Statistics education is an important focus of interest for the International Statistical Institute (ISI) since its foundation, as it is visible in the Education Committee started in 1948 and in the creation of the International Association for Statistical Education in 1991. The IASE influence is clear in the increasing presence of statistics at all the educational levels, and in the numerous publications, conferences and research related to statistics education, including doctoral dissertations which are being carried out in departments of statistics, mathematics education, psychology or education. It is, however, unusual that future statisticians take a statistics education course along their undergraduate studies. In this paper we suggest that some didactical training is an important part in the education of future statisticians and reflect about the components and suitable teaching strategies in the initial didactical training of statisticians.

Interest of statistics education competence for statisticians

The many different possible areas of work for future statisticians are visible in the papers presented at the ISI Sessions, as well as observing the widespread use of statistics in almost all human activities. A very simple classification of professional statisticians is as follows (although a same person might play several of these roles simultaneously or at different stages of his/her professional life):

1. Statistics teachers/lecturers at school, professional training or university (undergraduate or post graduate training).
2. Theoretical statisticians who take part in research teams and develop new statistical theories, methods and procedures.
3. Applied statisticians working as consultants for professionals who have specialized in other topics and to whom they provide data analysis and sampling or experimental design services.
4. Producers of statistical data or reports for official agencies, financial or business institutions to which they provide information, guide in decision making and facilitate the planning of their functioning.
5. Statistics educators who analyze the teaching and learning of statistics, try to understand its rules and design didactical devices to help improving the functioning of didactical systems.

The interest of didactical knowledge is clear for teachers/lecturers and statistics educators. Moreover, those statisticians who create new methods or theoretical models should be able to communicate them at different complexity levels to other statisticians or scientists: e.g. in research papers, divulgation papers, training courses directed to applied statisticians or textbooks. Didactical abilities will contribute to this communication being clear and producing no misunderstanding in the potential users of statistics, since the experience shows that the use and interpretation of statistics are not always adequate (Morrison & Henkel, 1970; Harlow, Mulaik & Steiger, 1997; Batanero & Díaz, 2006).

The didactical problems and possibilities of statistical consultancy have been debated in the Tokyo 2000 IASE Round Table Conference on Training Researchers in the Use of Statistics and in many Invited Paper Sessions at different ICOTS or ISI Conferences. Statistical consultancy provide good training opportunities for both the statistician and the client. Not only do statisticians need to understand enough about their colleagues'
disciplines to be effective consultants, but they also can help researchers to learn more statistics and acquire enough appreciation of statistical concepts so that the association may be more productive.

On the one hand, the professional should be able to clearly communicate his/her problem to the statistician and provide him/her with a basic understanding of the topic, the aim of the study and the concepts involved in the same (Jolliffe, 2001). The statistician, in turn, can help clarify the questions that the researcher wishes to consider and both the researcher and the statistician will learn from each other as they move towards a common understanding. Statisticians need also to be trained to communicate statistical information and results from statistical analyses in a language accessible to the researcher and meaningful for the potential audience of the research report. They have a unique opportunity to help their clients acquire statistical literacy using their own data and the problems in which they are interested (Belli, 2001). All of this justify that the training of consultants include some didactical elements.

Finally, the shifting of the economy from tangible activities to intangible service-based activities has increased the difficulty of the work of statisticians producing public statistics and poses several challenges for statistical education, including the training of official statisticians, as new statistical concepts and methodology are being created, but also educating the users of these official statistics (Cheung, 1998). On one hand the statistical information, produced by these offices has a special role in modern societies in enabling people getting information and reacting to social, political and economic phenomena. Therefore, statisticians need to communicate and diffuse Statistics, not only as a technique for dealing with quantitative data, but also as a culture, in terms of capacity to comprehend the logical abstraction, which makes the quantitative study of collective phenomena possible (Murray & Gal, 2002). Statistical offices need moreover collaboration of citizens to collect their data since no statistical method, even if reliable and efficient can produce sound results from invalid data. This is why “to maintain high response rates, and therefore provide data collections that are widely regarded as providing high quality outputs, a statistical office needs to be trusted by the public, and in particular by its respondents” (McDonald, 2001, p. 121). All of these has increased the interest of statistical offices towards education, and even in some cases lead to collaborative projects such as Census at School.

Since didactical problems and situations seem to be pervasive in the work of statisticians, whatever this work is, it is interesting to offer future statisticians some didactical courses in their initial training. In this paper we discuss what the components of didactical knowledge, beyond the knowledge of statistics and probability itself could be, and analyse some activities used in the training of statisticians at the University of Granada. We hope these ideas may encourage other colleagues to organise courses of statistics education directed to statisticians at undergraduate or post graduate levels.

