Actividad formativa de Doctorado: Riemann-Penrose inequality

1. Motivation and aim of the course

In 1973, Penrose [4] proposed a celebrated geometric inequality, in order to test the *establishment viewpoint* on the theory of black holes. Such an inequality suggested an unexpected relation between apparently unrelated geometric elements in a class of Lorentzian (and Riemannian) manifolds. In a nutshell, the inequality says that the *ADM mass* of the involved physical spacetimes, computed from some integral involving the curvature of asymptotic surfaces, should be non-smaller than the mass of the black holes in that spacetime, computed from its area. For a particular class of spacetimes, this inequality is reduced to the *Riemann-Penrose inequality*, an inequality on classical (positive definite) Riemannian manifolds, namely: the ADM mass m and the area A of the outermost minimal surface in an asymptotically flat 3-manifold of nonnegative scalar curvature, satisfy

$$m \geq \sqrt{\frac{A}{16\pi}},$$

being the equality attained if and only if the 3-manifold is isometric to the canonical spacelike slice of the Schwarzschild spacetime.

The proof of the Riemann-Penrose inequality by Huisken and Ilmanen at the end of the past century (published in [2]) was a major breakthrough for both, Black Hole theory and Differential Geometry. For the former, such an inequality might be regarded as the best test of its physical consistency, due to the difficulty of direct observational evidences of black holes. For Differential Geometry, the proof showed the powerful and sharpness of the *inverse mean curvature flow*, including the weak solutions which allowed the flowing surfaces to "jump" at the singularities. Indeed, such a flow was shortly after used by Bray and Neves to compute the Yamabe invariant of three-dimensional real projective space [1].

The aim of the present activity is twofold. First, a course, where Huisken and Ilmanen's proof of the Riemann-Penrose inequality will be explained in detail, including an overview on its physical motivations and the background on flows related to the mean curvature in Differential Geometry. Second, a small meeting will provide some acquaintance with the recent progress on Penrose inequality and geometric flows, as well as contact with researchers specialized in these fields.

2. Structure

- Course on the Riemann-Penrose inequality
  - Professor: Francisco Martín Serrano, with the collaboration of Miguel Sánchez Caja (both at Dept. Geometría y Topología, UGR).
  - Schedule: 30 hours delivered in 10 weeks along the second semester (March/May 2017), in two sessions of 1.5 hours per week.
  - Contents
    1. Physical and mathematical framework
    2. Weak solutions of the Inverse Mean Curvature Flow (IMCF)
    3. Geroch Monotonicity Formula
    4. Huisken-Ilmanen *weak solutions of the IMCF*
    5. Schwarzschild example and rigidity.
- Meeting on the Riemann-Penrose inequality
  - Organizers: Francisco Martín Serrano and Miguel Sánchez Caja.

– Schedule: short meeting of two days at the end of May. It will contain two main talks, one about the technique of the proof and the other on the physical and mathematical framework (tentative speakers would be T. Ilmanen [2] and M. Mars [3], resp.) Additionally 2-4 talks about specific aspects of present-day research will be included. Interaction with participants will be encouraged.

References

[1] H. Bray, A. Neves, *Classification of prime 3-manifolds with Yamabe invariant greater than* RP^3. Annals of Mathematics **159** (2004), 407–424.

[2] G. Huisken, T. Ilmanen, *The Inverse mean curvature flow and the Riemannian Penrose inequality* J. Diff. Geom. **59** (2001), 353–437.

[3] M. Mars, *Present status of the Penrose inequality*, Classical Quantum Gravity **26** (2009), no. 19, 193001.

[4] R. Penrose, *Naked singularities*, Ann. New York Acad. Sci., **224** (1973) 125–134.