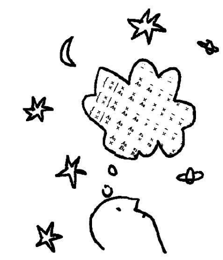

The baryon vertex with magnetic flux

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In collaboration with: Y. Lozano (U. Oviedo) and D. Rodríguez Gómez (U. Oviedo & U.A.M)

References: in preparation.

Outlook

1. Introduction
    - Brief review of AdS/CFT
    - Witten's baryon vertex

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2. The baryon vertex with magnetic flux
   - Construction
   - Stability
   - Supersymmetry

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   - Supersymmetry
3. Microscopic description
   - Brief review of the dielectric effect
   - Microscopic description in terms of D1-strings
   - F1's in the microscopic description
   - Comment on S-duality
4. Conclusions

1 Introduction

1.1 Brief review of AdS/CFT

Explicit example of gauge/gravity correspondence: [Maldacena]

Type IIB Supergravity (strings) on $AdS_5 \times S^5$
$\sim \mathcal{N} = 4$ Super Yang-Mills with gauge group $SU(N)$

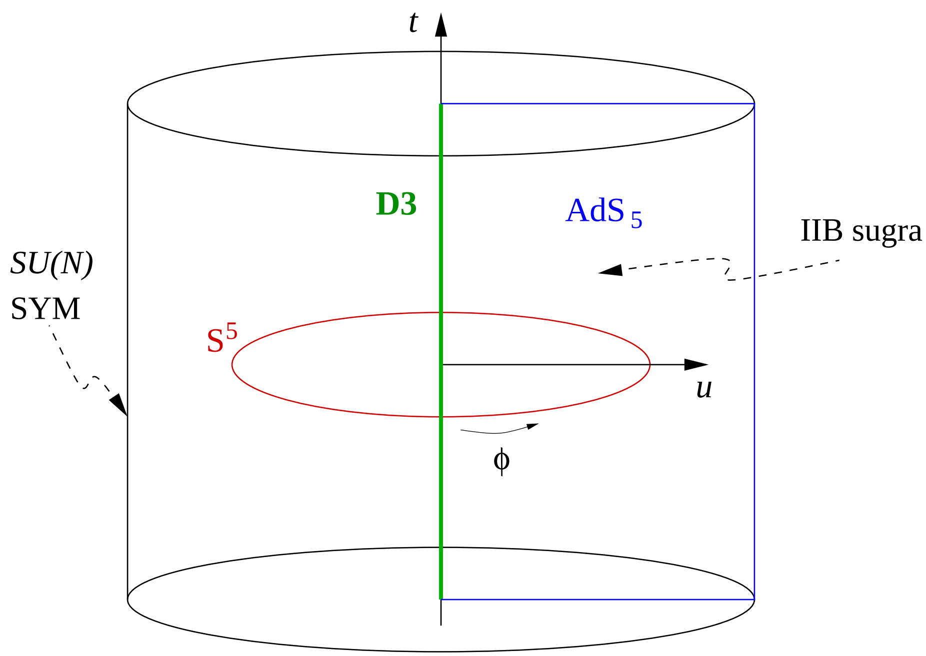
1 Introduction

1.1 Brief review of AdS/CFT

Explicit example of gauge/gravity correspondence: [Maldacena]

Type IIB Supergravity (strings) on $AdS_5 \times S^5$
$\sim \ \mathcal{N} = 4$ Super Yang-Mills with gauge group $SU(N)$

$$ds^2 = \frac{u^2}{L^2}\eta_{ab}dx^a dx^b \ + \ \frac{L^2}{u^2}du^2 \ + \ L^2 d\Omega_5^2, \qquad G_5 = 4L^{-1}\sqrt{|g_{AdS}|} + 4L^4\sqrt{|g_S|}$$



$AdS_5 \times S^5$: near horizon limit of N D3-branes
$\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills: Gauge theory on D3 worldvolume

$\longrightarrow$ $D = 4$ gauge theory captures holographically $D = 10$ gravity

$AdS_5 \times S^5$: near horizon limit of N D3-branes
$\mathcal{N} = 4\ SU(N)$ Super Yang-Mills: Gauge theory on D3 worldvolume

$\longrightarrow$ $D = 4$ gauge theory captures holographically $D = 10$ gravity

$\longrightarrow$ Dictonary between $D = 4$ gauge theory and $D = 10$ gravity:

- $N = \int_{S^5} F_5$: $\sharp$ of branes = rank of SYM gauge group
- Coupling constants: $g^2_{YM} N \ = \ 4\pi g_s N \ = \ (L/\ell_s)^4$
- $SO(4,2) \times SO(6)$ isometry group of $AdS_5 \times S^5$
  $\sim \ SO(4,2) \times SU(4)_R$ conformal group of $\mathcal{N} = 4$ SYM
- Fields in AdS_5 $\sim$ operators in SYM
- Quark (q) in SYM $\sim$ string stretched between boundary and horizon