What is Didactical Knowledge and how Can it be Taught?

A wide statistical knowledge, even when essential, is not enough for teachers/lecturers/statisticians to be able to teach statistics. A review of the literature on teachers’ professional knowledge (e.g. Shulman, 1986, Cooney, 1999, Llinares & Kraisner, 2006) show that teachers draw on three main interrelated knowledge bases that evolve with practice: knowledge of content, knowledge of teaching processes and knowledge of their students. Research focused on teacher's training is producing a great deal of information about this 'didactical knowledge', which includes the following complementary aspects:

- Epistemological reflection on the meaning of concepts to be taught (e.g. reflection on the different meanings of probability). A main point in preparing teachers is the epistemological reflection, which can help them to understand the role of concepts within statistics and other areas, its importance in students' learning and the students' conceptual difficulties in problem solving. Statistics and probability are young areas and the formal development of probability was linked to a large number of paradoxes, which show the disparity between intuition and conceptual development in this field. This comparative difficulty is also shown in the fact that, even when Kolmogorov axiomatic was generally accepted in 1933, professional statisticians still debate about different views of probability and different methodologies of
inference. Biehler (1990) also suggested that this “meta-knowledge” should include a historical, philosophical and cultural perspective on statistics and its relations to other domains of science.

- Critical capacity to analyse textbooks and curricular documents and experience in adapting this knowledge to different teaching levels, and students’ various levels of understanding. This include (Steinbring, 1990), organizing and implementing statistics projects, experiencing students’ multiple forms of work and understanding; experiments, simulations and graphical representations not just as methodological teaching aids, but rather as essential means of knowing and understanding statistics. It is important to remark that general principles that are valid for geometry, algebra or other areas of mathematics cannot always be applied (Batanero, Godino & Roa, 2004). For example, in arithmetic or geometry an elementary operation can be reversed and this reversibility can be represented with concrete materials. This is very important for young children, who still are very linked to concrete situations in their mathematical thinking. For example, when joining a group of two apples with another group of three apples, a child always obtain the same result (5 apples); if separating the second set from the total he/she always returns to the original set; no matter how many times this operation is repeated. These experiences are very important to help children progressively abstract the mathematical structure behind them. In the case of random experiment we obtain different results each time the experiment is carried out and the experiment cannot be reversed (we can not get the first result again when repeating the experiment).

- Capacity to develop and analyse assessment tests and instruments and interpret students’ responses to the same. Prediction of students' learning difficulties, errors, obstacles and strategies in problem solving (e.g., students strategies in comparing two probabilities; students' confusion between the two terms in a conditional probability).

- Experience with good examples of teaching situations, didactic tools and materials (e.g., challenging and interesting problems; Galton board, simulation, calculators, etc.) and knowing the role of them in instruction. For example, even when simulation or experimentation with random generators, such as dice and coins have a very important function in stabilizing children’s intuition and in materializing probabilistic problems, these experiences do not provide the key to how and why the problems are solved. Teachers should realize that it is only with the help of combinatorial schemes or tools like tree diagrams that children start to understand the solution of probabilistic problems, due to the complementary nature of classical and frequentist approaches to probability. Moreover, the teaching of stochastics should provide a pedagogical space, where processes are given more value than facts, ideas are preferred to techniques, and a great diversity of problems involving other areas are proposed to help students develop positive attitudes towards this topic (Erasadin, 2006).

**Examples of teaching situations oriented to teachers’ didactical training. The Experience at the University of Granada**

It is important to find suitable and effective ways to teach this "didactical knowledge" to teachers –in this case the future statisticians-. Since students build their knowledge in an active way, by solving problems and interacting with their classmates we should use this same approach in training the teachers especially if we want them later use a constructivist and social approach in their teaching (Even and Lappan 1994; Ball & Bass, 2000, Jaworski 2001). An important view is that we should give teachers more responsibility in their own training and help them to develop creative and critical thinking (Shulman 1986). That is why we should create suitable conditions for teachers to reflect on their previous beliefs about teaching and discuss these ideas with other colleagues. Below we describe some examples of possible didactical activities to train teachers in the didactic contain related to statistics. These activities are complementary from various viewpoints and can be used to provoke teachers' reflection about the meaning of elementary stochastic notions, students' difficulties and obstacles, didactical methodology and materials.
1. Solving paradoxical problem situations and reflecting on its content. The history of statistics is full of examples of apparently simple problems that challenged the minds of brilliant mathematicians (see for example, Székely, 1986). In solving these problems, the future statisticians might reflect on the complex meaning of stochastic notions, share and predict some learning difficulties in their future students and learn some principles of teaching and assessment. For example, we ask the future statisticians to find the best strategy in the following game that has been designed to teach probability at secondary level (this situation is analysed in detail in Batanero, Godino & Roa, 2004).

