creativity in problem solving by students is a progressively achievable goal, rather than a precondition for learning.

The OSA's epistemological and onto-semiotic assumptions provide a rationale for designing and implementing a mixed instructional model, including inquiry-based cooperative, dialogic and transmission-based phases (Figure 1). Students have to learn numerous rules (concepts, propositions, procedures) as well as the conditions under which they can be applied. The learner proceeds from known rules and produces new ones, which have to be shared and be compatible with those already established in the mathematical culture. Such rules (knowledge) have to be stored in the subject's long-term memory and put to work at the appropriate time in the short-term memory.

In the moments or phases of the student's first encounter with a specific meaning of a mathematical object, a dialogic-collaborative configuration can optimise learning. In this type of interaction, the teacher and the students work together to solve problems that bring the intended knowledge into play in a critical way. The first encounter with new knowledge should be supported by explicit explanations and input from the teacher. These transmissive (somewhat magisterial) didactic configurations can be meaningful if they refer to the problem situation they are studying collaboratively. The teaching-learning process could thus achieve higher epistemic and ecological suitability (Godino et al., 2007). Under an inquiry-based teaching model, with minimal teacher guidance, students are exposed to the risk of not finding the solutions in the first encounter phase, with the consequent rejection and frustration. “Even if the students find the solutions on their own, they do not know the most effective procedures as they have to wander around in the problem searching process, not to mention the cognitive loads they are imposed” (Zhang, 2016, p. 909).

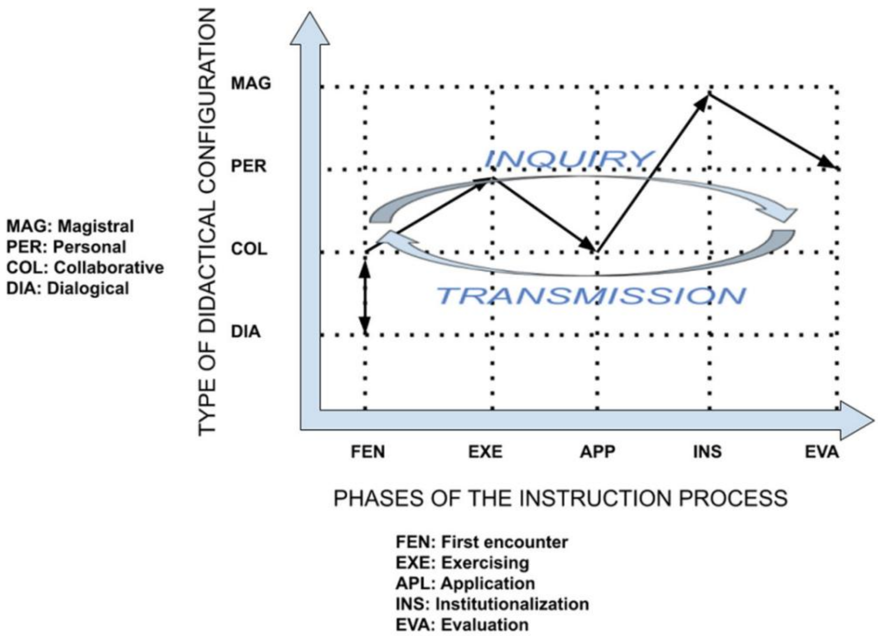

Figure 1. Transmission-inquiry dialectics according to the instructional process phases (Godino & Burgos, 2020, p. 16)

When the application rules and circumstances that characterise the object of learning are understood, it is possible to move towards higher levels of cognitive and affective suitability by offering more in-depth study of the content (exercise and application situations), through didactic configurations that progressively and in a controlled manner grant more autonomy to the learner.

CONCLUSSIONS

Instruction is usually understood as the combined activity of teaching and learning, i.e. Instruction = Teaching + Learning. In this paper, we add to this equation the content to be taught/learned, as we consider that how a content should be taught depends substantially on the nature of what is intended to be taught/learned. Moreover, it depends on the context and circumstances of the teaching process and the subjects involved, so that the optimisation of such activity has local components (actions of teacher and students in the classroom) and global components (didactic transposition processes of the specific content). A theory of learning does not suffice to understand and make decisions about instructional practices, rather a theory of content/knowledge is also required. Learning theories do not prescribe what instruction should look like, but that role should be played by content theories and theories of intended skills, or rather derive from the three-way dialectic between content, learning and teaching.

Hudson et al. (2006) also justify the implementation of mixed instructional models that adapt and mix explicit instruction (teacher-centred) with problem-based instruction (learner-centred) because of the need to make curricular adaptations to the diversity of students' abilities. As we can see, several authors propose blended instructional models that combine enquiry and knowledge transmission. In this paper we also defend the greater suitability of these mixed models, but we add some details on how the content type influences the greater or lesser presence of enquiry or transmission in the mixed model. The reasons given are basically linked to the onto-semiotic complexity of the intended mathematical activity, which complement the cognitive reasons highlighted by other authors (Kirschner et al., 2006).

The challenge of education is to optimise the teaching and learning process by balancing the learning of all students at their own pace and, at the same time, proposing educationally valuable content. This is a challenging task for the teacher and other educational agents who have to select and develop the right instructional resources for each circumstance. In our opinion, posing the dilemma between student-centred and teacher-centred education, between constructivist and objectivist learning theories, is naïve since the optimisation of the educational process requires the coherent and appropriate articulation of the principles of these educational theories and models.

Achieving mathematics education with high didactical suitability requires the coherent and balanced articulation of various facets and components (Godino et al., 2016), not only at the local level, i.e., within the classroom where the interaction between students, teacher and content takes place, but also at the global level. Two essential components of the ecological facet that crucially condition and support educational processes are the curriculum, where decisions are made about the content and educational resources that are made available to teachers, and the teacher education system, which is responsible for the preparation and ongoing support of teachers.

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