KK Monopoles as Giant Gravitons in $AdS_5 \times S^5$

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References: in preparation

Outlook

1. Introduction:
    - Giant gravitons as p-branes
    - Giant gravitons as gravitational waves

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2. A new giant graviton in $AdS_5 \times S^5$
   - Gravitational waves expanding into S^5

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   - The giant graviton solution
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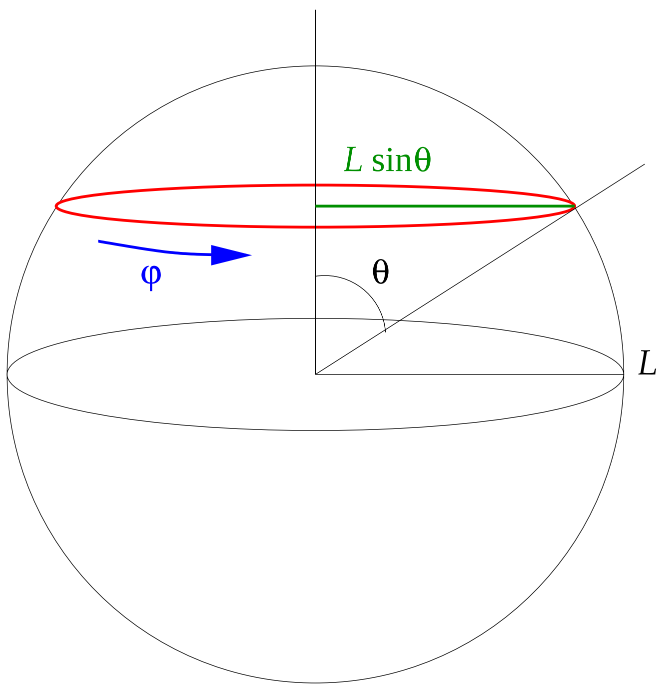
   - Generalisation to Sasaki-Einstein Manifolds
4. A possible CFT interpretation

1 Introduction to giant gravitons

1.1 Giant gravitons as p-branes

$(n-2)$-brane in $AdS_m \times S^n$, wrapped around $S^{(n-2)}$ and moving in ϕ:

$$ds^2 = ds^2_{AdS} + L^2\Big(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\Omega^2_{(n-2)}\Big)$$



$$E = \frac{P_\phi}{L}\sqrt{1 + \tan^2\theta\left(1 - \frac{\tilde{N}}{P_\phi}\sin^{n-3}\theta\right)^2}$$

Minima: $\sin\theta = 0$: Pointlike graviton

$\sin\theta = (\frac{P_\phi}{\tilde{N}})^{\frac{1}{n-3}}$: Giant graviton

Energy: $E(\text{minimum}) = \frac{P_\phi}{L}$

[McGreevy, Susskind, Toumbas]

$\longrightarrow$ Realisation of Stringy Exclusion Principle

1.2 Giant gravitons as gravitational waves

Abelian picture: $(n-2)$-brane with momentum in worldvolume

Non-Abelian picture: Gravitational waves expanding into fuzzy $(n-2)$-brane due to dielectric effect [McGreevy, Susskind, Toumbas] [BMN] ...

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\begin{aligned}
S &= -T_W \int d\tau \, \mathrm{STr}\left\{ k^{-1}\sqrt{-P[E_{00} + E_{0i}(Q^{-1}-\delta)^i_k E^{kj} E_{j0}] \ \det Q} \right\} \\
&+ T_W \int d\tau \, \mathrm{STr}\left\{ -P[k^{-2}k^{(1)}] + iP[(\mathrm{i}_X \mathrm{i}_X)C^{(3)}] + \tfrac{1}{2}P[(\mathrm{i}_X\mathrm{i}_X)^2 \mathrm{i}_k C^{(6)}] + \cdots \right\}
\end{aligned}
$$

$$
E_{\mu\nu} = g_{\mu\nu} - k^{-2}k_\mu k_\nu + k^{-1}(\mathrm{i}_k C^{(3)})_{\mu\nu}, \qquad Q^i_j = \delta^i_j + ik[X^i, X^k]E_{kj}
$$