**Game:** We take three counters of the same shape and size. One is blue on both sides, the second is red on both sides and the third is blue on one side and red on the other. We put the three counters into a box, and shake the box, before selecting a counter at random. After selecting the counter we show one of the sides. The aim of the game is to guess the colour of the hidden side. We repeat the process, putting the counter again in the box after each new extraction. We make predictions about the hidden side colour and win a point each time our prediction is right.

The lecturer organises the teaching time in several stages: a) playing the game; b) time to individually look for the best strategy; c) playing again to check the teachers’ conjecture against results in the experiment; d) general debate where students defend their different preferred strategies and try to give a mathematical proof about why one of them is optimum; e) didactical reflection on the game, the statistical content behind it, what was learnt in proving the best solution; stages in a didactical situation and predictable difficulties of students in the activity.

2. Analysing assessment items or tasks and some students responses to the same. The aim is to reflect on the complex meaning of stochastic notions, show the utility of the task in teaching and assessment and predict some learning difficulties. For example we ask the future statisticians imagine they want to try a treatment they suspect may alter performance on a certain task. After comparing the means of a control and an experimental groups (say 20 subjects in each sample) they get (t = 2.7, d.f. = 18, p = 0.01) in a simple independent means t-test. We ask future statisticians to discuss what is the meaning of this significant result and decide which of the following sentences (if any) are true (Krauss & Wassner, 2002):

- They have absolutely disproved the null hypothesis (the hypothesis that there is no difference between the population means).
- They have found the probability of the null hypothesis being true.
- They have absolutely proved your experimental hypothesis (that there is a difference between the population means).
- They can deduce the probability of the experimental hypothesis being true.
- If they decide to reject the null hypothesis, they know the probability that they are making the wrong decision.
- They have a reliable experimental finding in the sense that if, hypothetically, the experiment were repeated a great number of times, they would obtain a significant result on 99% of occasions.

In fact none of the previous statements is true, although many people, including students, scientists and even methodology instructions have erroneously believed some of this statements or other equivalent statements to be correct in Krauss & Wassner (2002) and other previous research. After analysing the sentence above as well as the correct definition of a p-value, we can introduce the future statisticians to some of the elements in what has been called: “the statistical tests controversy” and ask them read some summaries of the same (e.g. Batanero & Diaz, 2006). Future teachers might analyse the mathematical concepts involved in a statistical test (e.g., statistics and parameters, null and alternative hypotheses, significance level and p-value, sampling distribution, power, etc.) and build a concept map showing the relationships between them. They later can try to predict and classify possible errors related to each of these concepts and compare with errors described in the literature, a summary of which would be provided by the lecturer. A deeper level of
analysis would involve discussing the psychological and philosophical explanations for the persistence of these errors, including analysing the different approaches to statistical tests by Fisher and Neyman-Pearson, and the difference between reasoning by contradiction and the logical reasoning behind a statistical test.

3. **Analysing statistical data or graphs in published reports, media or research journals.** Future statisticians have a special interest in any activity related to the analysis of interpretation of data, so that we can also use these type of activities to provoke didactical reflection. For example, we can ask them to comment on the appropriateness of the graph presented in Figure 1, which was published in a major Spanish journal. The graph served to illustrate the possible reduction of deathly accidents as a consequence of applying the new traffic regulations. According this new law higher fines (€ 301 to € 1,500) might be imposed for committing the most serious traffic infractions that the Traffic Spanish law provides, such as driving in a negligent manner, speeding, driving without lights or parking in dangerous places. A driver who is fined three times for committing any of these infractions within a period of 2 years, may also have his driver's license definitively revoked. Incidentally, Spain has the highest road mortality rate in Europe. Independently of the accuracy or lack of accuracy of data and its interpretation, the graph is, in itself a compendium of frequent errors in a graphical display of data that students might discover. Other similar data and incorrect interpretation of statistics in press can be downloaded from the web server “Malaprensa” (bad press) at http://www.malaprensa.com. More sophisticated analyses include criticizing use of statistics in research reports published in scientific journals, such as using statistical procedures without taking into account their assumptions, too small sample sizes, incorrect multiple comparisons or incorrect interpretation of statistical analyses.