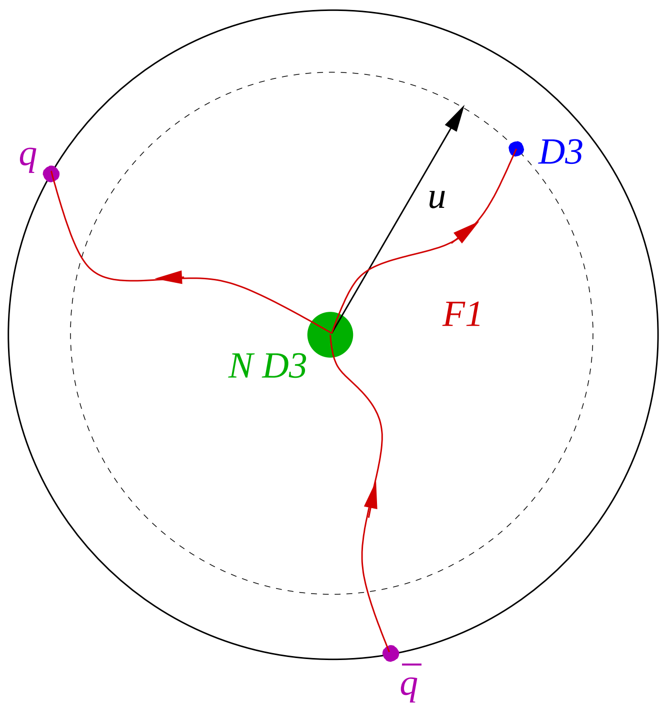
Probe D3-brane at radial distance u

→ massive string between D3 and horizon with $m \sim u$ in vector representation of $U(N)$

→ massive quark in gauge theory

→ $u \sim \infty \;\Leftrightarrow\; m \sim \infty \;\Leftrightarrow$ non-dynamical quark of SYM

$\sim$ F1 between horizon and boundary

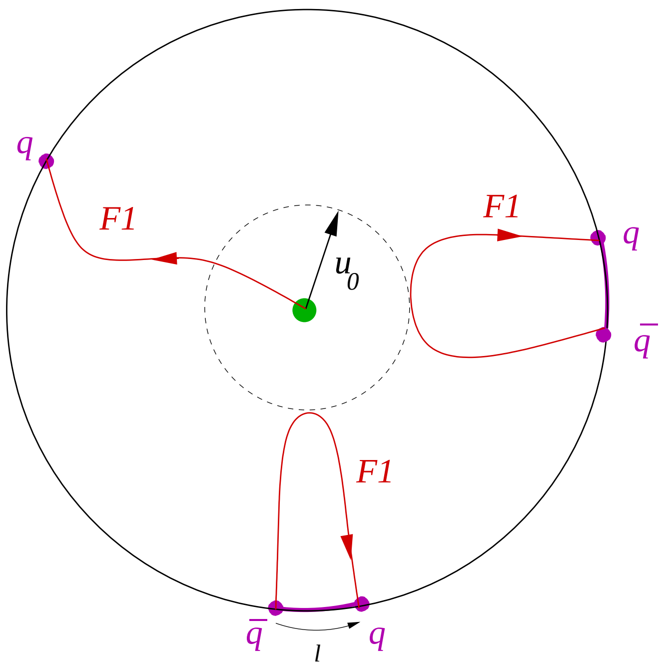


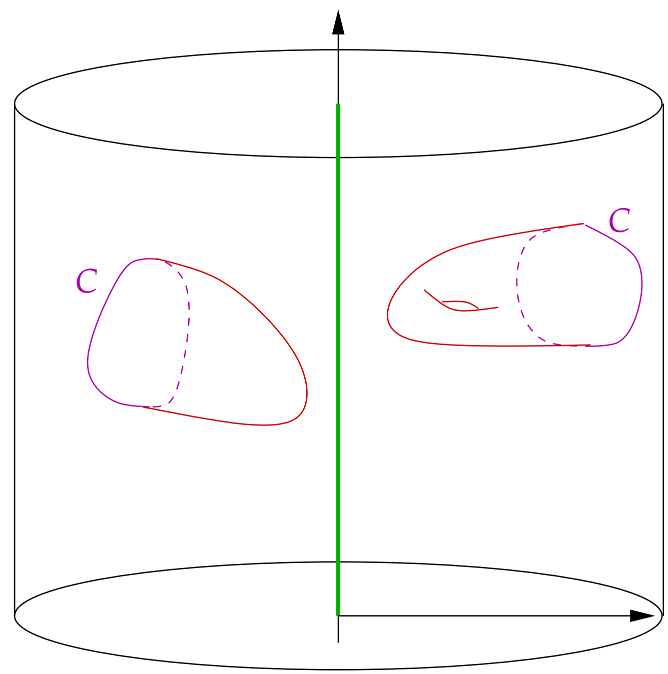
Quark physics in AdS

non-dynamical quark of SYM $\sim$ F1 between horizon and boundary

$q\bar{q}$-pair in SYM $\sim$ string "hanging" from boundary

Wilson line $\mathcal{C}$ in SYM $\sim$ string worldsheet ending on $\mathcal{C}$ [Maldacena]