[B.J., Lozano]

- Abelian wave action for $X^i \sim \mathbb{1}$
- Myers' action for multiple D0-branes after dim reduction
- First order: Matrix string theory in infinite momentum frame

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- Abelian wave action for $X^i \sim \mathbb{1}$
- Myers' action for multiple D0-branes after dim reduction
- First order: Matrix string theory in infinite momentum frame
- Gauged sigma model in k^μ (= propagation direction of waves): $\nexists$ dynamical embedding scalar for this direction

$\underline{AdS_7 \times S^4}$: $W \longrightarrow M2$ as fuzzy S^2

$$X^i = \frac{L \sin\theta}{\sqrt{N^2-1}} J^i, \quad \text{with } [J^i, J^j] = 2i\epsilon^{ijk} J^k$$

$$\Longrightarrow \quad \sum_i X^i X^i = L^2 \sin^2\theta \, \mathbb{1}$$

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$$E = \frac{T_W}{L\cos\theta} \mathrm{STr} \left\{ \sqrt{\mathbb{1} - \frac{4L\sin\theta}{\sqrt{N^2-1}} X^2 + \frac{4L^4\sin^2\theta\cos^2\theta}{N^2-1} X^2 + \frac{4L^2\sin^2\theta}{N^2-1} X^2 X^2} \right\}$$

$$= \frac{NT_W}{L} \sqrt{1 + \tan^2\theta \left(1 - \frac{2L^3}{\sqrt{N^2-1}} \sin\theta\right)^2} + \mathcal{O}(N^{-5})$$

$$\left[E = \frac{P_\phi}{L} \sqrt{1 + \tan^2\theta \left(1 - \frac{2L^3}{N} \sin\theta\right)^2} \right]$$

$\longrightarrow$ Agreement for large N, upon identification $P_\phi = NT_W$

[B.J., Lozano]

$\underline{AdS_5 \times S^5}$: $W \longrightarrow D3$ as fuzzy S^3 = $U(1)$ fibre bundle over fuzzy S^2

Type IIB wave action: extra isometry due to T-duality

$$E_{\mu\nu} = g_{\mu\nu} - k^{-2}k_\mu k_\nu - \ell^{-2}\ell_\mu \ell_\nu - k^{-1}\ell^{-1}e^{\Phi}(\mathbf{i}_k \mathbf{i}_\ell C^{(4)})_{\mu\nu},$$

$$Q^i_j = \delta^i_j + i[X^i, X^k]e^{-\Phi}k\ell E_{kj}.$$

$\longrightarrow$ Identify isometry direction with fibre direction of S^3

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$\underline{AdS_4 \times S^7}$: $W \longrightarrow M5$ as fuzzy S^5 = $U(1)$ fibre bundle over fuzzy CP^2

Technical reasons: Add momentum in fibre direction

Macroscopically: $S = -T_5 \int d^5\xi \Big\{ -P[i_k C^{(6)}] - \frac{1}{2} P[k^{-2}k^{(1)}] \wedge F \wedge F \Big\}$

with $\int_{CP^2} F \wedge F = 8\pi^2 N$

$$\Longrightarrow S \sim -T_5 \int d\tau P[k^{-2}k^{(1)}] \int_{CP^2} F \wedge F = N T_0 \int d\tau P[k^{-2}k^{(1)}]$$

[B.J., Lozano, Rodríguez-Gómez]

2 A new giant graviton in $AdS_5 \times S^5$

$$ds^5 = -(1+\frac{r^2}{L^2})dt^2 + (1+\frac{r^2}{L^2})^{-1}dr^2 + \frac{r^2}{4}\Big[d\Omega_2^2 + (d\chi + A)^2\Big]$$
$$+L^2\Big[d\Sigma_4^2 + (d\psi + B)^2\Big]$$

with

$$A = \cos\chi_1 d\chi_2 \qquad B = \frac{1}{2}\sin^2\varphi_1(d\varphi_4 + \cos\varphi_2 d\varphi_3)$$

$$d\Sigma_4^2 = d\varphi_1^2 + \frac{1}{4}\sin^2\varphi_1\Big(d\varphi_2^2 + \sin^2\varphi_2 d\varphi_3^2 + \cos^2\varphi_1\big(d\varphi_4 + \cos\varphi_2 d\varphi_3\big)^2\Big)$$

