# The relations among the prime numbers

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**Gracia Arredondo Fernández**

# The symmetry of the prime numbers

## 1. The relations among the prime numbers

All the prime numbers greater than 3 have, at least, two pairs of equidistant prime numbers. Here is a list of the first primes (in red) with their equidistant primes.

| | | |
|---|---|---|
| 3 | | 5 |
| | **11** | |
| 19 | | 17 |
| 3 | | 7 |
| | **13** | |
| 23 | | 19 |
| 5 | | 11 |
| | **17** | |
| 29 | | 23 |
| 7 | | 7 |
| | **19** | |
| 31 | | 31 |
| 3 | | 17 |
| | **23** | |
| 43 | | 29 |
| 11 | | 17 |
| | **29** | |
| 47 | | 41 |
| 3 | | 19 |
| | **31** | |
| 59 | | 43 |
| 13 | | 31 |
| | **37** | |
| 61 | | 43 |
| 23 | | 29 |
| | **41** | |
| 59 | | 53 |
| 13 | | 19 |
| | **43** | |
| 73 | | 67 |

5 and 17 are two prime numbers equidistant to **11**.

3 and 19 is the second pair of prime numbers equidistant to **11**

A tiling that relates any two primes will be built. The tiling is built from numbers obtained with the following computation:

$$2.\mathbf{p_1}.\mathbf{p_2} - \mathrm{e_1}.\mathrm{E_2} - \mathrm{E_1}.\mathrm{e_2}$$

$\mathbf{p_1}$ (its two equidistant primeson the right are $e_1$ and $E_1$, $e_1 < E_1$) and $\mathbf{p_2}$ (its two equidistant primes on the right are $e_2$ and $E_2$, $e_2 < E_2$) are the two primes used as coordinates in the tiling. The result of that computation is the number that appears in each tile of coordinates ($\mathbf{p_1}$, $\mathbf{p_2}$)

The product of the two diagonals of any square (an example in green) is the same:

144.72.144.576.72 = 144.144.144.288.72

| | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **7** | 32 | 48 | 48 | 48 | 96 | 48 | 96 | 96 | 48 | 96 | 192 | 48 |
| **11** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |
| **13** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |
| **17** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |
| **19** | 96 | 144 | 144 | 144 | 288 | 144 | 288 | 288 | 144 | 288 | 576 | 144 |
| **23** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |
| **29** | 96 | 144 | 144 | 144 | 288 | 144 | 288 | 288 | 144 | 288 | 576 | 144 |
| **31** | 96 | 144 | 144 | 144 | 288 | 144 | 288 | 288 | 144 | 288 | 576 | 144 |
| **37** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |
| **41** | 96 | 144 | 144 | 144 | 288 | 144 | 288 | 288 | 144 | 288 | 576 | 144 |
| **43** | 192 | 288 | 288 | 288 | 576 | 288 | 576 | 576 | 288 | 576 | 1152 | 288 |
| **47** | 48 | 72 | 72 | 72 | 144 | 72 | 144 | 144 | 72 | 144 | 288 | 72 |

There is a second tiling that stablishes relations among all the prime numbers. It is also built from the fact that every prime number has at least two pairs of equidistant primes.

For the tiling in the previous page the equidistant numbers on the right of each prime have been used. In the tiling that appears in this page those on the left of each prime will be used.

| | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 128 | 160 | 192 | 192 | 320 | 288 | 416 | 384 | 288 | 480 |
| 13 | 160 | 200 | 240 | 240 | 400 | 360 | 520 | 480 | 360 | 600 |
| 17 | 192 | 240 | 288 | 288 | 480 | 432 | 624 | 576 | 432 | 720 |
| 19 | 192 | 240 | 288 | 288 | 480 | 432 | 624 | 576 | 432 | 720 |
| 23 | 320 | 400 | 480 | 480 | 800 | 720 | 1040 | 960 | 720 | 1200 |
| 29 | 288 | 360 | 432 | 432 | 720 | 648 | 936 | 864 | 648 | 1080 |
| 31 | 416 | 520 | 624 | 624 | 1040 | 936 | 1352 | 1248 | 936 | 1560 |
| 37 | 384 | 480 | 576 | 576 | 960 | 864 | 1248 | 1152 | 864 | 1140 |
| 41 | 288 | 360 | 432 | 432 | 720 | 648 | 936 | 864 | 648 | 1080 |
| 43 | 480 | 600 | 720 | 720 | 1200 | 1080 | 1560 | 1140 | 1080 | 1800 |

