
Enseñar Matemáticas
en el siglo XXI

Centro Mediterráneo

Enseñar Matemáticas en el siglo XXI

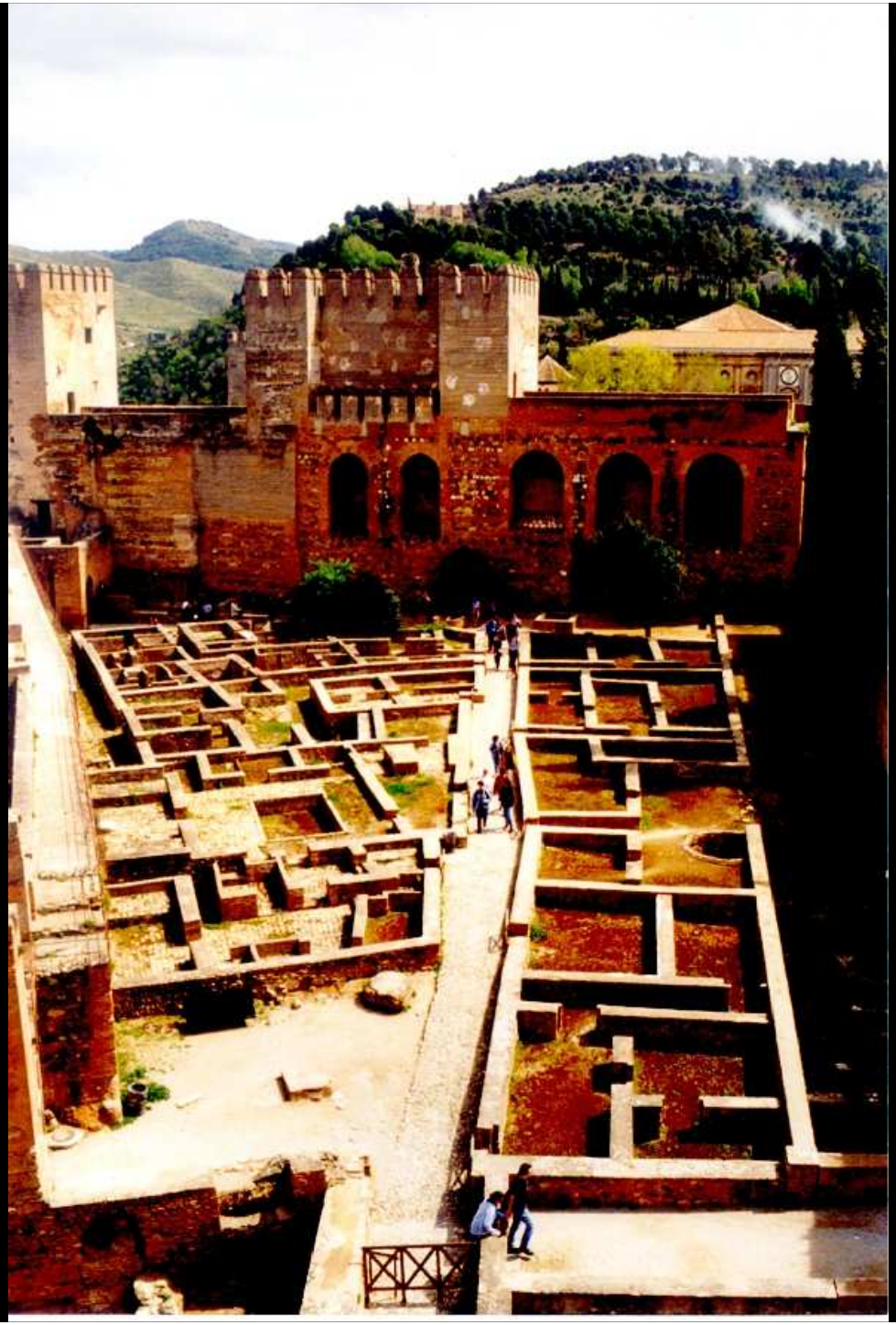
Visita matemática (virtual) a la Alhambra

