The powers of deconfinement

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The trace anomaly of gluodynamics encodes the breakdown of classical scale invariance due to interactions around the deconfinement phase transition. While it is expected that at high temperatures perturbation theory becomes applicable we show that current lattice calculations are far from the perturbative regime and are dominated instead by inverse even power corrections in the temperature, while the total perturbative contribution is estimated to be extremely small and compatible with zero within error bars. We provide an interpretation in terms of dimension-two gluon condensate of the dimensionally reduced theory which value agrees with a similar analysis of power corrections from available lattice data for the renormalized Polyakov loop and the heavy quark-antiquark free energy in the deconfined phase of QCD [1,2].

Introduction. The Lagrangian of gluodynamics is conformal invariant, reflecting the absence of an explicit scale. The divergence of the dilatation current equals the trace of the improved energy-momentum tensor $\Theta_\mu^\mu$ [3] and vanishes classically. Quantum-mechanically yields instead the so-called "trace anomaly" [4]. It reflects the breaking of scale invariance which introduces a single mass scale, $\Lambda_{\text{QCD}}$. The dimensionless "interaction measure" $\Delta = T\partial_T(p/T^4) = (\epsilon - 3p)/T^4$ quantifies the departure from the conformal limit $\epsilon = 3p$, which corresponds to a gas of free massless particles. At finite temperature, the energy density $\epsilon$ and the pressure $p$ enter as [5–8],

$$T^4\Delta \equiv \epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle \equiv \langle \Theta_\mu^\mu \rangle,$$

where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ is the field strength tensor and $\beta(g) = \mu \partial g/\partial \mu = -b_0 g^3 + \mathcal{O}(g^5)$ is the beta function, with $b_0 = 11N_c/(48\pi^2)$. A good knowledge of $\Delta$ is crucial to understand the deconfinement process, where the non perturbative (NP) nature of low energy QCD seems to play a prominent role [9]. In this contribution we analyze the highly NP behaviour of the trace anomaly just above the phase transition and describe it in a way that is consistent with other thermal observables (see [10] for further details).

Low and high temperatures. At low temperatures $\Delta$ is dominated by the lightest confined states in the spectrum. In gluodynamics the lightest glueball mass $m_G \approx 1.3$ GeV is much heavier than $T_c \approx 270$ MeV, and the pressure is $p \sim e^{-m_G/T}$, so $\Delta \sim e^{-m_G/T}$, $T \ll T_c$, indicating an exponentially small violation of scale invariance.\footnote{In full QCD for massless quarks one has a gas of weakly interacting massless pions and $\Delta \sim T^4/\Lambda_{\text{QCD}}^4$ as dictated by chiral symmetry [11].} At very high temperatures one also expects scale invariance to be restored while asymptotic freedom guarantees the applicability of perturbative QCD (pQCD). Actually, from the pressure to two loop one has [12]

$$\Delta = \frac{N_c(N_c^2 - 1)}{72} b_0 g^4(\mu) + \mathcal{O}(g^5), \quad T \gg T_c \quad (2)$$

where $1/g^2(\mu) = b_0 \log(\mu^2/\Lambda_{\text{QCD}}^2)$. It should be noted the ambiguity in this result, since generally...
one has both the temperature $T$ and the (MS)-renormalization scale, $\mu$, for which one takes the
reasonable but arbitrary choice $\mu \sim 2\pi T - 4\pi T$. Higher order corrections including up to $g^6 \log g$
can be traced from [13]. The infrared problems of the perturbative expansion yield poor
convergence at the lattice QCD available temperatures $T < 6T_c$. Perturbation theory contains
only logarithms in the temperature, suggesting a mild temperature dependence. This feature is
shared by hard thermal loops (HTL) and other resummation techniques of infrared divergencies
(see e.g. [14,15]). Actually, the value they find $\Delta_{\text{HTL}} = 0 \pm 0.5$ for $T > T_c$ is compatible with
zero within uncertainties. Furthermore, it is not clear at what temperatures is the pQCD result
dominating $\Delta$.

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valid up to $\mathcal{O}(g^5)$ in pQCD. $A_0$ is the gluon field in the (Euclidean) time direction.

