A bifurcation result for a critical fractional equation in $\mathbb{R}^n$

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Abstract. In this talk we will study the following critical problem,

$$(-\Delta)^s u = \varepsilon\, h\, u^q + u^p \quad \text{in } \mathbb{R}^n,$$

where $s \in (0,1)$, $n > 4s$, $\varepsilon > 0$ is a small parameter, $p = \frac{n+2s}{n-2s}$, $0 < q < p$ and h is a contionuous and compactly supported function.

In particular, we will prove existence of a solution u_ε by considering the problem as a perturbation of the equation

$$(-\Delta)^s u = u^p \quad \text{in } \mathbb{R}^n, \qquad p = 2_s^* - 1. \tag{1}$$

Moreover, we will see that u_ε tends to one of the solutions of (1) as $\varepsilon \to 0$, which are the minimizers of the Sobolev embedding.

To obtain this existence result, we will perform a Lyapunov-Schmidt reduction, taking advantage of the variational structure of the problem.

References

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