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- χ and ψ global isometry directions
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$\longrightarrow$ waves expand into S^5 via fuzzy CP^2?

$$S_W = -T_W \int d\tau \, \mathrm{STr}\Big\{ k^{-1} \sqrt{-P\Big[E_{\mu\nu} - E_{\mu i}(Q^{-1} - \delta)^i{}_j E^{jk} E_{k\nu}\Big] \det Q} \;\Big\}$$

$$+ T_W \int d\tau \, \mathrm{STr}\Big\{ -P[k^{-2} k^{(1)}] - iP[(\mathbf{i}_X \mathbf{i}_X)\mathbf{i}_\ell C^{(4)}] - \tfrac{1}{2} P[(\mathbf{i}_X \mathbf{i}_X)^2 \mathbf{i}_\ell \mathbf{i}_k N^{(7)}] + \ldots \Big\}$$

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$$Q^\mu{}_\nu = \delta^\mu_\nu + ik\ell \, [X^\mu, X^\rho] E_{\rho\nu} \qquad ((\mathbf{i}_X \mathbf{i}_X)\mathbf{i}_\ell C_4)_\lambda = [X^\rho, X^\nu]\ell^\mu C_{\mu\nu\rho\lambda}$$

$$(\mathbf{i}_\ell \mathbf{i}_k \Omega)_{\mu_1 \ldots \mu_n} = \ell^\rho k^\nu \Omega_{\nu\rho\mu_1\ldots\mu_n}$$

- double gauged sigma model:

  $\nexists$ dynamical embedding scalars for k^μ and ℓ^μ

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  $d(N^{(7)}) = {}^*d(\ell^{(1)})$
- Choose: $k^\mu = \delta^\mu_\psi$ $\quad \ell^\mu = \delta^\mu_{\chi 3}$ $\quad X^i \to$ fuzzy CP^2

CP^2 is coset manifold $SU(3)/U(2)$ or embedded in $\mathbb{R}^8$ via

$$\sum_{i=1}^{8} x^i x^i = 1 \qquad\qquad \sum_{j,k=1}^{8} d^{ijk} x^j x^k = \frac{1}{\sqrt{3}} x^i$$

Fuzzy CP^2 generated by $SU(3)$ generators J^i in (anti-)fundamental repres

$$X^i = \frac{J^i}{\sqrt{(2n^2-2)/3}} \qquad\qquad [X^i, X^j] = \frac{i f^{ijk}}{\sqrt{(2n^2-2)/3}} X^k$$

[Alexanian, Balachandran, Immirzi, Ydri]

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$$E(r) = \frac{N T_W}{L} \sqrt{1 + \frac{r^2}{L^2}} \ \Big[1 + \frac{L^6 r^2}{32(N-1)}\Big] \ + \ \mathcal{O}(N^{-5})$$

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$$\Longrightarrow H(0) = \frac{N T_W}{L} = \frac{P_\psi}{L}$$

(topological) giant graviton!

- Fuzzy brane with finite extension and massless dispersion relation
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Fuzzy CP^2: $(\mathrm{i}_X \mathrm{i}_X)^2 \neq 0$

$\longrightarrow$ KK-monopole with Taub-NUT in ℓ^μ and wrapped on k^μ

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- What is field theory dual on CFT side?

3 KK monopoles as giant gravitons

3.1 Effective actions for KK monopoles

Kaluza-Klein monopole:

$$ds^2 = \eta_{ab} dx^a dx^b + H^{-1}(dz + A_i dy^i)^2 + H \delta_{ij} dy^i dy^j$$

$$\text{with} \qquad \partial_i \partial^i H = 0 \qquad 2\partial_{[i} A_{j]} = \epsilon_{ijk} \partial_k H$$

[Sorkin] [Gross, Perry]

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Effective action:

Taub-NUT direction z is isometry: not dynamical embedding scalar!