![Figure 1. Example of misleading graph](image)

4. **Analysing textbooks, statistical software or didactic resources.** For example, we might ask future statisticians to compare curricular guidelines for the same educational level in different Spanish autonomies (geographical regions with self capacity in education) and reflect on their adequacy. We might ask them to suggest possible ways to introduce the concepts to specific students (e.g. secondary students who still have not a deep mathematical knowledge). Another activity is analysing the different sections included in the journal Teaching Statistics, oriented to teachers of 9-18 year-olds students; this analysis may provide an idea of the different types of knowledge required in the professional work of statistics teachers: history of statistics, didactical research, resources, problems, assessment, data sets, computer corner, etc. They might also visit some of the many didactical resources for teaching statistics at Internet, for example those listed in the links section at the IASE web page (http://www.stat.auckland.ac.nz/~iase/), at the ISI Statistical Literacy Project web page (http://www.stat.auckland.ac.nz/~iase/islp/home) or at other statistics education web servers. Unfortunately most of these resources are written in English language (so that they are not useful for
teachers who have to teach in a different language); although little by little the situation is changing and an increasing number of Internet resources in Spanish and other languages are being made available.

Other complementary activities are as follows: A) Reading and discussing short papers by prestigious statisticians where didactical problems are set; for example the paper by David Moore (1997), summarise the main points made in the paper by the author and decide with which of these points they agree and why. Other different students might read papers that present complementary positions about some of these ideas (e.g. Rossman & Short, 1995; Albert, 1995). B) The lecturer summarises a theoretical model described in statistics education research and a collective activity is organised to apply the model in the analysis of students’ statistical work. For example, the lecturer summarises Wild & Pfannkuch’s (1999) model for statistical thinking and students in pairs try to get examples of fundamental modes of statistical thinking (recognising the need for data, transnumeration, perception of variation, reasoning with statistical models and integrating statistics in the context) in some examples of statistical projects carried out by secondary school students. C) Some students interested in getting a better score in the course carry out a short bibliographical survey and a synthesis of a theme which interest them and prepare a short historical, epistemological or didactical essay. D) Finally, a compulsory activity for all the participants is preparing a didactic unit to introduce a statistical topic to a specific type of student, where future teachers elect the statistical topic and type of student (e.g. introducing correlation to psychologists). In this didactic unit, the future teachers have to apply the knowledge acquired along the course and include the objectives, contents, activities, teaching methodology, didactic resources and software needed and assessment criteria. Students can consult examples of didactic units in Spanish and foreign statistics textbooks and curricular materials available at the Department library.

These and other similar activities have been experimented along the past 10 years at different courses in Statistics Education at the University of Granada, Spain and in some Latin American countries. One of these courses has been included since 1996 as an optional topic within the Major in Statistics Sciences. The course has a length of 60 teaching hours and the majority of students are in the 4º or 5º year of studies, and, therefore, have a good training in statistics. Consequently this course is focused only in the didactical content and the types of activities described above. Since the content is very sparsely widespread in journals and books, a summary of the same, which also include possible activities for the course was developed in a basic textbook (Batanero, 2001) divided into 5 chapters:

5. **Teaching statistics through project work**: Principles in working with projects. Examples of statistical project to introduce statistics along secondary education.

**Final Remarks**

In this paper we have described our experience in training future statisticians in statistics education at the University of Granada. It is clear that the time available (60 teaching hours) is not enough to get even a basic mastering of didactic knowledge by future statisticians and moreover the structure of the course do not
include teaching practices which are the most important part of professional development. Even with these limitations we think this type of course can provide future statisticians with some didactical preparation that will be useful for their future work at the time it increases their interest and sensibility towards the good practice of statistics.

Another remark is the scarcity of teaching materials devoted to the didactical preparation of statistics teachers as compared with materials related to teaching mathematics or teaching other topics. The significant research efforts focusing on mathematics teacher education and professional development in the past decade have not been reflected in statistics education. This is evident in conferences (e.g., the ICMI Study 15), journals (e.g., *Journal of Mathematics Teacher Education*), surveys, and books that hardly take into account the particular case of statistics. This omission needs to be addressed by promoting research specifically focussed on the education and professional development of teachers to teach statistics (Shaughnessy, in press). This need was recognised by both the International Association for Statistical Education and the International Commission on Mathematical Instruction who are jointly organising a Study around this specific topic with two components: a Study Conference and the production of a book (http://www.ugr.es/~icmi/iase_study/). We hope this initiative lead to a better preparation of teachers which in turn serves to improve statistics education at all educational levels.

ACKNOWLEDGEMENT: This work was supported by the projects SEJ2004-00789, MEC, Madrid, and FQM 126, Junta de Andalucia, Spain

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ABSTRACT

In this presentation we first suggest that some didactical training is an important part in the education of future statisticians and reflect about what other components would be needed and what are suitable teaching strategies in the initial and on-going didactical training of statisticians. We finally present and analyse examples of activities used in courses of statistics education directed to statisticians. These reflections take into account the experience at the University of Granada, where an optional Statistics Education course was included ten years ago in the Major in Statistical Sciences and Techniques, as well as the experience in other courses given to in-service statistics lecturers in several Latin American countries.