The causal structure of cosmological models with two fluids

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Abstract

Studying the dynamics of the scale factor of a Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological model on a specific spatial geometry shows the evolution of the universe. Furthermore, the causal structure of observers that live in such universe can be studied with the help of Penrose diagrams. This project obtains both, the scale factors and the Penrose diagrams for universes that contain two fluids. First, universes with flat spatial geometry are studied, showing that independently of the type of fluids the universe starts with a Big Bang and expands forever.If one of the fluids causes deceleration and the other acceleration, the universe will be decelerating at early stages and accelerating at late stages. Observers in such universe have both a particle horizon and a cosmic event horizon. After that, spatially closed universes are studied, showing more a varied dynamics that depend on the type of fluids and their relative densities. Solutions indicate that universes might either have a Big Bang as an origin or they might not have an origin and be contracting at early times. The relative densities of fluids that lead to universes collapsing in a Big Crunch and expanding forever is also obtained. The existence of horizons is also dependent on the type of fluids and their relative densities.

Contents

1 Introduction

The origin, evolution, and fate of the universe are among the oldest questions in the history of science and philosophy. Newton's theory of gravity was not able to provide a satisfactory answer, but since the advent of Einstein's theory of General Relativity we are able to approach the problem from a scientifically rigorous way that is supported by observations. A cosmological solution to Einstein's equations is the Friedmann–Lemaître–Robertson–Walker (FLRW) solution that explains the behavior of the whole universe on large scales according to its content, modeled as a combination of perfect fluids. The goal of this project is to model FLRW solutions containing two perfect fluids.

The origin of relativistic cosmology goes back to the decade of 1920, when, based on Einstein's equations of General Relativity, Georges Lemaître [1] and Alexander Friedmann [2] independently performed theoretical studies of expanding universes. At that moment the scientific community believed in a static universe but that changed with the observations of Edwin Hubble, who showed that the universe is actually expanding [3]. This suggests that in the past it was smaller, denser and hotter, and it even had an origin, the so called Big Bang. Based on this, the theory of the hot Big Bang cosmology emerges, this model made some predictions that were later verified. One of them is the prediction of the Cosmic Microwave Background radiation (CMB). If the universe has an origin, then it has a finite age, so light had time to travel a finite distance from its origin until now. Furthermore, if the density of matter and radiation was bigger in the past, there must have been a time in the universe when it was opaque because any photon would be absorbed and reemitted by all surrounding matter, having thus a short mean free path. The expansion of the universe would make these densities smaller, so that at one point in time, called recombination, the photons would be redshifted enough to not ionize atoms anymore, being able to travel an arbitrary far distance, see [4] for a deeper analysis. Photons from recombination should be the furthest light that can reach us, they should be redshifted but they should be measurable. That is the origin of the CMB, which was first observed in 1965 by Penzias & Wilson [5]. Another success of this theory is the prediction of the abundances of the elements that were created during the primordial nucleosynthesis, explaining accurately the proportions that we see today [6, 7].

In spite of the predictions made, some problems were found in the hot Big Bang model. If the universe had a finite age, then light did only have time to travel a finite distance, namely that between us and the CMB, so parts opposite in the sky that we observe today can not have been in causal contact. The measurements of the CMB show that it is very homogeneous, coming with a temperature of $2,7K$, suggesting that such parts of the sky must have been in causal contact in the past. This is what is called the horizon problem. Another drawback is the flatness problem. For the universe to be flat, as observations indicate [8], the density of matter should be fine-tuned to a critical value in the past. Small deviations from this value magnify and give rise to big differences of the curvature at present. These and other problems can be solved if a period of exponential expansion took place at the beginning of the universe. This mechanism is called inflation, some proposals to model it as a quantum field like [9–15] have been done. It is not the only solution but it solves many problems in a very elegant way, therefore it is included as part of the description of the cosmological model that describes the early stages of the universe.

In addition to that, in 1998, it was measured that the universe is actually expanding at an accelerated rate [16, 17]. FLRW cosmological models can describe this expansion due to a special type of energy, called dark energy, that generates an accelerated expansion. We can see the effects of dark energy and model it as a perfect fluid but its nature remains unknown. Many studies like [18, 19] have been done on dark energy, recently also taking into account constraints from gravitational waves observations [20, 21].

In this context a standard model of Big Bang cosmology arises, the so-called ΛCDM model, named after the main fluids that are observed today: cosmologolical constant and cold dark matter. ΛCDM adds inflation at early stages to the hot Big Bang theory, solving its problems. It also modifies it at late stages by including dark energy, explaining this way an accelerated expansion of the universe today. The accuracy of this model has been evaluated in studies like [22]. This model states that the universe is currently composed of matter (both dark and luminous) and dark energy according to measured parameters. For this reason, a realistic picture of our universe should include at least two fluids. It is thus interesting to study different types of fluids; a mix of matter and cosmological constant seems to be the current stage of the universe but since different fluids dilute at a different rate, a universe containing radiation and matter can describe earlier stages of the universe. Different types of fluid combinations and their proportions can lead to a different evolution of the universe or show different causal relations between observers at different spatial locations. Some approaches to study the causality of flat universes with two fluids can be found, for example, in [23]. This project continues that research line.