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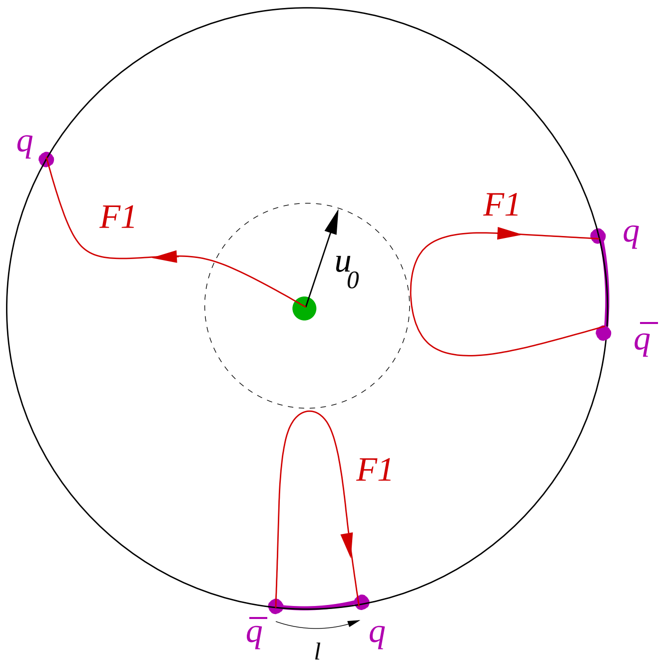
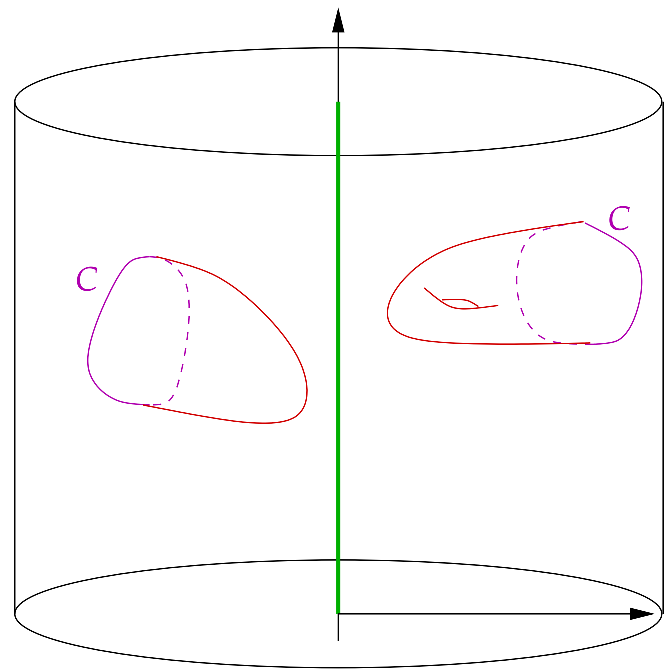




Solution:

$$x = \frac{L}{u_o}\int_1^{u/u_0}\frac{dz}{z^2\sqrt{z^2-1}} \qquad E = \frac{u_0}{\pi}\int_1^{\infty} dz\Big(\frac{z^2}{\sqrt{z^4-1}}-1\Big)-\frac{u_0}{\pi} \sim \frac{\sqrt{g_{YM}^2 N}}{\ell}$$

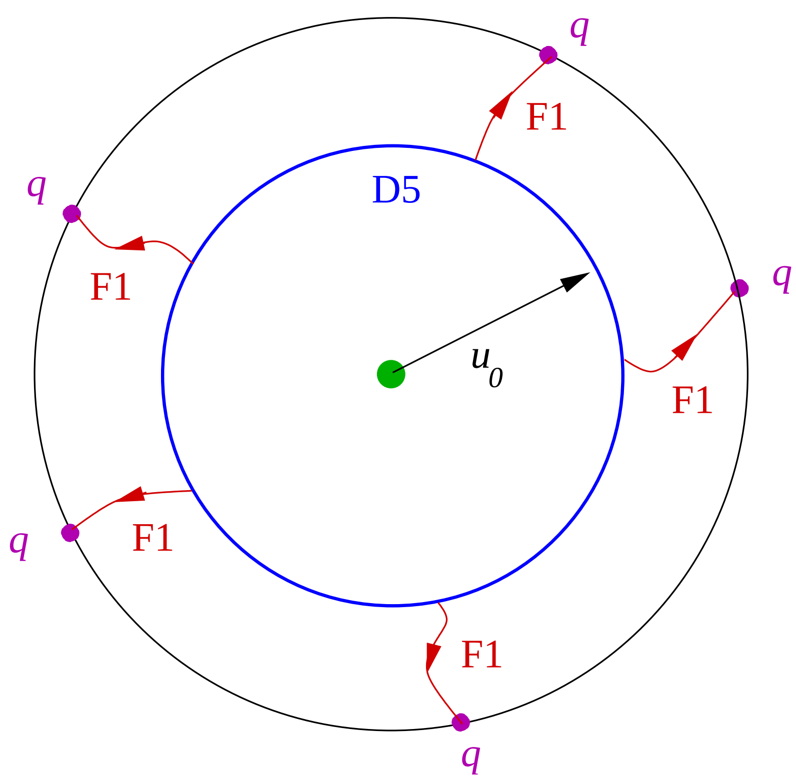
1.2 Witten's baryon vertex

$q\bar{q}$-pair (meson) in SYM $\sim$ string "hanging" from boundary

Does there exist a baryon configuration?

