

The relations among the prime numbers

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The symmetry of the prime numbers

1. The relations among the prime numbers

All the prime numbers greater than 3 have, at least, two pairs of equidistant prime numbers. Here is a list of the first primes (in red) with their equidistant primes.

3 5 11 19 17	<p>5 and 17 are two prime numbers equidistant to 11.</p> <p>3 and 19 is the second pair of prime numbers equidistant to 11</p>
3 7 13 23 19	
5 11 17 29 23	
7 7 19 31 31	
3 17 23 43 29	
11 17 29 47 41	
3 19 31 59 17	
13 31 37 61 43	
23 29 41 59 53	
13 19 43 73 67	

A tiling that relates any two primes will be built. The tiling is built from numbers obtained with the following computation:

$$2 \cdot p_1 \cdot p_2 - e_1 \cdot E_2 - E_1 \cdot e_2$$

p_1 (its two equidistant primes on the right are e_1 and E_1 , $e_1 < E_1$) and p_2 (its two equidistant primes on the right are e_2 and E_2 , $e_2 < E_2$) are the two primes used as coordinates in the tiling. The result of that computation is the number that appears in each tile of coordinates (p_1, p_2)

The product of the two diagonals of any square (an example in green) is the same:

$$144 \cdot 72 \cdot 144 \cdot 576 \cdot 72 = 144 \cdot 144 \cdot 144 \cdot 288 \cdot 72$$

	7	11	13	17	19	23	29	31	37	41	43	47
7	32	48	48	48	96	48	96	96	48	96	192	48
11	48	72	72	72	144	72	144	144	72	144	288	72
13	48	72	72	72	144	72	144	144	72	144	288	72
17	48	72	72	72	144	72	144	144	72	144	288	72
19	96	144	144	144	288	144	288	288	144	288	576	144
23	48	72	72	72	144	72	144	144	72	144	288	72
29	96	144	144	144	288	144	288	288	144	288	576	144
31	96	144	144	144	288	144	288	288	144	288	576	144
37	48	72	72	72	144	72	144	144	72	144	288	72
41	96	144	144	144	288	144	288	288	144	288	576	144
43	192	288	288	288	576	288	576	576	288	576	1152	288
47	48	72	72	72	144	72	144	144	72	144	288	72

There is a second tiling that establishes relations among all the prime numbers. It is also built from the fact that every prime number has at least two pairs of equidistant primes.

For the tiling in the previous page the equidistant numbers on the right of each prime have been used. In the tiling that appears in this page those on the left of each prime will be used.

	11	13	17	19	23	29	31	37	41	43
11	128	160	192	192	320	288	416	384	288	480
13	160	200	240	240	400	360	520	480	360	600
17	192	240	288	288	480	432	624	576	432	720
19	192	240	288	288	480	432	624	576	432	720
23	320	400	480	480	800	720	1040	960	720	1200
29	288	360	432	432	720	648	936	864	648	1080
31	416	520	624	624	1040	936	1352	1248	936	1560
37	384	480	576	576	960	864	1248	1152	864	1140
41	288	360	432	432	720	648	936	864	648	1080
43	480	600	720	720	1200	1080	1560	1140	1080	1800

The product of the two diagonals of any square is the same:
 $360 \cdot 576 \cdot 624 \cdot 720 \cdot 720 = 400 \cdot 432 \cdot 624 \cdot 960 \cdot 648$

2. The tiles of the natural numbers

There is also, at least, one pair of prime numbers **equidistant** to every natural number (in green) greater than 7:

5 3 8 11 13	← 8 = $\frac{5+11}{2}$ = $\frac{3+13}{2}$
7 5 9 11 13	
3 7 10 17 13	
3 5 11 19 17	
7 5 12 17 19	
3 7 13 23 19	
5 11 14 23 17	
11 13 15 19 17	
3 13 16 29 19	

Whenever there is more than one pair of equidistant primes, the closest pair to the natural numbers has been chosen, those that are at the right of the natural number in the squares of the previous page. With them a first tiling that relates any two natural numbers will be built. The tiling is formed with the numbers obtained from the following computation:

$$2 \cdot n_1 \cdot n_2 - e_1 \cdot E_2 - E_1 \cdot e_2$$

n_1 (its two equidistant primes are e_1 and E_1 , $e_1 < E_1$) and n_2 (its two equidistant primes are e_2 and E_2 , $e_2 < E_2$) are the two natural numbers used as coordinates in the tiling. The result of that computation is the number that appears in each tile of coordinates (n_1, n_2) .

