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Human Processing of Colour Information in the Chromatic-frequency Domain

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On the basis of MacAdam's data, we have computed a psychophysical function which characterizes the transference of the colour information processed by the human visual system in the chromatic frequency domain. This function, obtained using chromatic-discrimination criteria, shows a cut-off frequency between 0.01375 and 0.02 c/nm, depending upon the colour-tolerance units adopted.

Colour vision   Chromatic frequency   Chromatic-contrast sensitivity function   Colour signal

INTRODUCTION

The term "chromatic frequency" was coined by Lennie (1991) to designate the variable resulting from the Fourier transform of functions that depend on wavelength, such as the spectral reflectance of a diffusing object, the spectral emission of an illuminant, or the spectral-sensitivity curve of a photoreceptor. The use of this type of variable is, however, not recent. Stiles, Wyszecki and Otha (1977) used limited-band functions to represent the spectral reflectance of opaque objects, which can be characterized by its limiting frequency expressed in c/nm. Buchsbaum and Gottschalk (1984) also used functions of the type

$$E(\lambda) = 1 + m \sin(2\pi f\lambda + \phi) \quad (1)$$

where f is expressed in c/nm, to characterize the spectral-emission curve of colour stimuli (called colour signals) and locate metamers of these in the chromatic diagram.

Interest in working in the chromatic-frequency domain stems from our consideration that the human colour-vision system is an information processor which samples colour signals and processes this information. We can apply information-theory criteria to an analysis of colour vision in a similar way to that in which they can be applied to studies of spatial vision or to the temporal aspects of vision.

Barlow (1982) obtained the Fourier-transform module from the action spectra of the cones deduced by Smith and Pokorny (1975). The results show that the transform module decays to almost zero at certain frequencies, at which the photoreceptors appear to function as chromatic information filters of the colour signals.

The responses of the colour-vision mechanisms to colour signals containing only one chromatic frequency have been analysed by Benzschawel, Brill and Cohn (1986). Through computer simulation they compared different colour-vision models on the basis of these responses. The colour signals used (comb-filtered spectra) are described by the equation

$$E(\lambda) = E_0[1 + m \sin(fp(\lambda) + p_0)] \quad (2)$$

where f is the chromatic frequency of the signal, m the relative amplitude (with values within the interval [0,1]) and p_0 the initial phase. Figure 1 shows different examples of colour signals of this type.

An initial experimental measurement of the human chromatic-contrast sensitivity curve (CCSF) was obtained by Barlow, Gemperlein, Paul and Steiner (1983). They used an interferometric device that allowed them to obtain both frequency and phase characterizations of the human CCSF, though these were not independent of each other. More recently Bonnardel and Varela (1991) measured the contrast sensitivity curve experimentally using periodic spectral distribution stimuli, within the visible interval of 400–700 nm. These stimuli were produced using a combination of polarized filters that permitted them, through square periodic modulations, to select spectral power distributions of frequencies within 0.5–3.6 c/300 nm, and a phase from 0 to 180 deg.

Our theoretical study attempts to characterize the human visual system as an information processor of chromatic frequencies. To do this we have chosen colour signals of the type used by Benzschawel *et al.* (1986) and studied the behaviour of the entire colour-vision system in the chromatic-frequency domain, employing a strictly computational procedure. It should be emphasized that we have made no attempt to study the individual responses of either the photoreceptors or the opponent and non-opponent mechanisms to colour signals of any particular chromatic frequency, but that our aim has been to look into the mechanisms of subsequent chromatic discrimination. Our computational procedure

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was somewhat analogous to the experimental one of Bonnardel and Varela (1991), the difference being that our simulation was made at constant luminance and a narrower sample frequency gap (0.0005 as opposed to 0.0016 c/nm). Our sampling was also extended to frequency values of between 0.2 and 19.8 c/400 nm and phases between 0 and 315 deg. The experimental simulation we developed allowed us to deal with pure harmonic functions, unlike Bonnardel and Varela (1991), who used a square-wave light modulation the spectral composition of which was not entirely flat. Despite the devices proposed by Goodman (1982) there still remain today considerable experimental difficulties in obtaining either sinusoidal or periodic spectral energy distributions. The studies of Barlow *et al.* (1983) and Bonnardel and Varela (1991) have pioneered in this direction.

