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Higher order derivative gravitational theories in the metric and Palatini formalisms

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Abstract

Lovelock and galileon theories are known extensions of general relativity that have actions containing second order derivatives, but do not produce field equation with third or higher order derivatives. We show that the dimensional reduction of Lovelock results in a generalized covariant galileon theory. Then we explain the Palatini formalism, a way to consider gravity in non-Riemannian geometry. It is known general relativity can be obtained in both the metric and Palatini formalism. This gives us a motivation to study other higher order derivative theories in the Palatini formalism as well. The possibility of using dimensional reduction as a tool to learn more about the Palatini formalism is explored in this thesis but does not lead to interesting results. Then Palatini formalism is applied to the cubic covariant galileon. The field equation for the connection can be solved and is found to be the Weyl connection. Furthermore, we show that the physics change in the Palatini formalism and point out the importance of projective invariance and some kind of duality between torsion and non-metricity in the cubic covariant galileon framework.

Chapter 1

Introduction

1.1 Motivation

In physics, we usually consider theories that have actions with at most first order derivatives. Actions that depend on second or higher order derivatives give in general rise equations of motion containing higher than second order derivatives. Ostrogradsky proved that as a result, more initial conditions than degrees of freedom are needed and so-called *ghost*states appear, see appendix A for an outline of his argument. Theories with ghosts are not stable and theoretically give infinite energy or particle production for example. However, there are some special second order derivative theories that give rise to at most second order derivative equations of motions. For this reason, these theories circumvent Ostrogradsky's theorem and are free of ghosts. Probably the best-known example of such a theory this is general relativity.

Since the beginning of the civilization, people have tried to explain why objects fall. The Greek philosopher Aristotle was one of the first people that we know of that tried to come up with a theory for this. His explanation was that all bodies move to their natural place, the center of the (geocentric) universe. Thousands of years after him, Newton wrote down a more mathematical description of gravity, the gravitational force that we all know as $F = G\frac{m_1 m_2}{r^2}$. His theory was a great success and able to predict a lot of phenomena, such as the movement of planets, very well. However, when astronomical observations improved, small deviations from this theory were found in the orbit of Mercury [1]. First, this problem in the discrepancy between theory and observations was tried to be solved by some ad-hoc solutions, such as the existence of a small planet closer to the sun. This would slightly modify the theoretical prediction to account for the observations. However, the hypothetical planet was never observed and it this puzzle wasn't solved by observing new matter.

Instead, a new theory solved the problem of Mercury. In the 20th century Einstein invented general relativity (GR). He came with the idea that space-time is a dynamical entity that interacts with the particles living in it. So GR describes gravity as a distortion of space-time and this is a huge conceptual jump from Newtonian gravity. Einstein's theory predicted a lot of new physics, such as the existence of black holes and gravitational waves. These were indeed detected long after Einstein came up with GR. The predictions of GR are in agreement with experiments in scales that range from millimeters to astronomical units [2]. However, there are a few problems with general relativity. To explain the rotation curves of spiral galaxies for example, we need the existence of an enormous amount of dark matter. On the other hand, to explain the distribution of matter at large scales, we need another source of energy with repulsive gravitational properties, dark energy. In total, dark matter and dark energy should constitute 96% of the total amount of total matter in the universe and we have no idea what it is.

One thing that could solve this problem is the discovery of this matter, but so far, all the searches for dark matter and energy didn't help us any further. However, just as the orbit of Mercury could only be explained by a new theory, maybe this time we should investigate modifications of general relativity. Another problem of GR is that we can not make predictions with it on quantum scales. So it's a good idea to look at other gravitational theories. We went from Aristotle to Newton and Einstein, who knows what's next?

Modifying general relativity

If we want to stay as close to the original Einstein-Hilbert Lagrangian as we can without breaking any symmetry, there are a few ways to modify gravity with a Lagrangian formulation [1]. If we want to keep a single massless metric, we are forced to go to *higher dimensions.* Then we will end up with Lovelock theory, which we will explain in the next chapter. We can also stay in 4 dimensions and consider *extra fields.* If we consider scalar fields, these theories are called galileon theories.

These two modifications of gravity have in common that they are described by actions containing second derivatives, but their field equations do not produce third or higher order derivatives. We will discuss them and their relations in the first part of this thesis. Furthermore, we may think of *other geometric constructions* such as a different connection than Levi-Civita to modify gravity. If we don't assume the shortest path to be equal to the path of parallel transport, a whole new way of calculating things open. This formalism is called the *Palatini* or *first order formalism* and will form the main idea of the second part of the thesis.

Objectives of the thesis

In this thesis we are going to investigate relations between the different modifications shorlty mentioned. It is known for GR it does not matter if we use the Palatini or metric formalism. So it is interesting to see what the effects of the Palatini formalism are on other higher derivative theories. In order to do this, we will split the thesis into two parts. The first half of the thesis (chapter 2-4) is dedicated to explaining different higher order derivative theories that give at most second order field equation and the relation between them. In this part we will try to answer the following research questions:

- We will explore modifications of General Relativity by considering Ostrogradsky-ghosts free theories. Which higher order derivative theories with second-order derivatives field equations do exist?
- Since Lovelock gives at most second-order field equations, its dimensionally reduced counterpart should do the same. How does dimensional reduction work and which specific scalar-tensor theory do we obtain by the dimensional reduction of Lovelock?