For example, the tile of coordinates $(11, 14) = 2 \cdot 11 \cdot 14 - 5 \cdot 17 - 17 \cdot 11 = 36$

The product of the two diagonals of any square (an example in green) is the same:

$$70 \cdot 72 = 84 \cdot 60$$

	7	8	9	10	11	12	13	14	15	16	17	18	19
7	32	40	32	24	48	56	48	24	16	24	48	88	96
8	40	50	40	30	60	70	60	30	20	30	60	110	120
9	32	40	32	24	48	56	48	24	16	24	48	88	96
10	24	30	24	18	36	42	36	18	12	18	36	66	72
11	48	60	48	36	72	84	72	36	24	36	72	132	144
12	56	70	56	42	84	98	84	42	28	6	60	110	120
13	48	60	48	36	72	84	72	36	24	36	72	132	144
14	24	30	24	18	36	42	36	18	12	18	36	66	72
15	16	20	16	12	24	28	24	12	8	12	24	44	48
16	24	30	24	18	36	6	36	18	12	18	36	66	72
17	48	60	48	38	72	60	72	36	24	36	72	132	144
18	88	110	88	66	132	110	132	66	44	66	132	242	24
19	96	120	96	72	144	120	144	72	48	72	144	24	288

There is a second tiling for the natural numbers greater than 7, obtained with the same computation but choosing now the other equidistant primes, those more distant to the natural number, those on the left column on page 23:

Again, the products of the two diagonals of any square are the same:

$$30 \cdot 40 \cdot 126 \cdot 64 \cdot 130 \cdot 240 = 100 \cdot 100 \cdot 144 \cdot 56 \cdot 52 \cdot 72$$

	8	9	10	11	12	13	14	15	16	17	18	19
8	18	12	42	48	30	60	54	24	78	72	66	72
9	12	8	28	32	20	40	36	16	52	48	44	48
10	42	28	98	112	70	140	126	56	182	168	154	168
11	48	32	112	128	80	160	144	64	208	192	176	192
12	30	20	70	80	50	100	90	40	130	120	110	120
13	60	40	140	160	100	200	180	80	260	240	220	240
14	54	36	126	144	90	180	162	72	234	216	198	216
15	24	16	56	64	40	80	72	32	104	96	88	96
16	78	52	182	208	130	260	234	104	338	312	286	312
17	72	48	168	192	120	240	216	96	312	288	264	288
18	66	44	154	176	110	220	198	88	286	264	242	264
19	72	48	168	192	120	240	216	96	312	288	264	288

3. The G1-G2 pattern

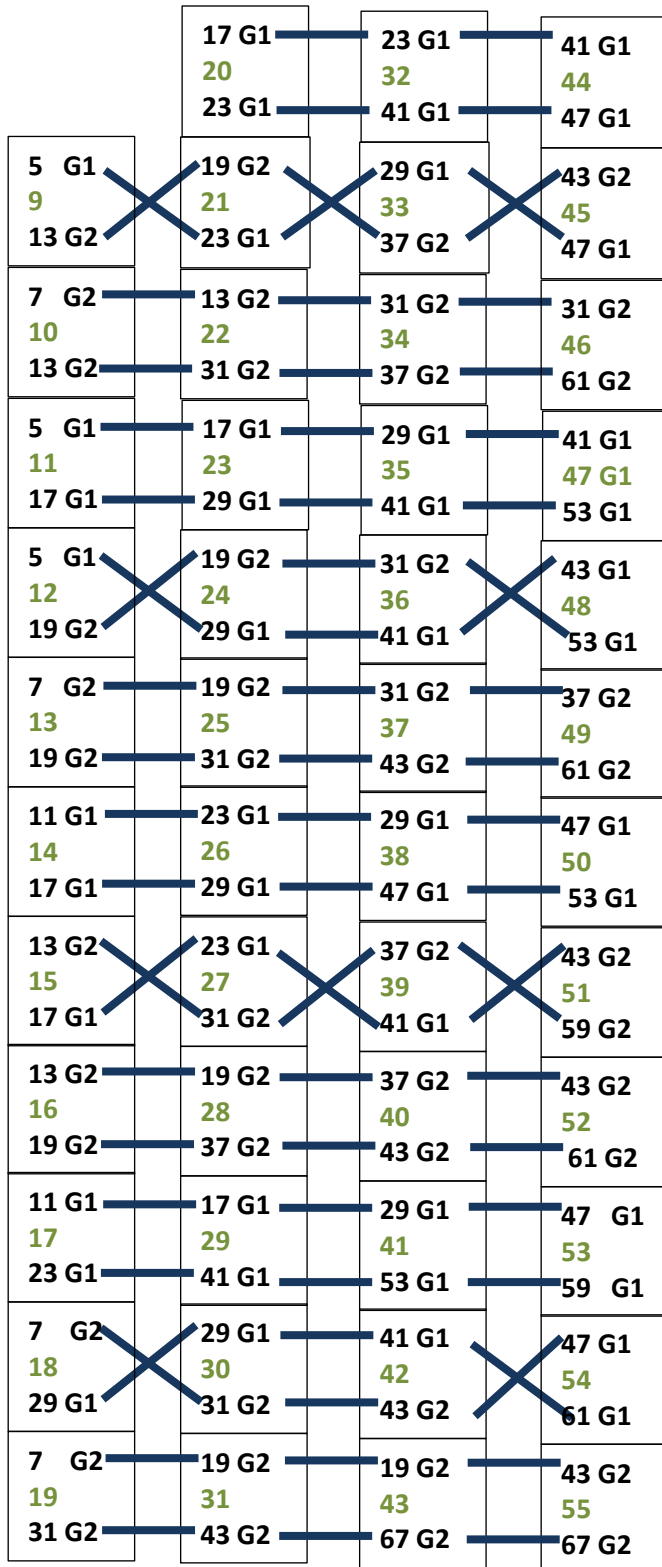
The triads of the primes with their equidistant primes are homogenous in the following sense. All the prime numbers can be classified in these two groups, G1 and G2:

5 G1 11 G1 17 G1	41 G1 47 G1 53 G1
7 G2 13 G2 19 G2	47 G1 53 G1 59 G1
11 G1 17 G1 23 G1	11 G1 17 G1 23 G1
7 G2 19 G2 31 G2	7 G2 19 G2 31 G2
17 G1 23 G1 29 G1	17 G1 23 G1 29 G1
17 G1 29 G1 41 G1	17 G1 29 G1 41 G1
19 G2 31 G2 43 G2	19 G2 31 G2 43 G2
31 G2 37 G2 43 G2	31 G2 37 G2 43 G2
29 G1 41 G1 53 G1	29 G1 41 G1 53 G1
19 G2 43 G2 67 G2	19 G2 43 G2 67 G2

- G1
- The smaller prime number in each twin pair: 11, 17, 29, 41...
 - The prime numbers in the triads that are the semisum of two equidistant smaller G1 twin primes: 23, 47, 52...Once a prime number results to be the semisum of two G1 primes, it remains G1 onward.
- G2
- The greater prime number in each twin pair: 13, 19, 31, 43...
 - The prime numbers in the triads that are the semisum of two equidistant greater G2 twin primes: 37, 67, 79...Once a prime number results to be the semisum of two G2 primes, it remains G2 in subsequent triads.

The three prime numbers that form each triad are homogeneous as to their G1 - G2 character.

A curious thing about the triads of the natural numbers and their primes is that, if they are displayed in columns of twelve, G1 and G2 draw a pattern:



4. The Goldbach conjecture

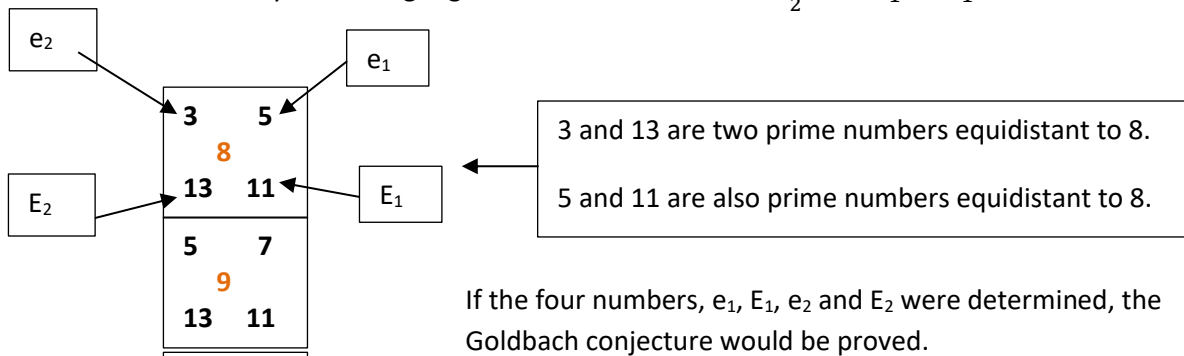
The strong Goldbach conjecture states that every even integer greater than 2 can be written as the sum of two primes.

To prove that the conjecture is true it would be more than enough to prove that any natural number greater than 7 has, at least, two equidistant primes E_1 and e_1

$$\exists E_1, e_1 \in P : n = \frac{E_1 + e_1}{2} \forall n \in N$$

Every even integer greater than 2 = 2.n, $n \in N$

$$\text{Every even integer greater than 2} = 2.n = 2 \cdot \frac{E_1 + e_1}{2} = E_1 + e_1$$



$$\left\{ \begin{array}{l} n = \frac{e_1 + E_1}{2} \\ n = \frac{e_2 + E_2}{2} \\ E_2 - E_1 + (e_1 - e_2) = \frac{E_2 \cdot e_1 - e_2 \cdot E_1}{n} \\ E_1 - e_1 + (E_2 - e_2) = \frac{E_2 \cdot E_1 - e_2 \cdot e_1}{n} \end{array} \right.$$

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