Our method, in which we have adopted a procedure parallel to that used in obtaining the curve of spatial-contrast sensitivity (CSF), has led to us being able to obtain a psychophysical function for chromatic-information transference, which indicates the capacity of the human colour-vision system as a whole to distinguish colour signals of different frequency contents. The method is based on experimental results of chromatic discrimination and is slightly different from that used by Bonnardel and Varela (1991) as far as the discrimination criterion applied to the colour signals is concerned.

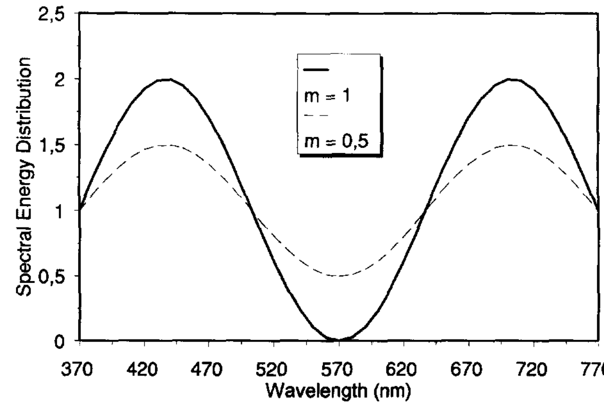

FIGURE 1. Examples of colour signals of the type described by equation (2); here for $f = 1.5$ c/400 nm, $p_0 = 0$ deg, amplitudes $m = 1$ and $m = 0.5$. These curves show the application of the chromatic-discrimination criteria adopted: the decrease in m is equivalent to a progressive disminution in the initial contrast.

METHODS

We simulated an experiment in which two stimuli are compared, as would be done with a bipartite field, one being a fixed, equal-energy, white stimulus and the other one having the spectral radiance given in equation (2).

The function $p(\lambda)$ permits us to change a wavelength spectrum measured in nanometers to a phase angle characterization of between 0 and 360 deg; since we have taken the visible spectrum between 370 and 770 nm, the expression for $p(\lambda)$ is given by

$$p(\lambda) = (0.9\lambda - 333)\text{deg}. \quad (3)$$

From equations (3) and (2), it can be deduced that f is expressed in *c/400 nm*, and thus it is possible to simulate the curves shown in Fig. 1.

We initially decided upon a discrimination tolerance in terms of the MacAdam units. Then, having fixed a frequency f and a phase p_0, we took the value $m = 1$ and $E_0 = 1$. We calculated the chromaticity coordinates (x, y) for this stimulus in the CIE 1931 system and determined [from the MacAdam (1943) data] whether the point represented in the chromaticity diagram was within the discrimination ellipse associated with the equal-energy stimulus (0.333, 0.333).

If the point falls outside the ellipse we consider the test and equal-energy stimuli to have been perceived as different and the value of m decreases by intervals of 0.001 units, the calculation being repeated until we arrive at coordinates that reach the edge of the ellipse. We worked at constant luminance and so the value E_0 is adjusted in each calculation of the chromaticity coordinates. In the simulation we use chromatic-discrimination criteria to discern between nonequi-energetic stimuli and equi-energetic ones. The procedure followed by Bonnardel and Varela (1991) was based on chromatic-perception criteria, i.e. the observers always began their experimental sessions from an unmodulated stimulus and increased the contrast until they obtained a just-noticeable chromatic perception.

Our process is equivalent to desaturating the initial stimulus with an equal-energy white one until reaching a spectral radiance that generates a stimulus indistinguishable from the initial one (see Fig. 1). For this stimulus we calculate the threshold contrast as

$$C_u = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = m \quad (4)$$

and, similar to the CSF, we define

$$V = \frac{1}{C_u} = \frac{1}{m}. \quad (5)$$

We determined the chromatic-contrast sensitivity curves for eight p_o phases, corresponding to the interval [0, 360] deg, with $\Delta p_0 = 45$ deg. For each curve we calculated the value of V for 100 frequencies from 0.2 to 19.8 c/400 nm and for the tolerance values 3 and 10 MacAdam units. Results of chromatic-discrimination experiments such as those of Boynton and Kambe (1980) or those obtained in our laboratory (Romero, García, Jiménez del Barco & Hita, 1993), show colour-difference values of around 10 units, just perceptible when the observation conditions and the measurement methods are not so restricted as with MacAdam's conditions. In fact 3 units are habitually used as colour tolerances in practical work.