por

Ceferino Ruiz Garrido

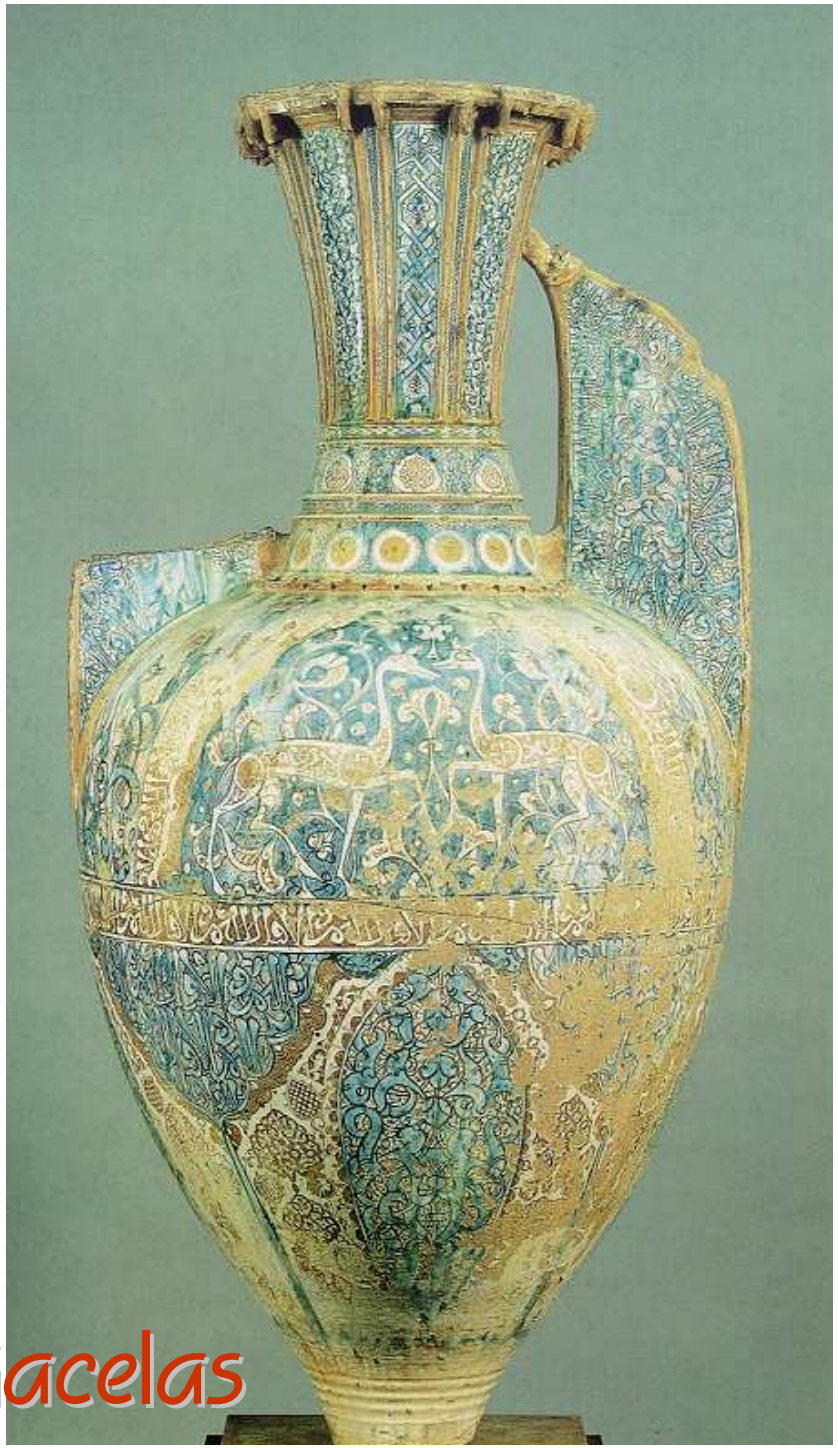
Catedrático de Geometría y Topología,

9 de julio de 2009

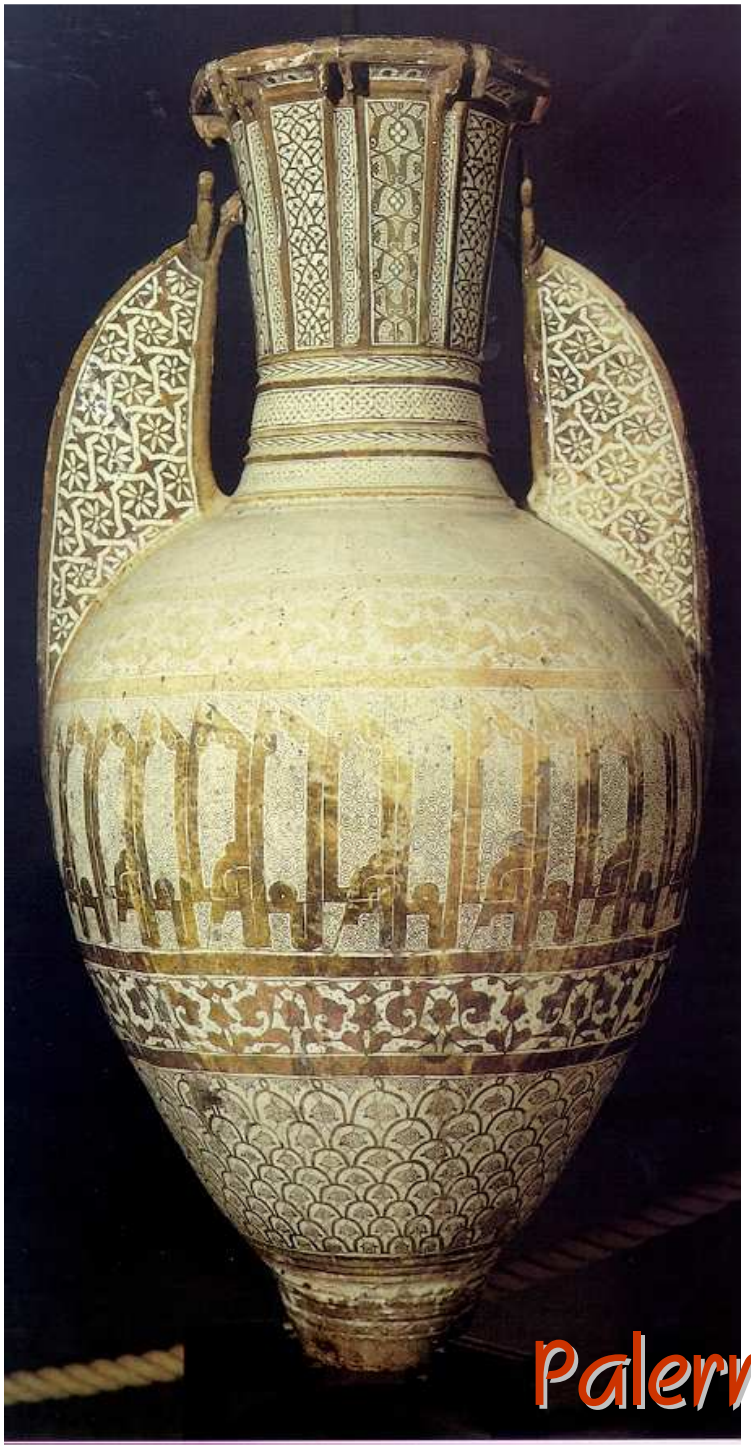


La Alhambra. 1, Patio de Machuca; 2, Torre de Machuca; 3, Mexuar; 4, Patio del Cuarto Dorado; 5, Cuarto Dorado; 6, Patio de Comares; 7, Sala de la Barca; 8, Sala de Comares; 9, Baño Real; 10, Patio de la Reja; 11, Habitaciones de Carlos V; 12, Torre del Peinador de la Reina; 13, Jardín de Daraxa; 14, Mirador de Daraxa; 15, Sala de Dos Hermanas; 16, Patio de los Leones; 17, Sala de los Mocárabes; 18, Sala de los Reyes; 19, Aljibe; 20, Sala de los Abencerrajes; 21, Capilla del Palacio de Carlos V.

