International Workshop on Nonlinear Analysis and Differential Equations

A celebration of the 60th birthday of Antonio Cañada

Abstracts

David Arcoya

Title: Regularizing effect of lower order terms in elliptic problems

Abstract: We study the regularizing effect of lower order terms ($h(x)|u|^{p-1}u$) in elliptic problems. Precisely, contrarily with the case $h \equiv 0$, we prove (joint work with **Francisco Odair de Paiva** and **José M. Mendoza**, Universidade Federal de São Carlos, Brazil) that for a bounded domain Ω, a bounded Carathéodory function g in $\Omega \times \mathbb{R}$, $p > 1$ and a nonnegative locally integrable function h in Ω which is strictly positive in a set of positive measure, there exists a solution of the semilinear elliptic problem

$$\begin{cases} -\Delta u + h(x)|u|^{p-1}u = \lambda u + g(x,u) + f(x), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

for every $\lambda \in \mathbb{R}$ and $f \in L^2(\Omega)$.
We also consider (joint work with **Alexis Molino** and **Lourdes Moreno** the case $0 \in \Omega$ and a Hardy potential is introduced in the problem

$$\begin{cases} -\Delta u + h(x)|u|^{p-1}u = \lambda \dfrac{u}{|x|^2} + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

being in this case $\lambda > 0$ and $f \in L^m(\Omega)$ for some adequate $m > 1$.

José Carmona

Title: Principal eigenvalue for a quasilinear elliptic singular problem and applications

Abstract: We characterize in [1] the principal eigenvalue λ^* for the following problem

$$\begin{cases} -\Delta u = \lambda u + \mu(x)\dfrac{|\nabla u|^q}{u^{q-1}}, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

in a regular bounded domain $\Omega \subset \mathbb{R}^N$, $1 < q \leq 2$ and $0 \leq \mu \in L^\infty(\Omega)$.
We use this characterization to describe, for each $0 \leq f(x) \in L^\infty(\Omega)$, the set of values of the parameter $\lambda \in \mathbb{R}$ such that the problem

$$\begin{cases} -\Delta u = \lambda u + \mu(x)\dfrac{|\nabla u|^q}{u^{q-1}} + f(x)\,, & \text{in } \Omega, \\ u > 0\,, & \text{in } \Omega, \\ u = 0\,, & \text{on } \partial\Omega, \end{cases}$$

admits a solution. Actually, the interior of this set is $(-\infty, \lambda^*)$. This latter problem has been studied in [2] in the case $q = 2$. We complete some of their results concerning with existence, nonexistence, comparison principle and bifurcation from infinity.

Bibliography

[1] J. Carmona, T. Leonori, S. López-Martínez and P. J. Martínez-Aparicio, *Quasilinear elliptic problems with singular and homogeneous lower order terms*. Submitted.

[2] D. Arcoya and L. Moreno-Mérida, *The effect of a singular term in a quadratic quasi-linear problem*. J. Fixed Point Theory Appl. **19** (2017), 815-831.

Pavel Drábek

Title: Convergence to higher-energy stationary solutions of a bistable equation with nonsmooth reaction term

Abstract: In this talk, we discuss the local stability of critical points with energy that is higher than the ground-state energy of the functional J associated with the bistable equation under suitable assumptions on the diffusion constant and double-well potential. In particular, we assume that a double-well potential F lacks C^2 regularity at the global minimizers -1 and +1. An instructive example is $F(u) = |1 - u^2|^\alpha$ for $\alpha \in (1, 2)$. For sufficiently small diffusion coefficient J possesses continua of critical points at high energy levels and that the relative interior of these continua are local minimizers. The local geometry of J at these points is convex in directions perpendicular to the continua, and thus has a trough-like shape locally, with critical points at the base of the trough. For each such continuum, we show that there is an open set containing the interior of the continuum that is a subset of the basin of attraction for the continuum. This stability result allows for a different perspective on the so-called slow dynamics associated with functionals such as J . The talk is based on two joint papers with S. B. Robinson, Z. Angew. Math. Phys. (2011) 62, 609–622 and Z. Angew. Math. Phys. (2017) 68:67.