RESULTS AND DISCUSSION

Figure 2 shows an example of the curves obtained. The different maxima and minima observed are a conse-

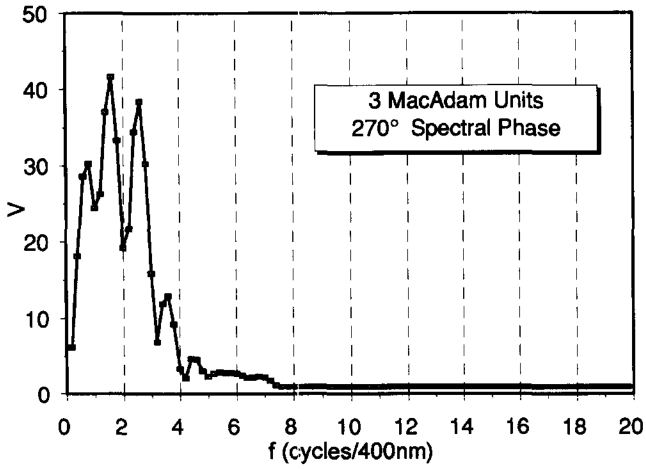

FIGURE 2. Chromatic contrast sensitivity curve for colour stimuli characterized by a 270 deg spectral phase and 3 MacAdam units.

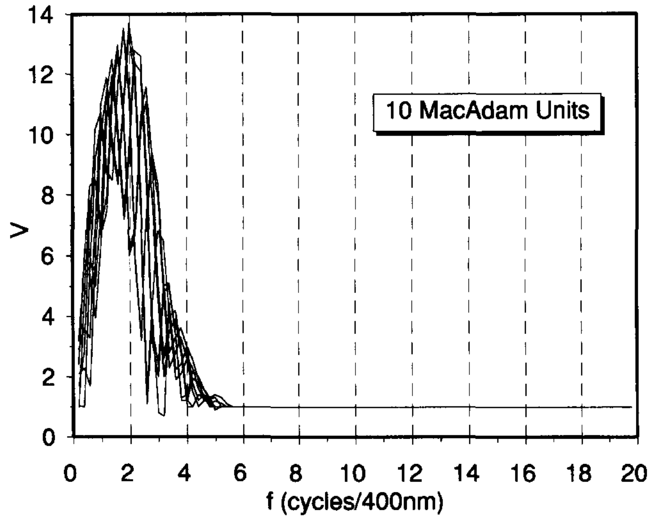

FIGURE 4. Chromatic contrast sensitivity curves for the eight phases computed and 10 MacAdam units.

quence of the fact that for each frequency as m decreases we approach the point representing the equal-energy stimulus along a straight line that originates at a distinct point in the diagram. For this reason it makes a difference whether the ellipse is arrived at via a point of minimum or of maximum elongation.

In Figs 3 and 4 we have superimposed the results for the different phases and can thereby deduce a mean curve of smooth variation. The curve associated with 3 units of colour difference shows that frequency stimuli of over 8 c/400 nm (0.02 c/nm), even for $m = 1$, are processed by the visual system in a similar way to that in which stimuli with an equal-energy spectral radiance would be processed. For 10 units, the cut-off frequency is reduced to 5.5 c/400 nm (0.01375 c/nm), though the form of the curve remains similar, showing an increase for low and intermediate frequency values and decreasing later to the value of the cut-off frequency.