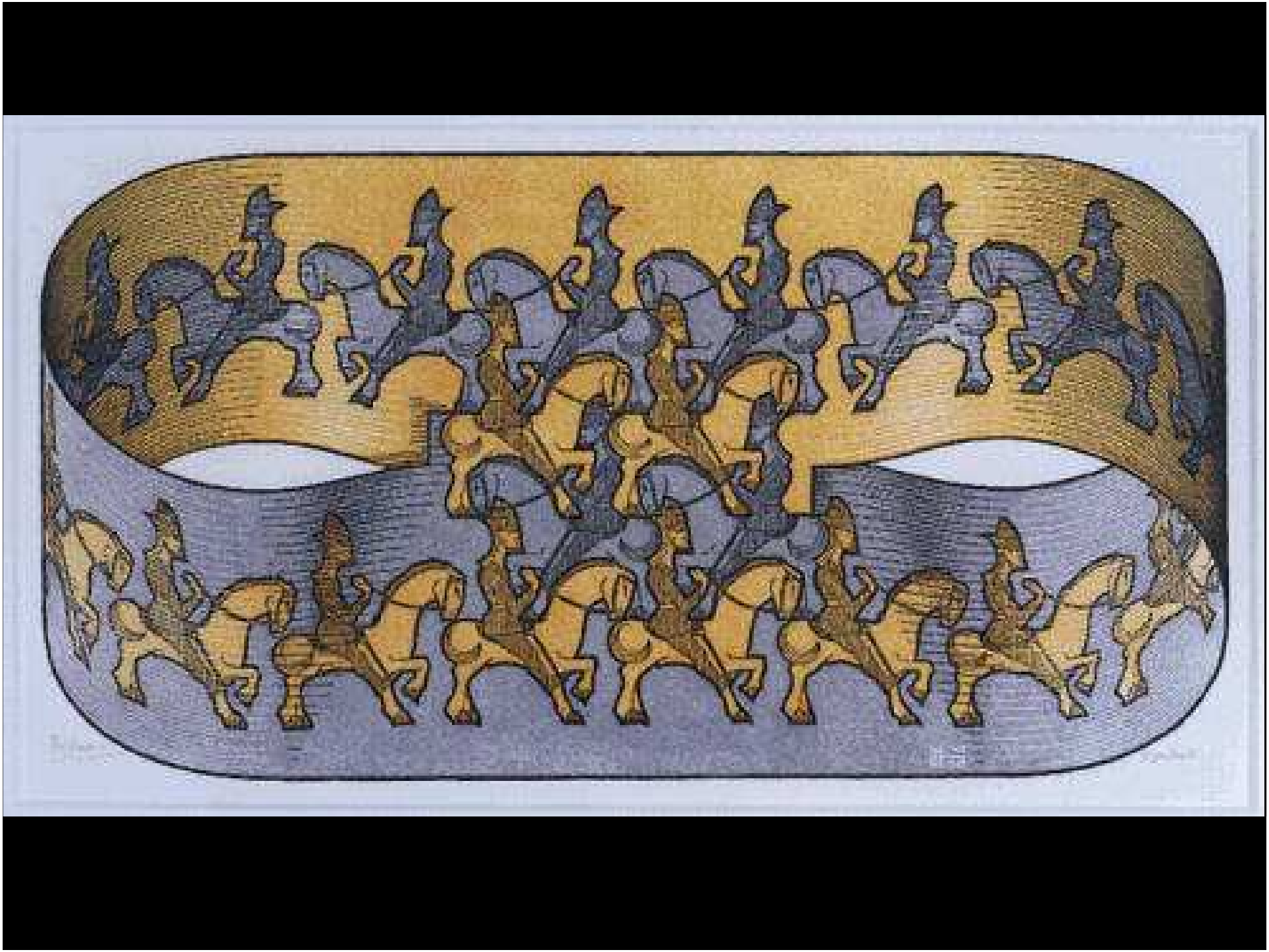
Decoración Nazarí