The product of the two diagonals of any square is the same:

360.576. 624.720.720 = 400.432.624.960.648

## 2. The tiles of the natural numbers

There is also, at least, one pair of prime numbers **equidistant** to every natural number (in green) greater than 7:

| | | |
|---|---|---|
| 5 | | 3 |
| | 8 | |
| 11 | | 13 |
| 7 | | 5 |
| | 9 | |
| 11 | | 13 |
| 3 | | 7 |
| | 10 | |
| 17 | | 13 |
| 3 | | 5 |
| | 11 | |
| 19 | | 17 |
| 7 | | 5 |
| | 12 | |
| 17 | | 19 |
| 3 | | 7 |
| | 13 | |
| 23 | | 19 |
| 5 | | 11 |
| | 14 | |
| 23 | | 17 |
| 11 | | 13 |
| | 15 | |
| 19 | | 17 |
| 3 | | 13 |
| | 16 | |
| 29 | | 19 |

3 and 13 are equidistat from 8

5 and 11 are also equidistat from 8

Whenever there is more than one pair of equidistant primes, the closest pair to the natural numbers has been chosen, those that are at the right of the natural number in the squares of the previous page. With them a first tiling that relates any two natural numbers will be built. The tiling is formed with the numbers obtained from the following computation:

$$\mathbf{2.n_1.n_2 - e_1.E_2 - E_1.e_2}$$

$n_1$ (its two equidistant primes are $\mathbf{e_1}$ and $\mathbf{E_1}$, $\mathbf{e_1} < \mathbf{E_1}$) and $n_2$ (its two equidistant primes are $\mathbf{e_2}$ and $\mathbf{E_2}$, $\mathbf{e_2} < \mathbf{E_2}$) are the two natural numbers used as coordinates in the tiling. The result of that computation is the number that appears in each tile of coordinates ($n_1$, $n_2$).

For example, the tile of coordinates (11, 14) = 2.11.14 – 5.17-17.11 = 36

The product of the two diagonals of any square (an example in green) is the same:

70.72 = 84.60

| | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 32 | 40 | 32 | 24 | 48 | 56 | 48 | 24 | 16 | 24 | 48 | 88 | 96 |
| 8 | 40 | 50 | 40 | 30 | 60 | 70 | 60 | 30 | 20 | 30 | 60 | 110 | 120 |
| 9 | 32 | 40 | 32 | 24 | 48 | 56 | 48 | 24 | 16 | 24 | 48 | 88 | 96 |
| 10 | 24 | 30 | 24 | 18 | 36 | 42 | 36 | 18 | 12 | 18 | 36 | 66 | 72 |
| 11 | 48 | 60 | 48 | 36 | 72 | 84 | 72 | 36 | 24 | 36 | 72 | 132 | 144 |
| 12 | 56 | 70 | 56 | 42 | 84 | 98 | 84 | 42 | 28 | 6 | 60 | 110 | 120 |
| 13 | 48 | 60 | 48 | 36 | 72 | 84 | 72 | 36 | 24 | 36 | 72 | 132 | 144 |
| 14 | 24 | 30 | 24 | 18 | 36 | 42 | 36 | 18 | 12 | 18 | 36 | 66 | 72 |
| 15 | 16 | 20 | 16 | 12 | 24 | 28 | 24 | 12 | 8 | 12 | 24 | 44 | 48 |
| 16 | 24 | 30 | 24 | 18 | 36 | 6 | 36 | 18 | 12 | 18 | 36 | 66 | 72 |
| 17 | 48 | 60 | 48 | 38 | 72 | 60 | 72 | 36 | 24 | 36 | 72 | 132 | 144 |
| 18 | 88 | 110 | 88 | 66 | 132 | 110 | 132 | 66 | 44 | 66 | 132 | 242 | 24 |
| 19 | 96 | 120 | 96 | 72 | 144 | 120 | 144 | 72 | 48 | 72 | 144 | 24 | 288 |