Enrique Fernández Cara

Title: On the Navier-Stokes equations: three major open problems and three modest contributions

Abstract: In the first part of this talk, I will recall the formulations of the uniqueness, regularity and controllability problems for the 3D Navier-Stokes equations. I will also mention some recent partial achievements. In the second part, I will present some contributions to the analysis, control and numerical solution of the equations.

Alessandro Fonda

Title: An infinite-dimensional version of the Poincaré–Birkhoff theorem on the Hilbert cube

Abstract: We propose a version of the Poincaré–Birkhoff theorem for infinite-dimensional Hamiltonian systems, which extends a recent result by Fonda and Ureña. The twist condition, adapted to a Hilbert cube, is spread on a sequence of approximating finite-dimensional systems. Some applications are proposed to pendulum-like systems of infinitely many ODEs. We also extend to the infinite-dimensional setting a celebrated theorem by Conley and Zehnder.

Rafael López Soriano

Titl: New existence and compactness results for a singular mean field problem of Liouville type

Abstract:
This talk is concerned with a singular mean field problem of Liouville type on compact surfaces. This equation appears in differential geometry and it is also connected with some current physical theories.
We will focus on the existence and compactness of solutions for this type of problems, which have been extensively studied for positive potentials. However, the case of sign changing potentials has not been much considered. For the latter case, we will establish some results on the solvability by means of variational methods. In addition, we will derive a suitable compactness criterion.

Bibliography

[1] F. De Marchis, R. López-Soriano, D. Ruiz. *Compactness, existence and multiplicity for the singular mean field problem with sign-changing potentials,* accepted for publication on J. Math. Pures Appl.

Jean Mawhin

Title: Bolzano theorems for holomorphic mappings

Abstract: The existence of a zero for a holomorphic functions on ball or on a rectangle under some sign conditions on the boundary generalizing Bolzano's ones for real functions on an interval is deduced in a very simple way from Cauchy's theorem for holomorphic functions.
A more complicated proof, using Cauchy's argument principle, provides uniqueness of the zero when the sign conditions on the boundary are strict. Applications are given to corresponding Brouwer fixed point theorems for holomorphic functions.
Extensions to holomorphic mappings from $\mathbb{C}^n$ to $\mathbb{C}^n$ are obtained using Brouwer degree.

Rafael Ortega

Title: Periodic solutions and regularization in a perturbed Kepler problem

Abstract: This is joint work with Alberto Boscaggin and Lei Zhao. We consider a perturbed Kepler problem in dimension two or three. It is also assumed that the equation is time dependent and has a period T. We prove that for each integer $N \geq 2$ there exist at least N periodic solutions (with period T) if the perturbation is small enough. The solutions are understood in a generalized sense since they can have collisions. The concept of generalized solution is defined intrinsically but we show that it coincides with the notion obtained in Celestial Mechanics via the theory of regularization of collisions.

Antonio Suárez

Title: Some elliptic equations with non-local diffusion coefficient

Abstract: In this talk we present some theoretical results concerning to non-linear elliptic equations with non-local terms. Specifically, the non-local term appears in the diffusion coefficient. From the biological point of view, this means that the species does not move randomly, but its movement (and velocity) depends on the amount of population in the entire habitat. We focus our attention in the classical logistic equation, showing the main differences between the case of linear and non-local diffusion.

Antonio J. Ureña

Title: Instability of all planar closed orbits obtained by minimization

Abstract: We study the dynamics around closed orbits of autonomous lagrangian system in orientable surfaces. Our main result states that if a orbit minimizes the free-period action functional, then it must be orbitally unstable. This applies to possibly degenerate, or even nonisolated minimizers (in the nondegenerate case already Poincare showed that minimizers must be hyperbolic). The result can be extended to higher dimensions in a symmetric framework. Applications to geodesics and Celestial Mechanics are given.