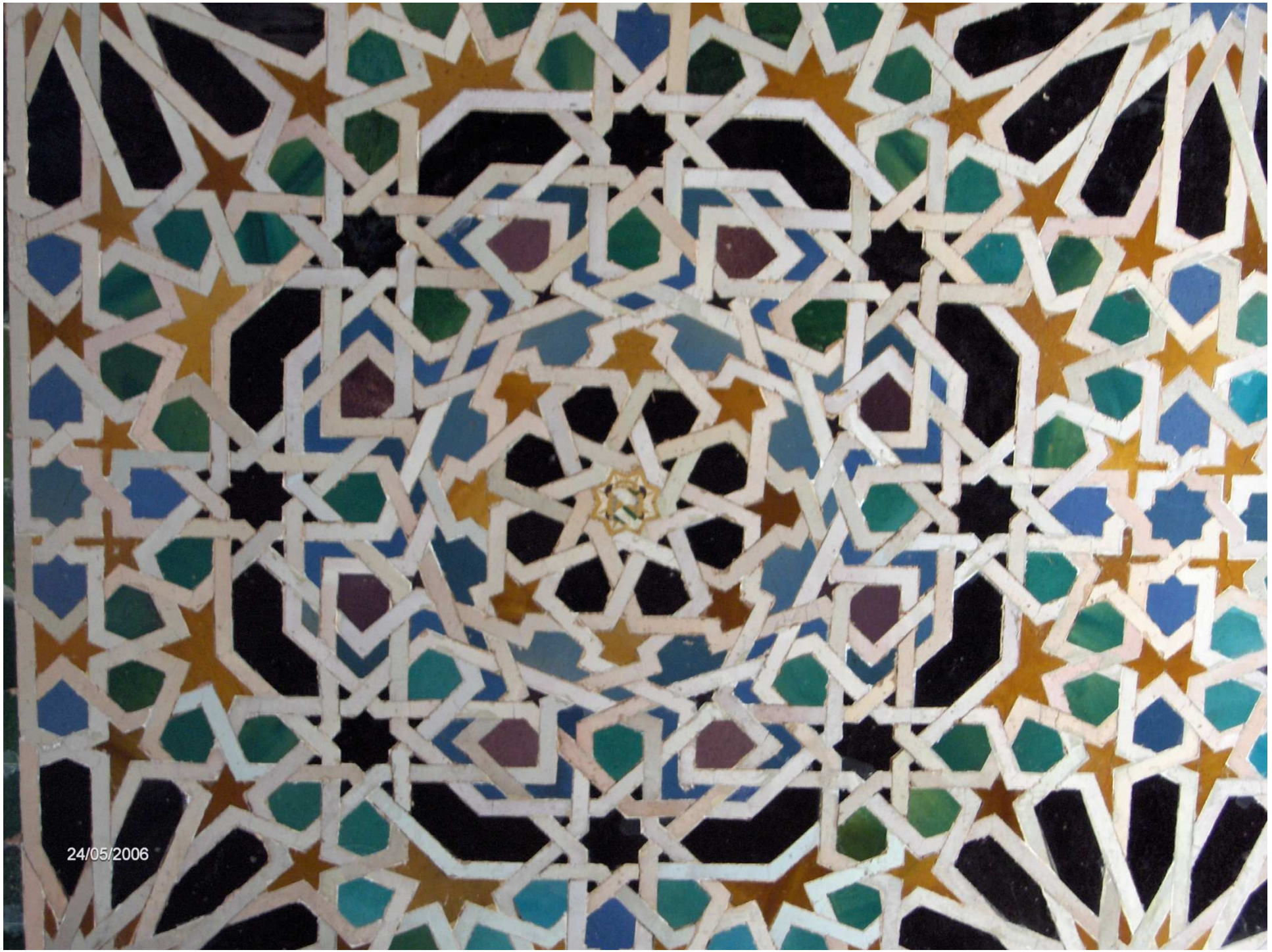
Gacelas

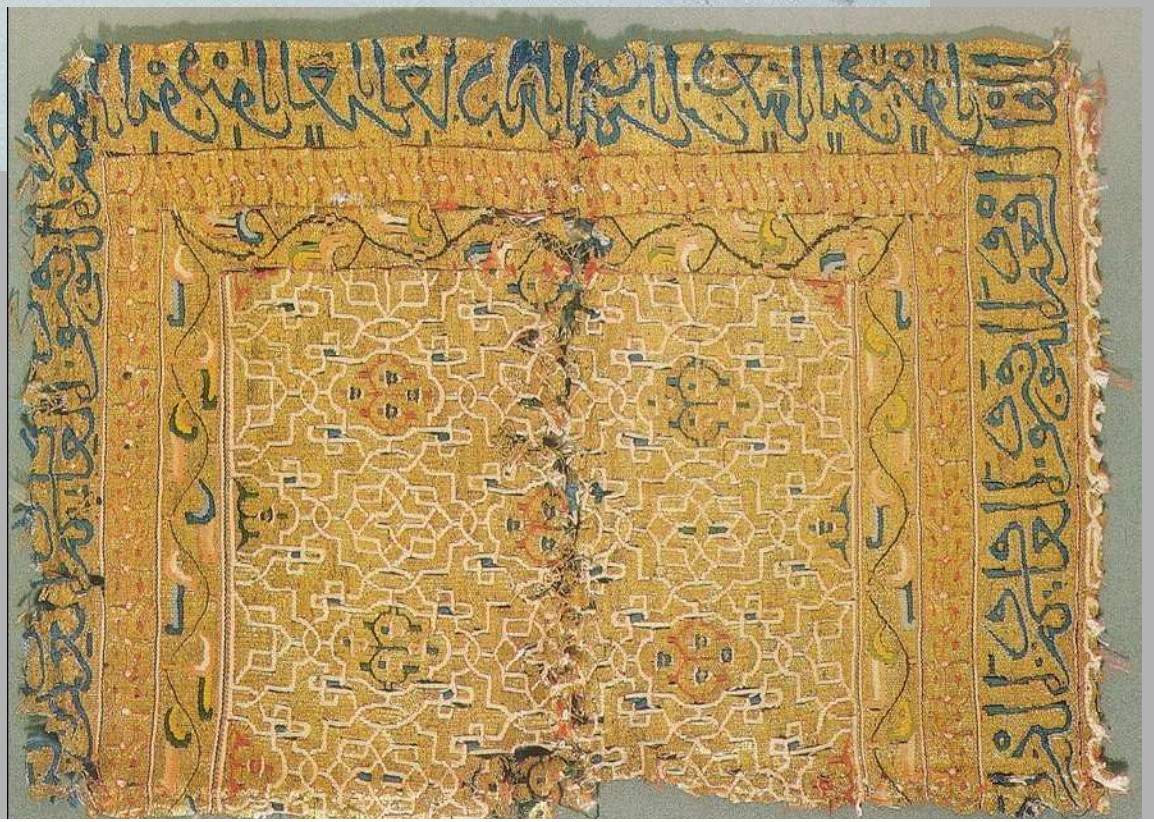