After this, we will introduce the Palatini formalism. We will apply this formalism to the theories considered in the first part. Specifically, the following subjects and research questions are explained in chapter 5-7:

- What is the Palatini formalism and what are its physical implications?
- We investigate the probability of dimensionally reducing a theory in the Palatini formalism. Can we learn more about the first order formalism of Lovelock and maybe even link it to the Palatini formalism of galileon theories?
- The last question that will be treated is what happens if we apply the Palatini formalism to galileons that live in curved space-time. How does it look like and to what extent is it different than the galileon in the metric formalism?

1.2 Outline

In the second chapter, a short summary of General Relativity is given. Its generalization to higher dimensions, Lovelock theory, is treated as well in this chapter and we will give an idea of how it avoids ghost states. In the third chapter, galileon theories are introduced we aim to give an overview of the differences, similarities and relations between different galileon theories. We will see that Lovelock and galileons have some similarities and in the fourth chapter we will link the Lovelock and galileon theories. Dimensional reduction is introduced and we will try to discover which galileon theory we obtain by the dimensional reduction of Lovelock.

In the fifth chapter, we explain the Palatini formalism. We will show how this formalism is equivalent to using the metric formalism for General Relativity. The consequences of using the Palatini formalism on Lovelock and galileon theory are discussed.
The possibilities to learn more about Palatini formalism for Lovelock and galileon theories are explored in chapter 6 and 7. In chapter 6 we start by calculating the dimensional reduction of the first two Lovelock terms in the Palatini formalism. In chapter 7 we will apply the Palatini formalism to the cubic galileon. The last chapter serves as a short summary and conclusion.

Chapter 8

Conclusions and outlook

We started by motivating the search for extended theories of gravity. Since there exist observations that do not match with the predictions of GR and we still do not know how to combine gravity with quantum physics, extensions to GR are worth studying. Two characteristics of GR are that (i) it is a higher order derivative theory without ghosts and (ii) that we obtain the same theory if we formulate it in the metric or Palatini formalism. These two subjects have been studied throughout this thesis in order to better understand their implications and to find which other gravitational theories they point out.

In the second chapter, we gave a recap of GR and its founding principles and introduced Lovelock theory as its extension to higher dimensions. Just as GR, Lovelock Lagrangians depend on second order derivatives of the metric but give only up to second order derivative field equations. In four dimensions GR and Lovelock are equivalent. In the third chapter we have seen if we require second order derivative equations of motion for *scalar* field, we end up with theories that are called galileons. So in a way, galileons can be regarded as a scalar portrayal of Lovelock theory. In this chapter we discussed the different types of galileon theories that exist and tried to clarify the distinction between galileons, generalized galileons and covariant galileons. After that, in chapter four, we introduced Kaluza-Klein reduction and demonstrated that the dimensional reduction of the Lovelock Lagrangian gives rise to *generalized* covariant galileons. A remarkable result was that in the flat space limit, the dimensional reduction of the sum of the Lovelock Lagrangians only gives rise to the kinetic scalar term. It could be interesting to study what extra terms appear if we consider dimensional reduction over more than one dimension.

After the introduction and study of higher order derivative theories, in chapter five we introduced another way to describe GR: in the Palatini formalism. In the Palatini formalism the connection is taken as an independent quantity as opposed to related to the metric in the metric formalism. We saw that in Einstein-Hilbert gravity, the first-order formalism and metric formalism give rise to exactly the same physics in vacuum. We also discussed the importance of including matter for the geometrical meaning of the connection. For Lovelock gravity, the first-order formalism is equivalent to the metric formalism in the way that the different sets of equations of motion give the same information when demanding the connection to be Levi-Civita. However, the general solution for the connection is not found yet. The Palatini formalism for galileon theories is largely an unknown domain. The possibility to learn more about both the first-order formalism for Lovelock as the formalism for galileons are explored in chapter 6 and 7. In chapter 6 we started by calculating the dimensional reduction of the first two Lovelock terms in the Palatini formalism. We succeeded in finding a way to reduce a theory in the first-order formalism, by regarding that having an independent connection is equivalent to a Levi-Civita connection and tensor. For Einstein-Hilbert, we could again solve the equations of motions but did not gain any physical insight on the Palatini formalism. In the case of Gauss-Bonnet, we couldn't find a way to extract useful information of the formalism. Even equivalence between the metric and Palatini formalism turned out to be trivial in the reduced theory and did not give any relations between the scalar field and the connection.

Then finally in chapter 7 the Palatini formalism was applied to the cubic covariant galileon. We noticed a few different interesting concepts. Firstly we have seen that in order to apply the Palatini formalism on covariant galileons, it is crucial that they have the same projective invariance as the Ricci scalar. We found a projectively invariant notation for both the cubic and quartic covariant galileon. In contrast to the case of Einstein-Hilbert and Lovelock, for the covariant galileon the Palatini formalism is certainly not equivalent to the metric formalism. Therefore, a study of the physical and cosmological implications of the cubic galileon in the Palatini formalism would be a nice follow-up. Furthermore, in the cubic galileon framework, there seemed to be some duality between torsion and non-metricity. This duality has appeared in different theories as well. The extension to which this duality is preserved in situations where matter is included or the exact physical meaning remains unknown and deserves to be studied in more detail to really understand the relation between geometry and physics.

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