The latest measurements of the spatial curvature of the universe indicates that it is flat, [8]. Nevertheless, it is still interesting to understand the evolution and causality of open and closed universes for several reasons. First of all, future measurements could reveal that the curvature of the universe is so small that it looks flat in our range of accuracy. On the other hand, such a study gives a complete picture and it could deliver mathematical models of universes that can show interesting dynamics.

The goal of this project is to study the causality of flat, open and closed universes containing two fluids. A useful tool to do that are Penrose diagrams. Such diagrams compactify spacetime, showing the causal relations of observers during the existence of the whole universe. They can show the extent of light cones of events, even when they take place at the birth or the end of the universe. The existence of horizons can also be seen in Penrose diagrams; they bound the regions that an observer can receive information from, since the beginning of the universe, or the regions that the observer can influence in the future.

The present work starts introducing the basic concepts of cosmology in section 2. It shows some relevant solutions, including FLRW universes with one fluid and it gives the starting point for universes with two fluids. After that, section 3 explains how a Penrose diagram can be constructed and interpreted. An analysis of flat universes with two fluids is performed in section 4, showing its causality properties for different types of fluids. This section includes a realistic model for our universe, with matter and cosmological constant. In all cases, universes start with a Big Bang and expand forever. These results can be generalised to open universes. Finally, the same analysis is performed for closed universes in section 5. The causal structure differs according to what fluids are included and their densities, so different cases are studied.

6 Conclusions

The goal of this project is to study the evolution and causality of FLRW universes composed of two fluids. To achieve this we have used two tools. First of all the analytic function of the scale factor obtained from the Friedmann equations, either in comoving or conformal coordinates, which has allowed us to study the general evolution. We can see in this function whether the universe has an origin or an end and its periods of acceleration or deceleration. The second tool is the Penrose diagram, that allows to visualize the causal contact of the timelike geodesics, showing if the universe has a cosmic event horizon or a particle horizon during part or its whole existence.

An introduction to General Relativity and cosmology has been given in section 2. Before moving to universes with two cosmic fluids we have studied universes with one fluid for different curvatures. They show different behavior depending on whether the fluid causes acceleration ($w < -1/3$), deceleration ($w > -1/3$) or neither of that ($w = -1/3$). Conformal transformations have been explained in section 3. They show transformations between universes with the property that null geodesics do not change character, thus the light cones do not change. Exploiting this property, Penrose diagrams can be obtained as two-dimensional representations that bring infinities to finite coordinates, showing the whole causal structure of the universe. Penrose diagrams of universes with one fluid are shown in section 3.2 where we can see that the causality of closed universes and non-closed universes varies significantly.

In view of this, spatially flat universes with two fluids have been studied in section 4. A summary of the results can be found in table 1. First of all, a universe with radiation and matter has been analyzed. This case is representative of an early period of our universe. Since both fluids cause deceleration, the causality and evolution have been shown to be the same as for one decelerating fluid. After that, a universe with a cosmological constant and a generic fluid has been studied. A general solution for the scale factor in comoving coordinates has been given in equation (4.15), showing that all universes start with a Big Bang and expand forever. However, transforming to conformal coordinates to obtain the Penrose diagram is not possible without specifying w. Several cases have been studied. First, a universe with $w = -2/3$, i.e. a second accelerating fluid. This case shows that the causal structure is the same as the one obtained for a single decelerating fluid. Then universes where w corresponds to a decelerating fluid are studied, including the case of matter that is a realistic model for our own universe. They show a period of decelerated expansion followed by a transition to accelerated expansion, indicating that at early times the decelerating fluid dominates and at late times the accelerating fluid drives the evolution. These universes posses particle horizons and cosmic event horizons during their whole existence. Finally a case when the second fluid has $w = -1/3$ is studied, leading to a universe that expands accelerating. Thus, the contribution to acceleration due to cosmological constant is not countered by the second fluid since it causes neither deceleration nor acceleration.

Finally the case of a cosmological constant with another fluid in a closed universe is studied in section 5. This can be relevant for pre-inflationary periods [38, 39] or bouncing universes [40, 41] since at that time the curvature of the universe is unknown. A summary of the results is found in table 2. In this scenario an analytical solution for comoving coordinates is no longer possible, thus specific fluids must be studied. First, a second accelerating fluid is considered. In this case the solution is that of a universe that starts contracting until reaching a minimum size and then expanding to arbitrary large values. This result is similar to a closed universe with one accelerating fluid.