Palermo



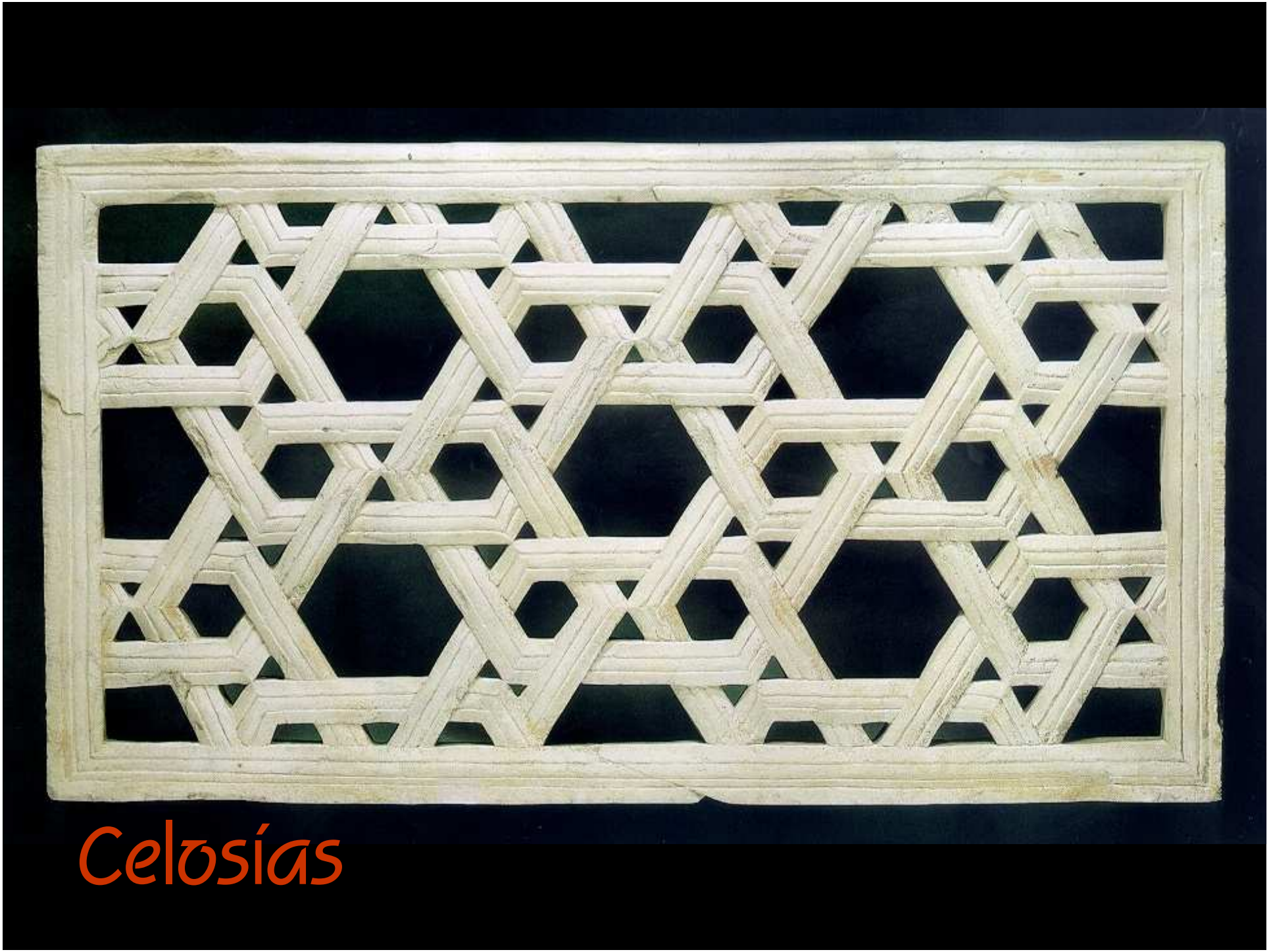

24/05/2006


24/05/2006