= colourless antisymmetric bound state of N quarks

$\rightarrow$ D5-brane wrapped around S^5 with N strings extending to boundary



Consider a D5-brane wrapped around S^5 at fixed point u_0 in AdS [Witten]

$$S_{CS} = -T_5 \int_{\mathbb{R}\times S^5} P[C^{(4)}] \wedge F$$

$$= T_5 \int_{\mathbb{R}\times S^5} P[G^{(5)}] \wedge A$$

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$$= T_5 \int_{S^5} P[G^{(5)}] \int_{\mathbb{R}} dt A_t$$

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Background: $\int_{S^5} G^{(5)} = 4\pi^2 N$

$$= N T_1 \int dt A_t,$$

$\rightarrow$ N units of string charge induced on D5 worldvolume

Consistency of Ansatz $A = A_t(t)dt$:

$$S = S_{DBI} + NT_1 \int dt A_t$$

Equation of motion:

$$0 \equiv \frac{\partial \mathcal{L}}{\partial A_t} = NT_1.$$

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However, we can add N fundamental strings, ending on D5:

$$\begin{aligned} S_{\text{total}} &= S_{DBI} + NT_1 \int dt A_t \;+\; NS_{F1} - NT_1 \int dt A_t \\ &= S_{DBI} \;+\; NS_{F1} \end{aligned}$$

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→ N F1 with same orientation (all q's), stretched between D5 and boundary

→ Configuration is antisymmetric under interchange of any two F1's

$\Longrightarrow$ Baryon vertex

[Witten]

2 The baryon vertex with magnetic flux

2.1 Construction

Baryon vertex = D5-brane wrapped around S^5

S^5 is $U(1)$ fibre bundle over CP^2

$$d\Omega_5^2 = (d\chi - B)^2 + ds^2_{CP^2},$$

$$B = -\frac{1}{2}\sin^2\varphi_1(d\varphi_4 + \cos\varphi_2 d\varphi_3),$$

$$ds^2_{CP^2} = d\varphi_1^2 + \frac{1}{4}\sin^2\varphi_1\Big(d\varphi_2^2 + \sin^2\varphi_2 d\varphi_3^2 + \cos^2\varphi_1\big(d\varphi_4 + \cos\varphi_2 d\varphi_3\big)^2\Big)$$

Fibre connection B satisfies

$$dB = {}^\star(dB), \qquad\qquad dB\wedge dB \;\sim\; \sqrt{g_{CP^2}} \;\sim\; \sqrt{g_{S^5}}$$

$\rightarrow$ B non-trivial gauge field on CP^2, with non-zero instanton number

Turn on magnetic Born-Infeld flux

$$F = \sqrt{2n}\, dB$$

$$\Rightarrow \int_{CP^2} F \wedge F = 8\pi^2 n.$$

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F is magnetic $\Rightarrow$ no extra terms in Chern-Simons action

New contributions to Born-Infeld action

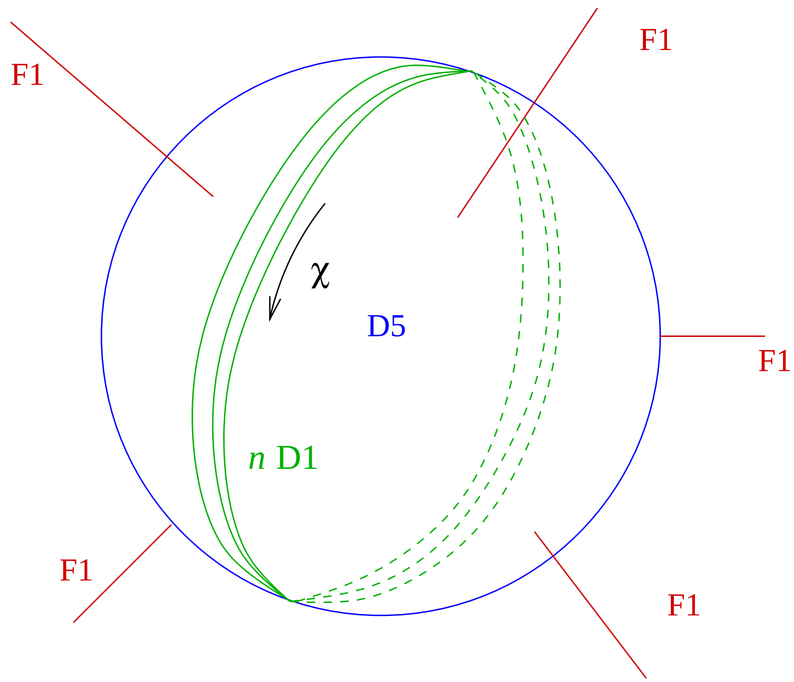
$$\begin{aligned} S_{DBI} &= -T_5 \int d^6\xi \, \frac{u}{L} \sqrt{\det\Big(g_{\alpha\beta} + F_{\alpha\beta}\Big)} \\ &= -T_5 \int d^6\xi \, u \, \sqrt{g_{S^5}} \, \Big(L^4 + 2F_{\alpha\beta}F^{\alpha\beta}\Big) \\ E &= 8\pi^3 T_5 \, u \left(n + \frac{L^4}{8}\right) \end{aligned}$$