$\longrightarrow$ gauged sigma model:

$$\mathcal{D}_a X^\mu = \partial_a X^\mu - \ell^{-2} \ell_\nu \partial_a X^\nu \ell^\mu \qquad \text{with} \quad \ell^\mu = \delta^\mu_z$$

$\longrightarrow$ effective metric:

$$P[g] \;=\; g_{\mu\nu} \mathcal{D}_a X^\mu \mathcal{D}_b X^\mu \;=\; \Big(g_{\mu\nu} - \ell^{-2} \ell_\mu \ell_\nu \Big) \partial_a X^\mu \partial_b X^\nu$$

[Bergshoeff, B.J., Ortín]

Intermezzo: Gauged sigma models

Eliminate embedding scalars in presence of isometry

$\longrightarrow$ no dymanical degree of freedom

E.g. Effective actions for gravitational wave from T-duality

$$S_{F1} = -T_1 \int d^2\sigma \, \sqrt{|\det(\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu})|}$$

Orientate strings as $X^\mu(\tau, \sigma) = (X^a(\tau), X^9(\sigma)) = (X^a(\tau),\ m\sigma)$

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Apply T-duality along X^9 $\longrightarrow$ wave propagating along X^9

$$S_W = \ -mT_0 \int d\tau \left\{ k^{-1}\sqrt{|\partial X^\mu \partial X^\nu\, \mathcal{G}_{\mu\nu}|} \ + \ k^{-2} k_a \partial X^a \right\}$$

with: $\mathcal{G}_{\mu\nu} = g_{\mu\nu} - k^{-2} k_\mu k_\nu$

$k^\mu = \delta^\mu_9$ is Killing vector along X^9

$$\longrightarrow \quad k_\mu = g_{\mu x}, \qquad\qquad k^2 = k_x = g_{xx}$$

X^9 is no dymanical degree of freedom (since Killing)

$$\begin{aligned} k^\mu \mathcal{G}_{\mu\nu} &= k^\mu g_{\mu\nu} - k^\mu k^{-2} k_\mu k_\nu \\ &= k_\nu - k_\nu = 0 \end{aligned}$$

$\partial X^\mu \partial X^\nu \, \mathcal{G}_{\mu\nu}$ has only contributions for $X^\mu \neq X^9$

Wave action S has only contributions for $X^\mu \neq X^9$

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Equivalently:

$$S_W = -mT_0 \int d\tau \, \Big\{ k^{-1} \sqrt{|\mathcal{D}X^\mu \mathcal{D}X^\nu \, g_{\mu\nu}|} + A - \partial X^9 \Big\}.$$

with $\mathcal{D}X^\mu \equiv \partial X^\mu - A k^\mu \equiv \partial X^\mu - k^{-2} k_\rho \partial X^\rho k^\mu$

S_W invariant under

$$\delta X^\mu = \Lambda(\tau) k^\mu \qquad \qquad \delta A = \partial \Lambda(\tau)$$

$\longrightarrow$ X^9 is pure gauge, not a dymanical degree of freedom

Type IIB KK monopole: $(2,0)$ tensor multiplet in 6 dim

$\longrightarrow$ field content: 3 embedding scalars X^i

2 worldvolume scalars ω, $\tilde{\omega}$

selfdual 2-form W^+_{ab}

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$\longrightarrow$ field content: 3 embedding scalars X^i

2 worldvolume scalars $\omega, \tilde{\omega}$

selfdual 2-form W^+_{ab}

KK wrapped around S^5 in $AdS_5 \times S^5$ with Taub-NUT in $\ell^\mu = \delta^\mu_\chi$

$U(1)$ fibre direction in KK worldvolume $\longrightarrow$ compactify over $k^\mu = \delta^\mu_\psi$

[Eyras, B.J., Lozano]

Type IIB KK monopole: $(2,0)$ tensor multiplet in 6 dim

$\longrightarrow$ field content: 3 embedding scalars X^i
2 worldvolume scalars $\omega, \tilde{\omega}$
selfdual 2-form W^+_{ab}