Tejidos

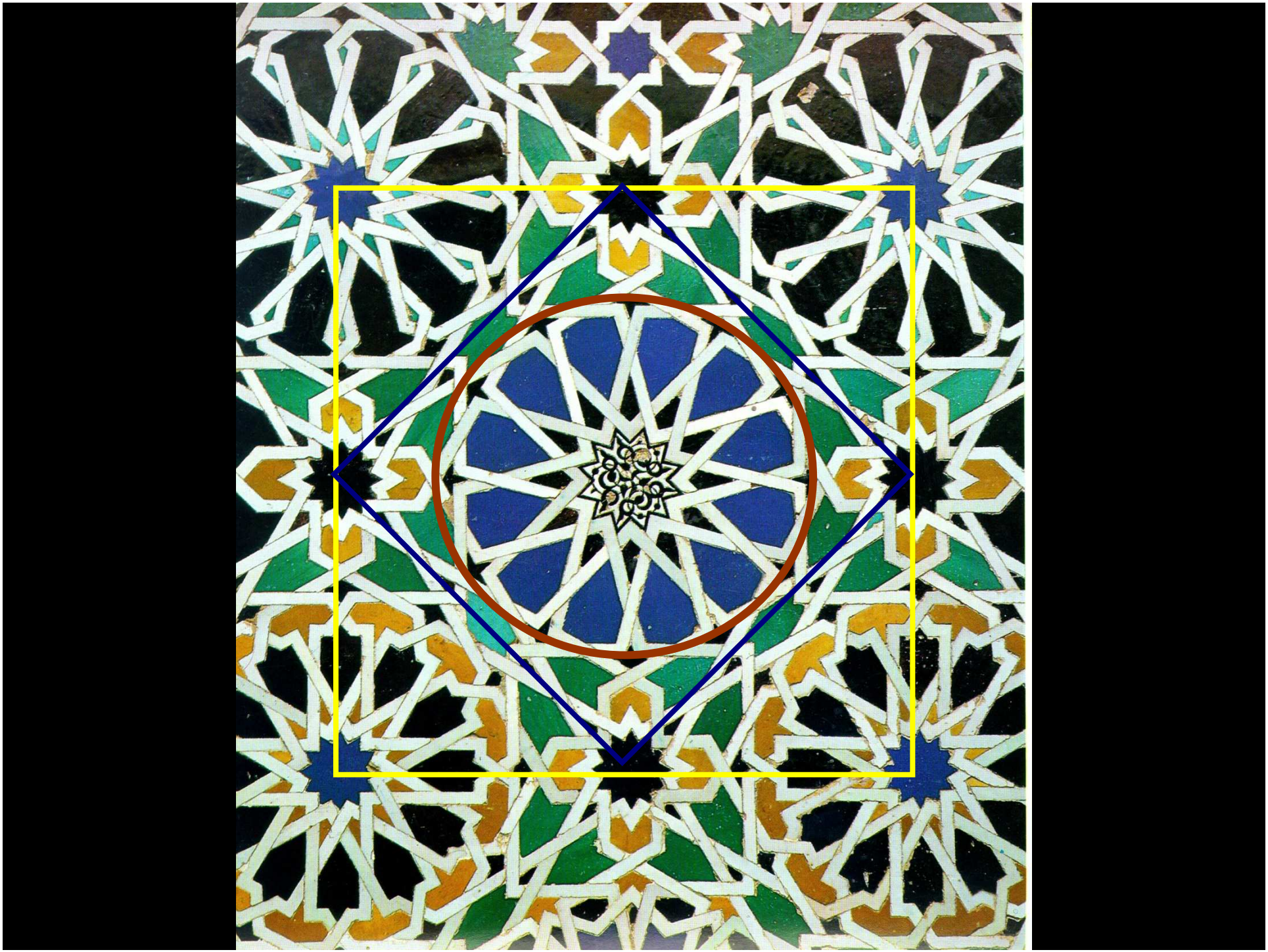
Libros

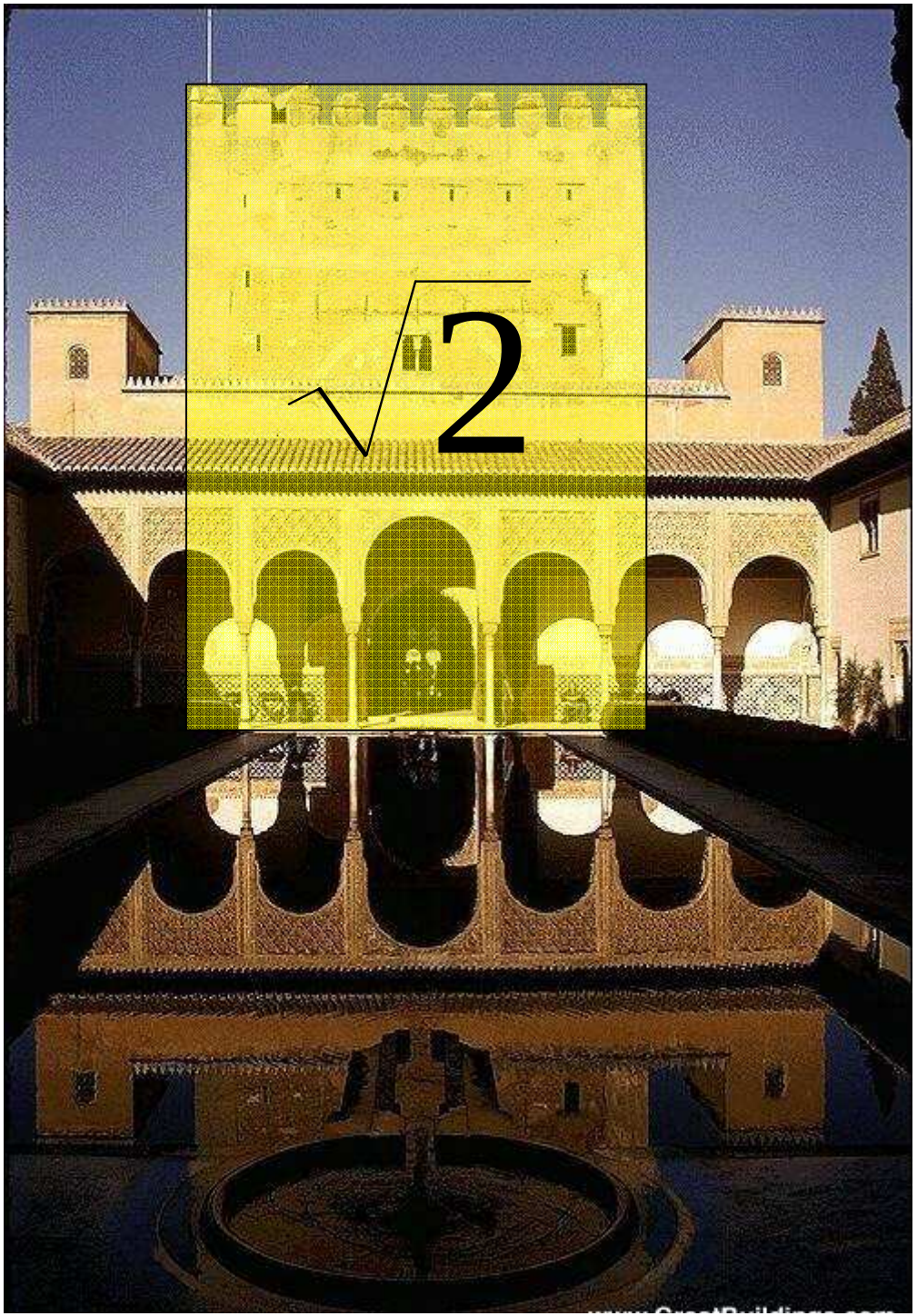