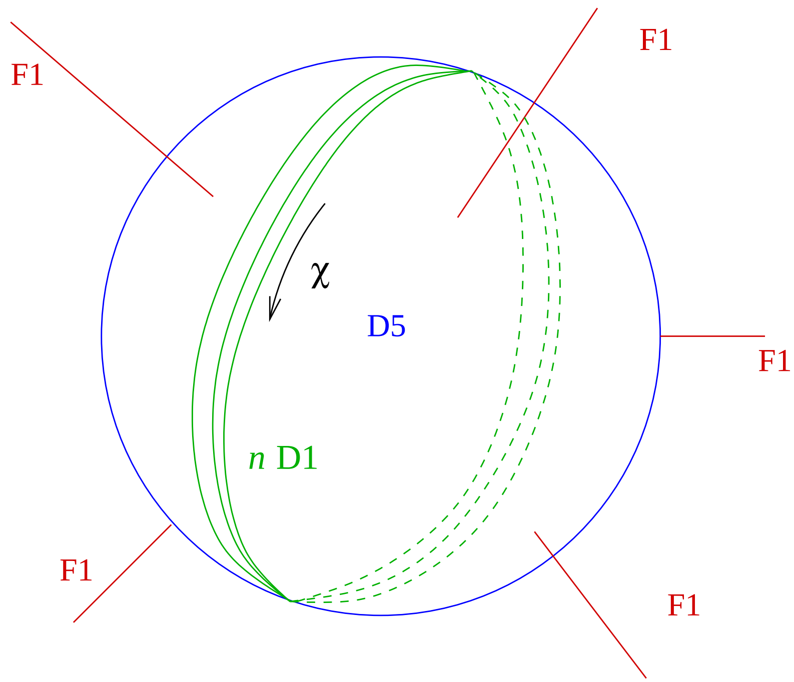
$F = \sqrt{2n}\, dB$ induces D1 charge in D5 worldvolume

$$
\begin{aligned}
S_{D5} &= \frac{1}{2} T_5 \int_{\mathbb{R}\times S^5} P[C^{(2)}] \wedge F \wedge F \\
&= nT_1 \int_{\mathbb{R}\times S^1} P[C^{(2)}] \\
&= n \;\; S_{D1}
\end{aligned}
$$

$\rightarrow n$ D1-branes: extended in t- and χ-directions
dissolved in D5 worldvolume

NB: n dissolved D-strings should not be confused
with the N baryon vertex F-strings!


F1
F1
χ
D5
F1
n D1
F1
F1

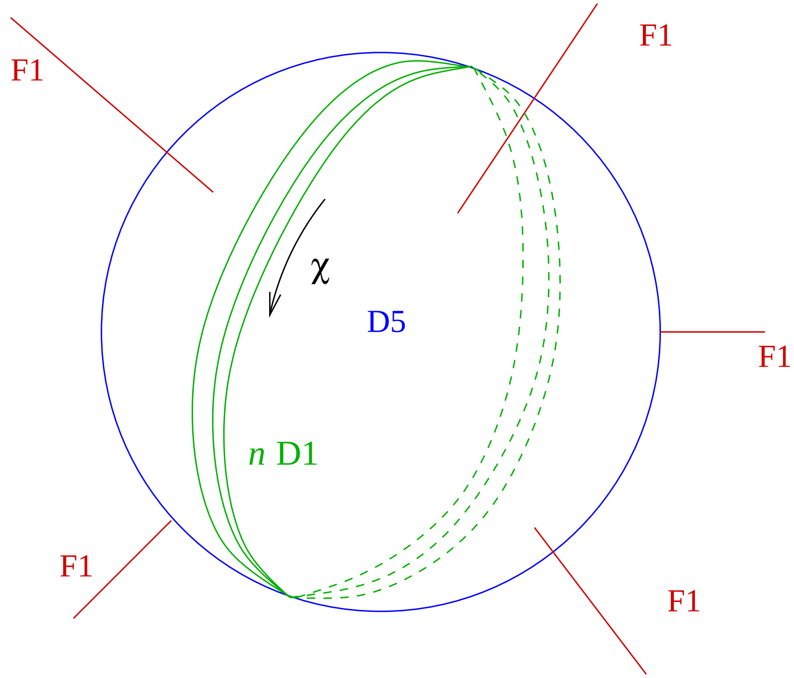


→ Alternative, microscopic description in terms of non-Abelian D1's?

Cfr Dielectric effect: [Emparan] [Myers]

Spherical D2-brane with dissolved D0-charge (Abelian)

∼ dielectric D0's expanding into fuzzy D2 (Non-Abelian)



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⟶ See section 3

2.2 Bound on n

baryon vertex with $n = 0$: stable under perturbations in x^i
stable under perturbations in u

$\rightarrow$ analysis of dynamics due to external F1's

[Brandhuber, Itzhaki, Sonnensshein, Yankielowicz]

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What is the influence of $n \neq 0$?