KK wrapped around S^5 in $AdS_5 \times S^5$ with Taub-NUT in $\ell^\mu = \delta^\mu_\chi$

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[Eyras, B.J., Lozano]

wrapped KK monopole: (1,1) vector multiplet in 5-dim

$\longrightarrow$ field content: 3 embedding scalars X^i
2 worldvolume scalars $\omega, \tilde{\omega}$ ($\longrightarrow$ truncate here)
1 vector V_a

double gauged sigma model:

$$\mathcal{D}_a X^\mu = \partial_a X^\mu - \ell^{-2}\ell_\nu \partial_a X^\nu \ell^\mu - k^{-2} k_\nu \partial_a X^\nu k^\mu$$

$\longrightarrow$ effective worldvolume: $\mathbb{R} \times CP^2$

3.2 The giant graviton solution

$$S = -T_4 \int d^5\sigma\, e^{-2\phi} k \ell^2 \sqrt{|\det(P[g]_{ab} + e^{\phi} k^{-1} \ell^{-1} \mathcal{F}_{ab})|}$$

$$-T_4 \int d^5\sigma \Big\{ P[\mathbf{i}_\ell \mathbf{i}_k N^{(7)}] - \frac{1}{2} P\Big[\frac{k^\mu}{k^2}\Big] \wedge \mathcal{F} \wedge \mathcal{F} + \ldots \Big\}$$

where

$$P[g] = g_{\mu\nu} \mathcal{D}_a X^\mu \mathcal{D}_b X^\mu = \Big(g_{\mu\nu} - \ell^{-2} \ell_\mu \ell_\nu - k^{-2} k_\mu k_\nu\Big) \partial_a X^\mu \partial_b X^\nu$$

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$$dB = {}^\star(dB), \qquad dB \wedge dB \sim \sqrt{g_{CP^2}} \sim \sqrt{g_{S^5}}$$

$$\Longrightarrow \int_{CP^2} \mathcal{F} \wedge \mathcal{F} = 8\pi^2 n^2 : \quad \text{non-trivial instanton number on } CP^2$$

[Trautman]

Chern-Simons:

$$\frac{T_4}{2}\int_{\mathbb{R}\times CP^2} P\left[k^{-2}k^{\mu}\right]\wedge F\wedge F \;=\; n^2 T_W \int dt\; P\left[k^{-2}k^{\mu}\right]$$

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$$E(r) \;=\; \frac{NT_W}{L}\sqrt{1+\frac{r^2}{L^2}}\;\Big[1+\frac{L^6r^2}{32(N-1)}\Big] + \mathcal{O}(N^{-5})$$

3.3 Generalization to Sasaki-Einstein manifolds

- Y is Sasaki-Einstein manifold $\Longleftrightarrow$ $\mathcal{M} = \mathcal{C}(Y)$ is Kähler & Ricci-flat

  with $ds^2_{\mathcal{M}} = dr^2 + r^2 ds^2_Y$

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$$ds^2_Y = d\Sigma^2_4 + (d\psi + B)^2$$

with: $d\Sigma^2_4$ metric on M_4

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- AdS/CFT conjecture:

  strings in $AdS_5 \times Y_5$ dual to $\mathcal{N} = 1$ CFT on boundary of AdS

$AdS_5 \times Y_5$:

$$ds^5 = -(1+\frac{r^2}{L^2})dt^2 + (1+\frac{r^2}{L^2})^{-1}dr^2 + \frac{r^2}{4}\Big[d\Omega_2^2 + (d\chi + A)^2\Big]$$
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Wrapped monopole with

- $\ell^\mu = \delta^\mu_\chi$ in Taub-NUT
- $k^\mu = \delta^\mu_\psi$ as extra isometry
- $\mathcal{F}$ such that

$$\int_M \mathcal{F} \wedge \mathcal{F} = 8\pi^2 n^2$$

NB: always possible thanks to $S^2 \times S^2$ topology of M_4

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$$\Longrightarrow \det(g + \mathcal{F}) \text{ is perfect square}$$