Then, the case of a fluid with $w = -1/3$ as a second fluid is studied. This situation presents two different possibilities. If $\Omega_\Lambda \leq 1$ the universe starts with a Big Bang and it expands to arbitrary large sizes. Observes in this universe have no particle horizon and close to the Big Bang they do not have a cosmic event horizon either. The later will appear at later stages of the universe. On the other hand if $\Omega_\Lambda > 1$ the universe possesses no Big Bang, it is a universe that starts contracting at early enough comoving times, it reaches a minimum and then expands with positive acceleration for the rest of its lifetime. This universe possesses a particle horizon and a cosmic event horizon, but depending on the content of the fluids, these horizons might exists for the whole existence of the universe or only on part of it. As we have seen in section 2.2.3, flat universes always present a Big Bang while a de Sitter universe does not have it and takes place when $\Omega_\Lambda > 1$. Since the fluid with $w = -1/3$ does not contribute to neither acceleration nor acceleration, the acceleration of the universe is solely determined by the cosmological constant. Thus if $\Omega_\Lambda > 1$ the acceleration of the universe is strong enough to turn a contracting universe to an expanding universe, like it happens on de Sitter universe.

The last companion fluid considered is radiation. This presents a closed universe with a decelerating fluid and an accelerating fluid. In this case the solutions are varied depending on the relative density of the fluids. As a first condition we find that when $\left(\Omega_k/\Omega_\Lambda\right)^2 > 4\Omega_R/\Omega_\Lambda$ two possibilities arise. By letting Ω_k constraint by the election of Ω_R and Ω_Λ, this condition takes place when the proportions of fluids are not close to each other. First, if $\Omega_R > \Omega_\Lambda$ we find a universe that starts with a Big Bang, expands up to a maximum size and then contracts to a Big Crunch. This scenario takes place if radiation is more abundant than cosmological constant and its behavior is similar to a closed universe with a decelerating fluid. Different relative densities of the fluids determine the rate of expansion and contraction with resulting universes that range from those where a light ray sent at the origin of the universe is, at most, able to reach its antipodes, when $\Omega_R \gg \Omega_\Lambda$, to universes where such light rays are able to cross the universe several times, for $\Omega_R \gtrsim \Omega_\Lambda$. On the other hand, if $\Omega_R < \Omega_\Lambda$ the universe reaches arbitrary large sizes at either arbitrary early times or arbitrary large times. This universe starts contracting up to a minimum size and then it expands. In such universes where the cosmological constant is dominant, the behavior is similar to a closed universe with a single accelerating fluid. Again, depending on the relative density of fluids, a light ray might have time to reach little more than the antipodes of the universe, if $\Omega_\Lambda \gg \Omega_R$; or to give a certain number of turn overs, if $\Omega_\Lambda \gtrsim \Omega_R$. Another type of solution arises if $\left(\Omega_k/\Omega_\Lambda\right)^2 = 4\Omega_R/\Omega_\Lambda$. In this case the universe is fine-tuned to be static, thus small perturbations of the fluids might either cause a collapse or a runaway expansion. This behavior is similar to Einstein's static universe with radiation playing the role of matter. Finally, under the condition $\left(\Omega_k/\Omega_\Lambda\right)^2 > 4\Omega_R/\Omega_\Lambda$, that takes place for relatively close values of both densities, the universe starts with a Big Bang and expands up to arbitrary large sizes, similar to a flat universe. This case has spherical spatial sections, so depending on the amount of fluids a light ray can cross the universe a specific number of times. This number has no maximum and it can also only be a small portion of the universe, being that the Penrose diagram can reach any height with the appropriate combination of fluids. For high densities of both fluids the diagram reaches big heights, since the curvature of the universe is big, and for low densities of both fluids it reaches small heights, since the curvature of the universe is small.

These results show that closed universes posses much richer dynamics that flat ones. However, very interesting results are found on flat universes when one fluids decelerates and the other accelerates.

Future research lines might investigate other cases whose dynamic are not a priori direct. For example when three fluids are involved or when phantom dark energy, a type of dark energy with $w < -1$, associated with negative kinetic energy, is taken into account. This last case might be specially interesting for several reasons. First of all observations of our universe are compatible with it [8]. Also the dynamics might reveal interesting results such as Big Rips [42, 43], singularities when the scale factor reaches infinite values at finite comoving times.

The causality of universes containing one fluid is widely studied in the literature. However, to our knowledge, this is the first time that the Penrose diagrams of universes containing two fluids are obtained. Since our universe is flat, the Penrose diagram of the flat universe with matter and a cosmological constant is a realistic model. But prior to inflation, we do not know the curvature of the the universe, thus pre-inflationary studies or studies of bouncing universes like [44] might make use these models in the future. Also the interaction between two fluids is an interesting topic, studies like [45–47] can make improvements in the models that we found.

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