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baryon vertex with $n = 0$: stable under perturbations in x^i

stable under perturbations in u

$\longrightarrow$ analysis of dynamics due to external F1's

[Brandhuber, Itzhaki, Sonnensshein, Yankielowicz]

What is the influence of $n \neq 0$?

$$S = S_{D5} - NT_1 \int dtdx \sqrt{(u')^2 + \frac{u^4}{L^4}}$$

Bulk eqn:

$$\frac{u^4}{\sqrt{(u')^2 + \frac{u^4}{L^4}}} = \text{const}$$

Boundary eqn:

$$\frac{u_0'}{\sqrt{(u_0')^2 + \frac{u_0^4}{L^4}}} = \frac{\pi L^4}{4N}\left(1 + \frac{8n}{L^4}\right)$$

Equations combine into

$$\frac{u^4}{\sqrt{(u')^2 + \frac{u^4}{L^4}}} = \beta\, u_0^2 L^2$$

with

$$\beta^2 = 1 - \frac{1}{16}\Big(1 + \frac{8\pi n}{N}\Big)^2$$

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Observation:

u is real $\Longrightarrow$ β should be real

$\Longleftrightarrow$ $0 \leq \frac{n}{N} \leq \frac{3}{8\pi}$

(Remember: $F = \sqrt{2n}\, dB \;\Rightarrow\; n > 0$)

$\rightarrow$ Upper bound on $\frac{n}{N}$

(relation to string exclusion principle?)

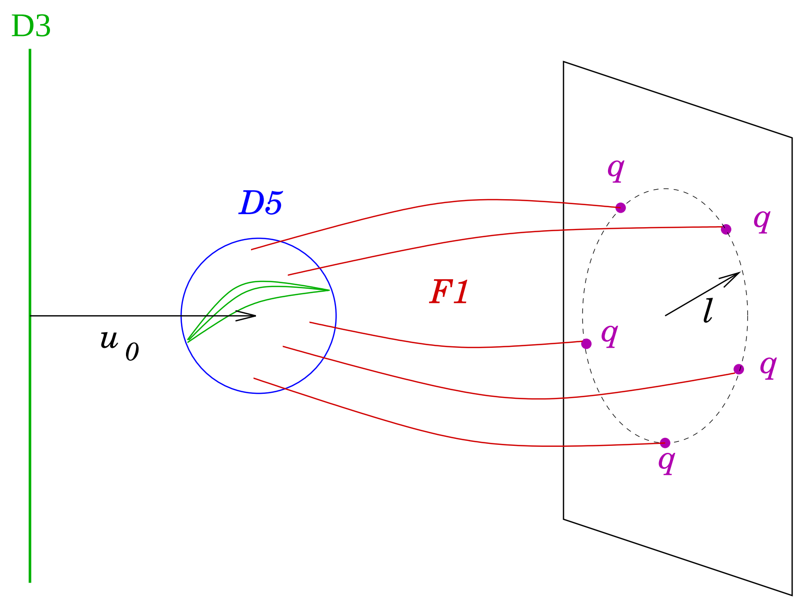
[Maldacena, Strominger]

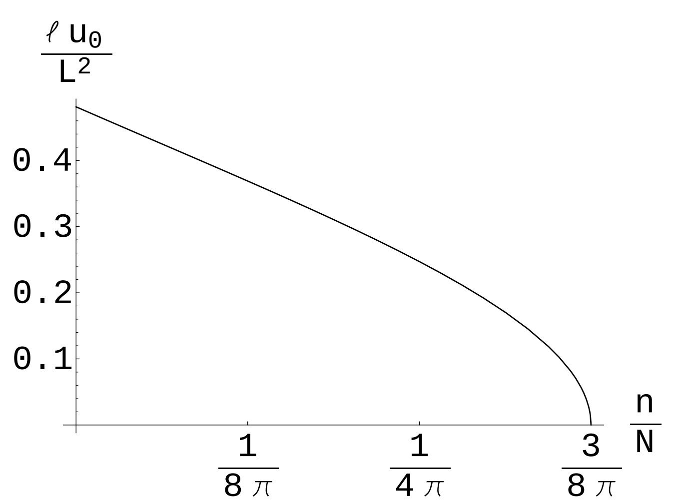
Solution: Size ℓ of the baryon vertex (in boundary)

$$\ell = \frac{L^2}{u_0} \int_1^\infty dy \frac{\beta}{y^2 \sqrt{y^4 - \beta^2}}$$

NB: Size of baryon vertex is inversely proporcional to u_0

Size of baryon vertex is function of n/N





Energy E of the baryon vertex

$$E = T_1 u_0 \Big\{ \int_1^\infty dy \Big[\frac{y^2}{\sqrt{y^4 - \beta^2}} - 1 \Big] - 1 \Big\}.$$

Energy E of the baryon vertex is:

- proportional to u_0 (conformal invariance)
- proportional to $\sqrt{g_{YM} N}$
- a function of n/N

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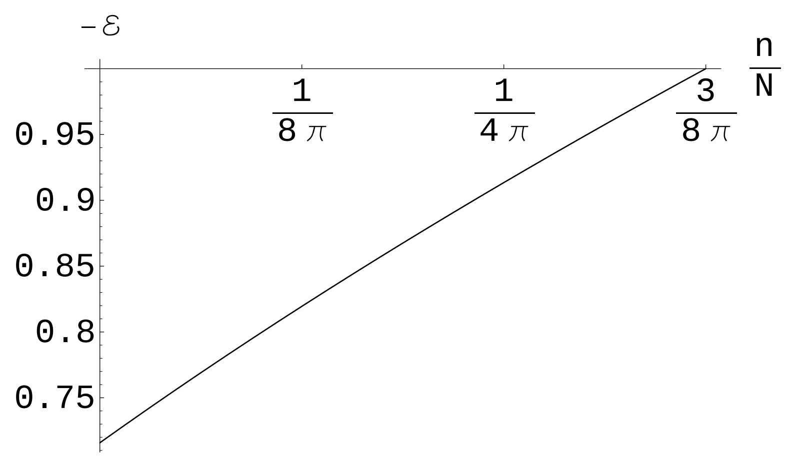