The dimension two gluon condensate $g^2(A_{0,a}^2)$ is obtained from the gluon propagator of the dimensionally reduced theory, $D_{00}$, by taking the coincidence limit. The perturbative propagator $D_{00}^P(k) = 1/(k^2 + m_D^2) + \mathcal{O}(g^2)$, being $m_D \sim T$ the Debye mass, leads to the known perturbative result [19] and fails to reproduce lattice data below $6 T_c$ [20]. A NP model is proposed in Ref. [1] to describe the lattice data of the Polyakov loop, and it consists in a new piece in the gluon propagator driven by a positive mass dimension parameter:

$$D_{00}(k) = D_{00}^P(k) + D_{00}^{NP}(k),$$

where $D_{00}^{NP}(k) = m_G^2/(k^2 + m_D^2)^2$. This ansatz parallels a zero temperature one [21], where the dimension two condensate provides the short-distance NP physics of QCD and at zero temperature this contribution yields the well known NP linear term in the $\bar{q}q$ potential. A justification of Eq. (7) based on Schwinger-Dyson methods has been given [22]. The Gaussian approximation has also been used in Ref. [2] to compute the singlet free energy of a heavy $\bar{q}q$ pair [20,23], through the correlation function of Polyakov loops.

**Non perturbative contribution to the Trace Anomaly.** The model of Eq. (7) can easily be used to compute the trace anomaly Eq. (1) in gluodynamics. Assuming the leading NP contribution to be encoded in the $A_{0,a}$ field and taking $A_{i,a} = 0$ yields [24]

$$\langle (G_{\mu\nu}^{a})^2 \rangle_{NP} = -6m_D^2\langle A_{0,a}^2 \rangle_{NP}. \quad (8)$$

Note that the NP model is formulated in the dimensionally reduced theory, so the gluon fields are static. This formula produces the thermal power behaviour of Eq. (3) with

$$b_{tra}T_c^2 = -(3\beta(g)/g)m_D^2\langle A_{0,a}^2 \rangle_{NP}, \quad (9)$$

where $m_D \equiv m_D/T$. If we consider the perturbative value of the beta function $\beta(g) \sim g^3 + \mathcal{O}(g^5)$, the r.h.s. of Eq. (9) shows a factor $g^2$ in addition to the dimension two gluon condensate $g^2\langle A_{0,a}^2 \rangle_{NP}$. So the fit of the trace anomaly...
data is sensitive to the value of the smooth $T$-dependent $g$, without jeopardizing the power correction. When we consider the perturbative value $g_P$ up to 2-loops, we get from the fit of the trace anomaly a value of $g^2(A^2_{0,a})_{NP}$ which is a factor 1.5 smaller than from other observables. This disagreement could be partly explained on the basis of certain ambiguity of $g$ in the NP regime. A better fit of the Polyakov loop and heavy quark free energy lattice data in the regime $T_c < T < 4T_c$ is obtained for a slightly smaller $g$ than $g_P$, i.e. $g = 1.26 - 1.46$ [2]. Taking this value we get from Eq. (9) $g^2(A^2_{0,a})_{NP} = (2.86 \pm 0.24 T_c)^2$, a better overall agreement, see Table 1. From Renormalization Group requirements it is possible to observe that the consistency of the model is warranted if the coupling constant $\alpha_s(\mu) \equiv g^2(\mu)/(4\pi)$ has a behaviour $\sim 1/\mu^2$ at low enough temperature [10]. This behaviour is just what is obtained within the Analytic Perturbation Theory formalism, after extracting the Landau pole [25], with the corresponding decrease of the perturbative value of $\alpha_s(\mu)$. This decrease could approximately explain the best-fit value of $g$.

While there might exist other explanations to the observed thermal power corrections our results of Table 1 suggest an unified and coherent description of observables in the non perturbative regime of the deconfined phase (sQGP) in terms of the dimension two gluon condensate [10].

<table>
<thead>
<tr>
<th>Observable</th>
<th>$g^2(A^2_{0,a})_{NP}$</th>
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<tbody>
<tr>
<td>Polyakov loop [1]</td>
<td>$(3.22 \pm 0.07 T_c)^2$</td>
</tr>
<tr>
<td>Heavy $q\bar{q}$ free energy [2]</td>
<td>$(3.33 \pm 0.19 T_c)^2$</td>
</tr>
<tr>
<td>Trace Anomaly</td>
<td>$(2.86 \pm 0.24 T_c)^2$</td>
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Table 1

Values (in units of $T_c$) of the dimension two gluon condensate from a fits in the deconfined phase of gluodynamics: Polyakov loop, singlet free energy of heavy quark-antiquark and trace anomaly. Lattice data are with $N_c = 8$. The error reflects an uncertainty in the coupling constant $g = 1.26 - 1.46$, being the highest value the perturbative $g_P$ up to 2-loops at $T = 2T_c$. The critical temperature in gluodynamics is $T_c = 270 \pm 2$ MeV [20].

REFERENCES