To obtain a clearer view of the smooth mean curve we plot maxima of visibility as a function of chromatic frequency for the optimum phase (Fig. 5). The frequency of the maximum for the optimum phase curve varies slightly with the tolerance adopted. Thus for the wider tolerance it is located at 2 c/400 nm ($p_0 = 180$ deg) and for the narrower one at 1.8 c/400 nm ($p_0 = 45$ deg). This type of curve is similar to those obtained when the CSF is evaluated (Campbell & Robson, 1968) or similar to the temporal contrast function (TMTF) (DeLange, 1958). In this same way, while always bearing in mind that our study is strictly computational, a clear similarity exists between these curves and the experimental ones analysed by Bonnardel and Varela (1991); in both studies the shapes of the curves show an irregular distribution of maximum and minimum values, though in our results the former are more tightly distributed because of our finer sampling, and no secondary peaks appear. On the other hand, the cut-off frequency values obtained by Bonnardel and Varela (1991) are lower than ours (0.012 c/nm regardless of phase) and are somewhat similar to our results with 10 MacAdam units. Due to

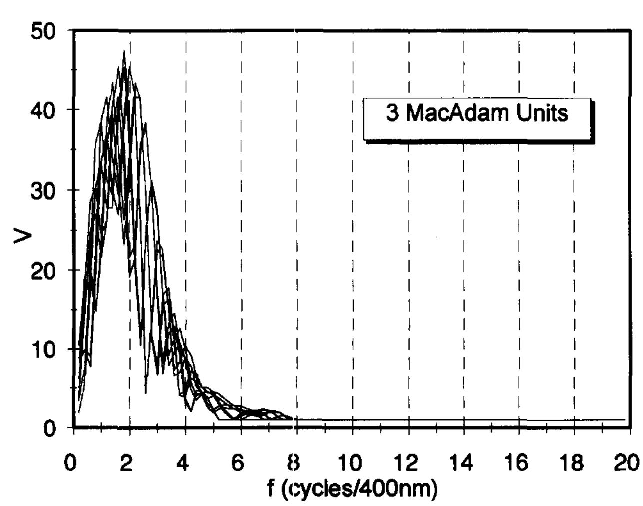

FIGURE 3. Chromatic contrast sensitivity curves for the eight phases computed and 3 MacAdam units.

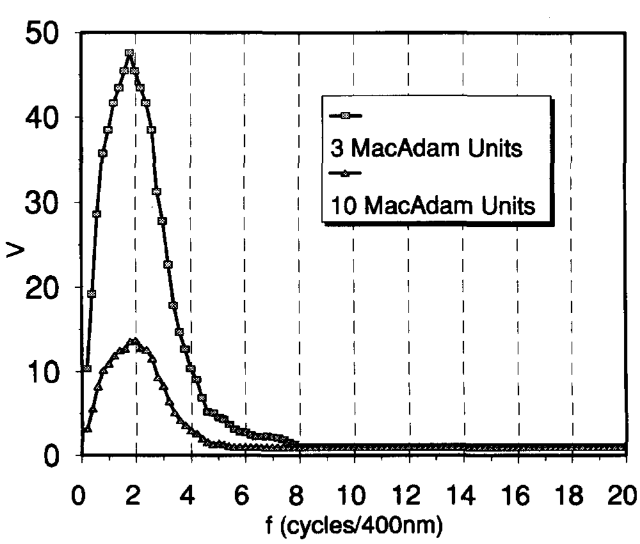

FIGURE 5. Upper envelopes of the superimposed curves in Figs 3 and 4; the curves show the maxima of sensitivity for optimum phases at any given chromatic frequency.