2.3 Supersymmetry

Witten's baryon vertex: $\frac{1}{2}$ (D5-brane) $\times$ $\frac{1}{2}$ (N F1) = $\frac{1}{4}$ total

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Generalised vertex: N F1 break $\frac{1}{2}$ (no influence of $F = \sqrt{2n} dB$)

D5-brane supersymmetry due to $F = \sqrt{2n} dB$?

D5-brane κ-symmetry:

$$\Gamma_n \epsilon \;=\; \mathcal{L}_{BI}^{-1} \Big(\Gamma_{(6)} + F \wedge F \; \Gamma_2 \Big) \epsilon \;=\; \epsilon$$

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D5-brane κ-symmetry:

$$\Gamma_n \epsilon \;=\; \mathcal{L}_{BI}^{-1}\Big(\Gamma_{(6)} + F \wedge F\, \Gamma_2\Big)\epsilon \;=\; \epsilon$$

in S^5 fibre coordinates:

$$\Gamma_n \epsilon \;=\; \mathcal{L}_{BI}^{-1}\Big(\Gamma_{ijkl} + (F \wedge F)_{ijkl}\Big)\Gamma_{t\chi}\epsilon \;=\; \epsilon$$

$\rightarrow$ Operator with $\mathrm{Tr}(\Gamma_n) = 0$ and $\Gamma_n^2 = \mathbb{1}$ in $AdS_5 \times S^5$

[Bergshoeff, Kallosh, Ortín, Papadopoulos]

$\rightarrow$ Generalised baryon vertex breaks preserves same susy as original

3 The microscopical description

3.1 Brief review of the dielectric effect

n coinciding D-branes $\Longrightarrow$ $U(1)^n \rightarrow U(n)$ gauge enhancement [Witten]

$\Longrightarrow$ non-Abelian action [Myers]

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$\Longrightarrow$ non-Abelian action [Myers]

$$S_{nD1} \;=\; -T_1 \int d^2\xi \,\mathrm{STr}\Big\{ \sqrt{\Big| \det\Big(P[g_{\mu\nu} + g_{\mu i}(Q^{-1} - \delta)^i{}_j g^{jk} g_{k\nu}] \Big) \det Q \Big|} \,\Big\}$$

$$+ T_1 \int d^2\xi \,\mathrm{STr}\Big\{ P[i(\mathbf{i}_X \mathbf{i}_X) C^{(4)} \;-\; \tfrac{1}{2}(\mathbf{i}_X \mathbf{i}_X)^2 C^{(4)} \wedge \mathcal{F} \Big\}$$

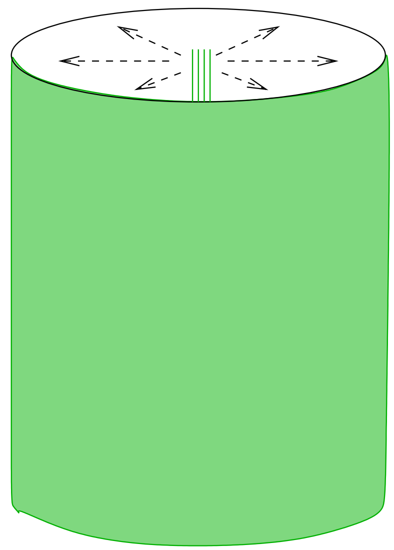
with

$$Q^i{}_j = \delta^i_j + i[X^i, X^k] g_{kj} \qquad \Big((\mathbf{i}_X \mathbf{i}_X) C^{(4)} \Big)_{\mu\nu} = \tfrac{1}{2}[X^\lambda, X^\rho] C^{(4)}_{\rho\lambda\mu\nu}$$

$$\mathcal{F} = 2\partial\mathcal{A} + i[\mathcal{A}, \mathcal{A}] \qquad (\mathbf{i}_X \mathbf{i}_X)^2 C^{(4)} = \tfrac{1}{4}[X^\lambda, X^\rho][X^\nu, X^\mu] C^{(4)}_{\mu\nu\rho\lambda}$$

$\frac{1}{2}[X^\lambda, X^\rho]C^{(4)}_{\rho\lambda\mu\nu}$ is dipole coupling

- Flat space: n D1's expand into fuzzy D3 [Myers]