There is a second tiling for the natural numbers greater than 7, obtained with the same computation but choosing now the other two equidistant primes, those more distant to the natural number, those two prime numbers that were on the left of the natural numbers:

Again, the products of the two diagonals of any square are the same:

30.40.126.64.130.240 = 100.100.144.56.52.72

| | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 18 | 12 | 42 | 48 | 30 | 60 | 54 | 24 | 78 | 72 | 66 | 72 |
| 9 | 12 | 8 | 28 | 32 | 20 | 40 | 36 | 16 | 52 | 48 | 44 | 48 |
| 10 | 42 | 28 | 98 | 112 | 70 | 140 | 126 | 56 | 182 | 168 | 154 | 168 |
| 11 | 48 | 32 | 112 | 128 | 80 | 160 | 144 | 64 | 208 | 192 | 176 | 192 |
| 12 | 30 | 20 | 70 | 80 | 50 | 100 | 90 | 40 | 130 | 120 | 110 | 120 |
| 13 | 60 | 40 | 140 | 160 | 100 | 200 | 180 | 80 | 260 | 240 | 220 | 240 |
| 14 | 54 | 36 | 126 | 144 | 90 | 180 | 162 | 72 | 234 | 216 | 198 | 216 |
| 15 | 24 | 16 | 56 | 64 | 40 | 80 | 72 | 32 | 104 | 96 | 88 | 96 |
| 16 | 78 | 52 | 182 | 208 | 130 | 260 | 234 | 104 | 338 | 312 | 286 | 312 |
| 17 | 72 | 48 | 168 | 192 | 120 | 240 | 216 | 96 | 312 | 288 | 264 | 288 |
| 18 | 66 | 44 | 154 | 176 | 110 | 220 | 198 | 88 | 286 | 264 | 242 | 264 |
| 19 | 72 | 48 | 168 | 192 | 120 | 240 | 216 | 96 | 312 | 288 | 264 | 288 |

## 3. The G1-G2 pattern

The triads of the primes with their equidistant primes are homogenous in the following sense. All the prime numbers can be classified in these two groups, G1 and G2:

| | |
|---|---|
| 5 G1<br>11 G1<br>17 G1 | 41 G1<br>47 G1<br>53 G1 |
| 7 G2<br>13 G2<br>19 G2 | 47 G1<br>53 G1<br>59 G1 |
| 11 G1<br>17 G1<br>23 G1 | 11 G1<br>17 G1<br>23 G1 |
| 7 G2<br>19 G2<br>31 G2 | 7 G2<br>19 G2<br>31 G2 |
| 17 G1<br>23 G1<br>29 G1 | 17 G1<br>23 G1<br>29 G1 |
| 17 G1<br>29 G1<br>41 G1 | 17 G1<br>29 G1<br>41 G1 |
| 19 G2<br>31 G2<br>43 G2 | 19 G2<br>31 G2<br>43 G2 |
| 31 G2<br>37 G2<br>43 G2 | 31 G2<br>37 G2<br>43 G2 |
| 29 G1<br>41 G1<br>53 G1 | 29 G1<br>41 G1<br>53 G1 |
| 19 G2<br>43 G2<br>67 G2 | 19 G2<br>43 G2<br>67 G2 |