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\begin{aligned}
S &= -T_4 \int d^5\sigma \, e^{-2\phi} k \ell^2 \sqrt{|\det(P[g]_{ab} + e^{\phi} k^{-1} \ell^{-1} \mathcal{F}_{ab})|} \\
&= T_4 \int dt \, d\Sigma_4 \, k \ell^2 \sqrt{\left(1 + \frac{r^2}{L^2}\right) \left[L^4 + \frac{8n^2}{k^2 \ell^2}\right]^2 |g_\Sigma|} \\
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Microscopically: Gravitational waves expanding into fuzzy M_4...

4 A field theory interpretation?

S^3 fibre χ is Taub-NUT direction of KK-monopole

$\longrightarrow \quad AdS_5$ in global coordinates

$\longrightarrow \quad \mathcal{N} = 4 \;\; SU(K)$ SYM on $\;\; S^3 \xrightarrow{S^1} S^2$

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$\mathcal{N}=4$ scalar action:

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with Φ_a in adjoint repres of $SU(K)$

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with Φ_a in adjoint repres of $SU(K)$

in fundamental repres of $SU(3)$ (explicit R-symmetry)

independent of ψ (Taub-NUT in gravity dual)

$\longrightarrow \Phi_a$ live on S^2

$$S \sim \int dt\, d\Omega_2\, \mathrm{Tr}\Big\{-\partial_t \Phi_a^* \partial_t \Phi_a \;-\; 4\Phi_a^*\nabla^2(S^2)\Phi_a \;+\; \frac{1}{L^2}\Phi_a^*\Phi_a \;+\; \frac{1}{4g^2}[\Phi_a, \Phi_b^*]^2\Big\}$$

Ansatz:

$$\Phi_a = e^{if(t)} \otimes \mathcal{M}_a \otimes J_a$$

with: $f(t)$ arbitrary function of time

J_a generators of $SU(2)$: constant on S^2

$\mathcal{M}_a = \mathrm{diag}(v^a, -\frac{v^a}{K-1}, \ldots, -\frac{v^a}{K-1})$: $SU(K)$ gauge dependence

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Hamiltonian:

$$H = \frac{p^2}{2v^2} + \frac{v^2}{2L^2} \qquad \text{with } p = \frac{\partial S}{\partial f'} = v^2 f'$$

Minimum for $v_0^2 = pL$

$$\Longrightarrow \quad H(v_0^2) = \frac{p}{L}$$

Massless particle!

$\mathcal{N} = 4$ scalar action with $SU(3)$ R-symmtry

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- Scalar sector with R-symmetry group G_R
- Φ_a independent of fibre and constant in S^2

$$\Phi_a = e^{if(t)} \otimes \mathcal{M}_a \otimes J_a$$

- Remaining R-symmetry $\;\; G_R/U(2) \;\simeq M_4 \;$ = worldvol of wrapped KK

Explicit example: $AdS_5 \times S^3 \times S^2$

Gravity side:

wrapped KK-monopoles on $S^2 \times S^2$

Gauge theory side:

scalars with $SU(2) \times SU(2) \times U(1)$ R-symmetry

Ansatz $\longrightarrow$ remaining symmetry

$$\frac{SU(2) \times SU(2) \times U(1)}{U(2)} \sim \frac{SU(2) \times SU(2)}{SU(2)}$$

$\longrightarrow$ diagonal $SU(2)$: same winding number over each S^2!

5 Conclusions

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  $\longrightarrow$ add momentum by $\int P[k^{-2}k^\mu] \wedge \mathcal{F} \wedge \mathcal{F}$ with $\int \mathcal{F} \wedge \mathcal{F} = 8\pi n^2$

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- Field Theory: scalars $\Phi_a = e^{if(t)} \otimes \mathcal{M}_a \otimes J_a$

  $\longrightarrow$ preserve $SU(3)/U(2) \simeq CP^2$

- Easily generalisable to $AdS_5 \times Y_5$ with Y_5 Sasaki-Einstein