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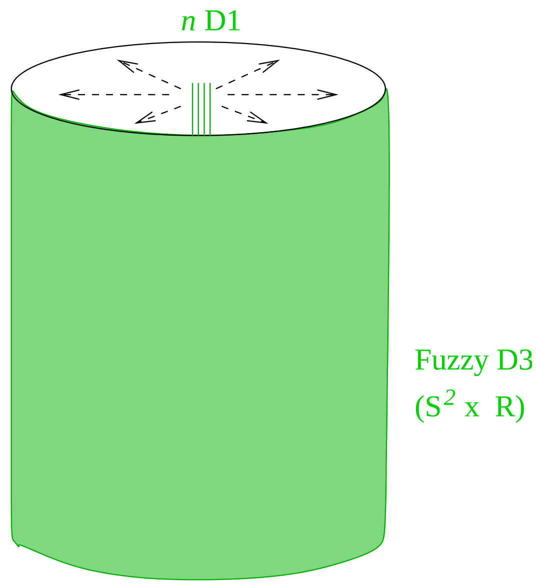

- $AdS_5 \times S^5$: D1's expand into fuzzy S^5

$\rightarrow$ Fuzzy S^5 is Abelian $U(1)$ fibre over fuzzy CP^2 [B.J., Lozano, Rodr.-Gómez]

NB: Chern-Simons couplings are zero $\Rightarrow$ purely gravitational effect

CP^2 is coset manifold $SU(3)/U(2)$
embedded in $\mathbb{R}^8$ via

$$\sum_{i=1}^{8} x^i x^i = 1 \qquad\qquad \sum_{j,k=1}^{8} d^{ijk} x^j x^k = \frac{1}{\sqrt{3}} x^i$$

Fuzzy CP^2 generated by $SU(3)$ generators T^i in (anti-)fundamental repres

$$X^i = \frac{T^i}{\sqrt{(2n-2)/3}} \qquad\qquad [X^i, X^j] = \frac{i f^{ijk}}{\sqrt{(2n-2)/3}} X^k$$

[Alexanian, Balachandran, Immirzi, Ydri]

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[Alexanian, Balachandran, Immirzi, Ydri]

Substituting in D1 action:

$$S_{nD1} = -T_1 \int dt d\chi \, u \, \mathrm{STr}\Big\{ \mathbb{1} + \frac{L^4}{4(2n-2)} \mathbb{1} \Big\}$$

$$E_{nD1} = 2\pi u T_1 \left(n + \frac{nL^4}{8(n-1)} \right)$$

$$\left[E_{D5} = 8\pi^2 u T_5 \left(n + \frac{L^4}{8} \right), \qquad T_1 = 4\pi^2 T_5 \right]$$

3.2 F1's in microscopic description

D5-brane in baryon vertex is expanded D1-branes

Where are F1's that form vertex?

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Where are F1's that form vertex?

→ Chern-Simons coupling:

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$$= -\frac{T_1}{4} \int dt d\chi \, \mathrm{STr}\Big\{ [X^i, X^j][X^k, X^l] G^{(5)}_{\chi ijkl} \mathcal{A}_t \Big\}$$

$$\rightarrow G^{(5)}_{\chi ijkl} = L^4 f^m_{[ij} f^n_{kl]} X^m X^n$$

$$= \frac{L^4 T_1}{2(n-1)} \int dt d\chi \, \mathrm{STr}\Big\{ \mathcal{A}_t \Big\}$$

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$$= \frac{L^4 T_1}{2(n-1)} \int dt d\chi \, \mathrm{STr}\Big\{ \mathcal{A}_t \Big\}$$

$$\rightarrow \mathcal{A} = A_t(t) \, \mathbb{1} dt$$

$$= \frac{n}{n-1} N T_1 \int dt \, A_t$$

3.2 F1's in microscopic description

D5-brane in baryon vertex is expanded D1-branes

Where are F1's that form vertex?

$\rightarrow$ Chern-Simons coupling:

$$S_{CS} = T_1 \int dt d\chi \, \mathrm{STr}\Big\{ P[(\mathrm{i}_X \mathrm{i}_X) C^{(4)}] \; - \; P[(\mathrm{i}_X \mathrm{i}_X)^2 C^{(4)}] \wedge \mathcal{F} \Big\}$$

$$= -\frac{T_1}{4} \int dt d\chi \, \mathrm{STr}\Big\{ [X^i, X^j][X^k, X^l] G^{(5)}_{\chi ijkl} \mathcal{A}_t \Big\}$$

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$\Rightarrow$ N BI charges as $n \rightarrow \infty$, cancelled by N extrnal F1's

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$$S_{nF1} = -T_1 \int d\tau d\sigma \,\mathrm{STr}\Big\{ \sqrt{\Big|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i{}_j E^{jk} E_{k\nu}]\Big) \det Q\Big|} \;\Big\}$$
$$-T_1 \int d\tau d\sigma \,\mathrm{STr}\Big\{ P[B^{(2)}] + \, iP[(\mathrm{i}_X \mathrm{i}_X) C^{(4)}] - \frac{1}{2} P[(\mathrm{i}_X \mathrm{i}_X)^2 B^{(6)}] \Big\}$$

with

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$\longrightarrow E_{nF1} = 2\pi u T_1 \left(n + \dfrac{nL^4}{8(n-1)} \right)$ $\quad$ N D1 strings via WV scalar

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- Microscopic description: D1-branes expanding into S^5

  $\Rightarrow$ agreement for $n \gg 1$

  (Witten's baryon vertex not included)

Outlook

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- $W \rightarrow S^3$ in $S^7 \subset AdS_4 \times S^7$: S^3 is non-contractable!
- ...