G1

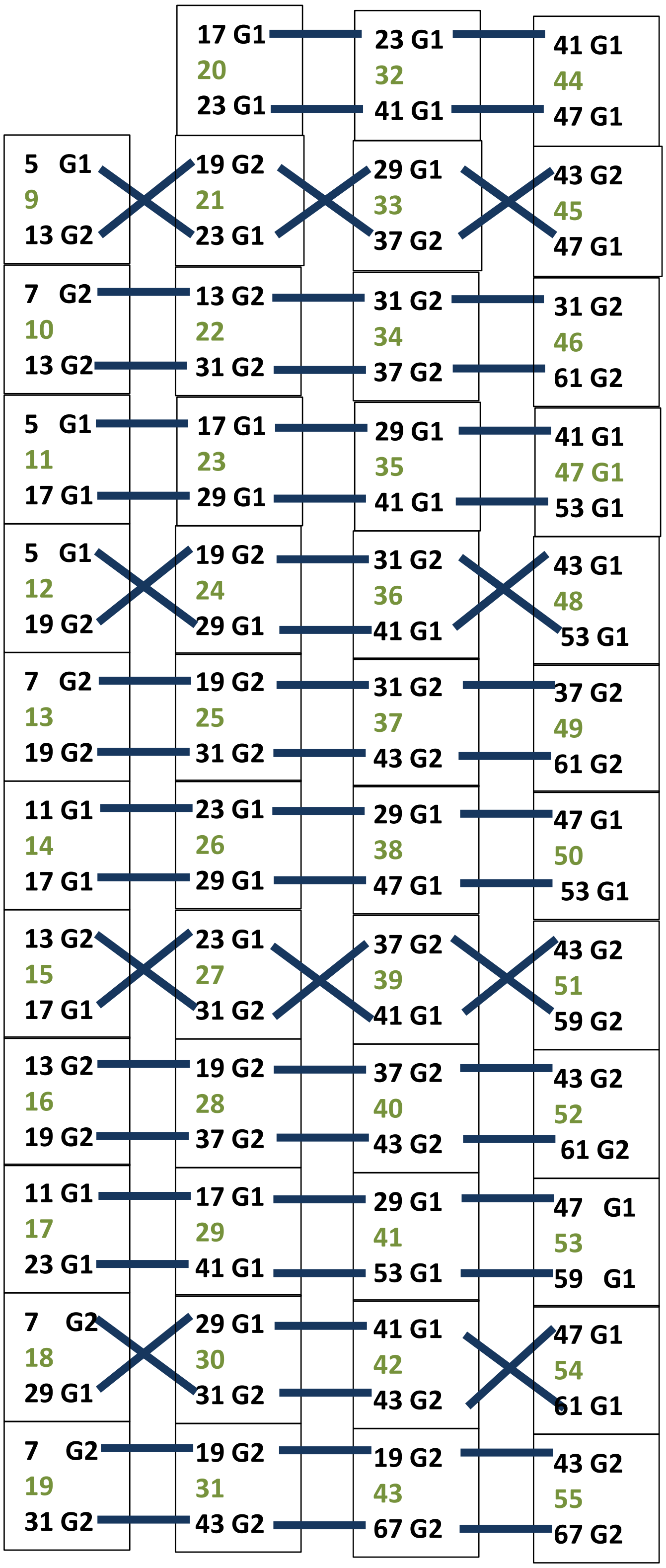
- The smaller prime number in each twin pair: 11, 17, 29, 41…
- The prime numbers in the triads that are the semisum of two equidistant smaller G1 twin primes: 23, 47, 52…Once a prime number results to be the semisum of two G1 primes, it remains G1 onward.

G2

- The greater prime number in each twin pair: 13, 19, 31, 43…
- The prime numbers in the triads that are the semisum of two equidistant greater G2 twin primes: 37, 67, 79…Once a prime number results to be thesemisum of two G2 primes, it remains G2 in subsequent triads.

The three prime numbers that form each triad are homogeneous as to their G1 - G2 character.

A curious thing about the triads of the natural numbers and their primes is that, if they are displayed in columns of twelve, G1 and G2 draw a pattern:



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