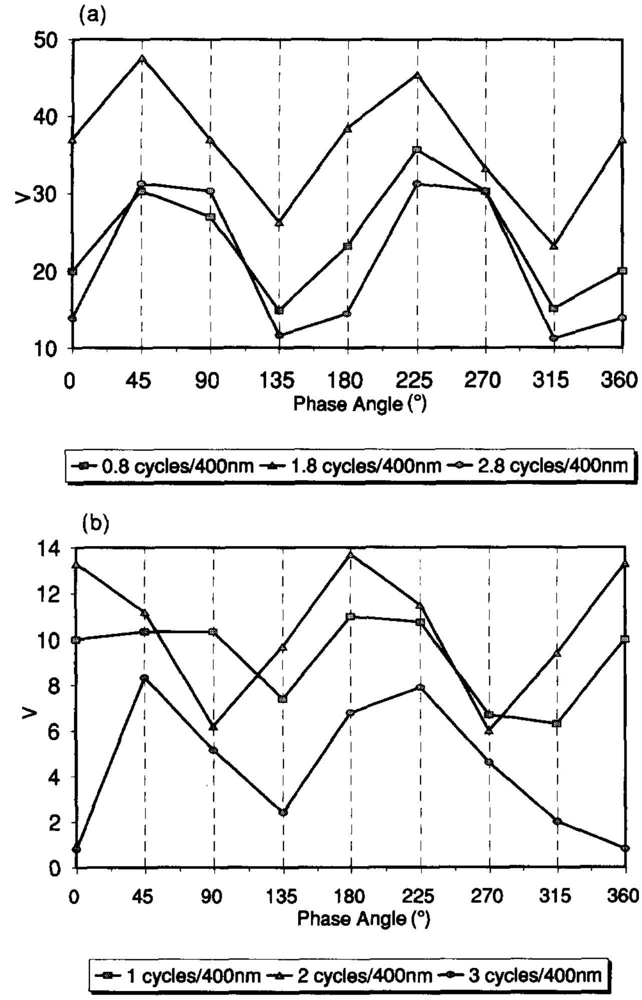

FIGURE 6. Curves showing the change of sensitivity with phase; here for (a) 3 and (b) 10 MacAdam units. The chromatic frequencies analysed included those of the maxima for optimum phase curves.

our discrimination criterion, the use of pure harmonic colour signals and the greater range of frequencies studied, our results lead to a broad and detailed characterization of the human colour-vision system in the chromatic-frequency domain.

Finally, Fig. 6 shows the change in sensitivity as a function of phase angle at different chromatic frequencies. The oscillatory behaviour of all the curves analysed is very similar whatever the tolerance units adopted, although this is not the case with the minimum peaks. Due to the significance of the initial phase (Buschbaum & Gottschalk, 1984), a study of the CCSF in dichromatic observers and the locations of these minima may prove useful in providing information about discrimination between colour anomalies. At the moment, however, we have no experimental measurements to confirm this hypothesis.

It should be pointed out here that the results of our analysis do not coincide with those of Barlow (1982), who attempts to demonstrate that the Fourier-transform module of the cone-action spectra decays in such a way that, according to the assumptions of the sampling theorem, three different photoreceptors are enough to obtain all the information from the corresponding colour signal.

Taking the photoreceptors to be elements which sample colour signals according to traditional assumptions goes against the fact that their spectral sensitivity is limited to a certain bandwidth, which, moreover, is different for each of them, and also the fact that the sensitivity maxima are unequally spaced within the visible spectrum. Furthermore, the spectral sensitivity of a photoreceptor can never be a strictly limited-band function, as it takes on values other than zero in only one interval of the electromagnetic spectrum.

Maloney's (1986) statement that there should be three different photoreceptors in colour vision is not incompatible with the fact that their spectral sensitivities show limiting frequencies over 0.01 or 0.02 c/nm (Barlow, 1982; Romero, Jiménez del Barco & Hita, 1992). In fact, if we consider each photoreceptor separately the colour signal will be filtered, according to the sensitivity characteristics of that photoreceptor. In the first step, we carry out the following operation

$$I(\lambda) = K(\lambda)O(\lambda) \quad (6)$$

where $O(\lambda)$ is the colour signal and $K(\lambda)$ the spectral sensitivity of the photoreceptor.

In the chromatic-frequency space, equation (6) is converted into a convolution product, whereupon the Fourier transform of the photoreceptor's spectral sensitivity cannot be directly associated with a transfer function of the type used in linear image-formation systems. In addition, in a second step, the visual system integrates the exit signal $I(\lambda)$, giving a particular value for each photoreceptor.

Our treatment of the problem implies considering the final response of the receptor mechanisms described and also the subsequent transformations of the signals generated in the photoreceptors, including the discrimination mechanisms. This has meant our obtaining a psychophysical transfer function to describe the processing of colour information by the human colour-vision system, in terms of the chromatic-frequency content of the visual stimulus.

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