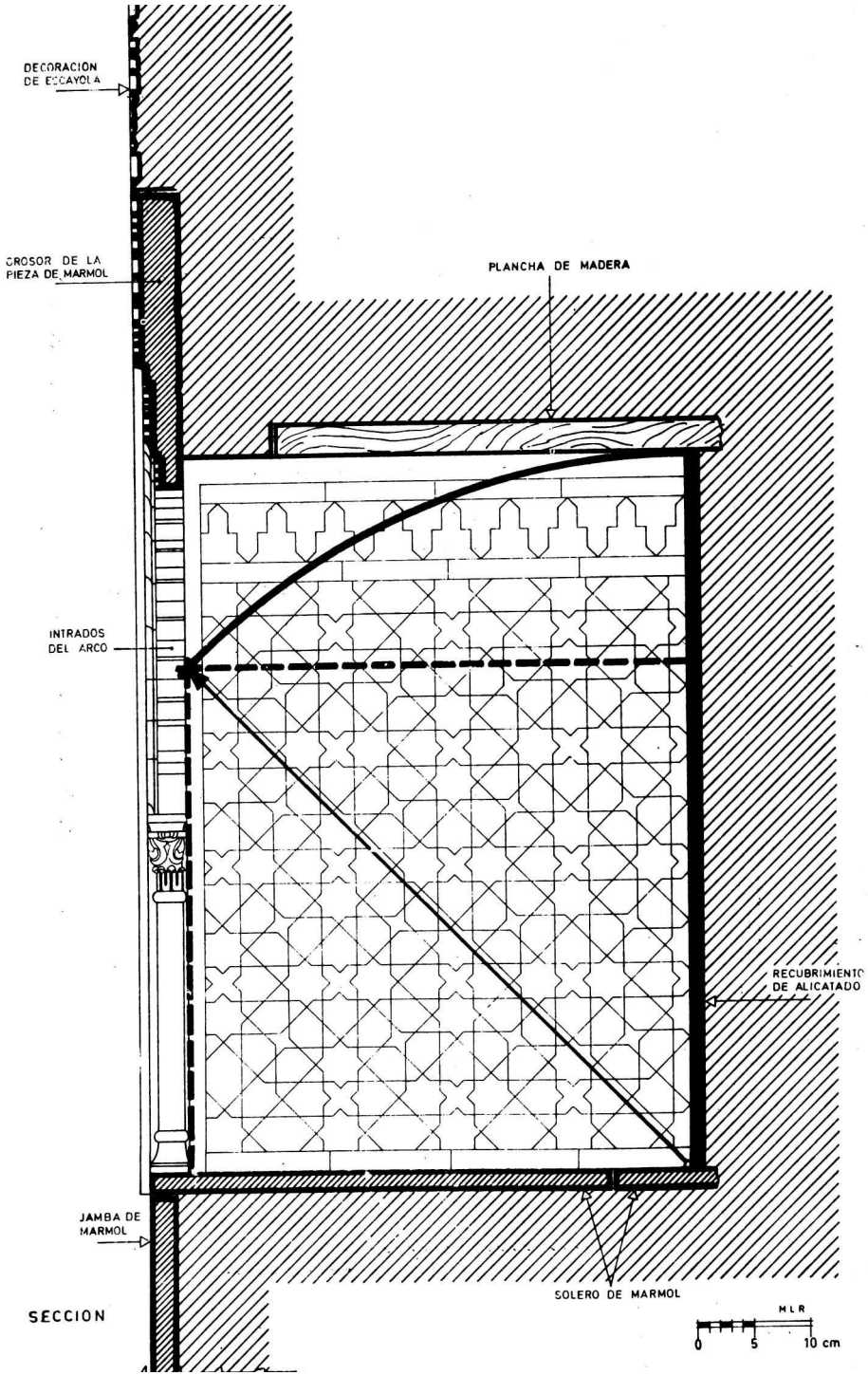
Celosías


Taraceas

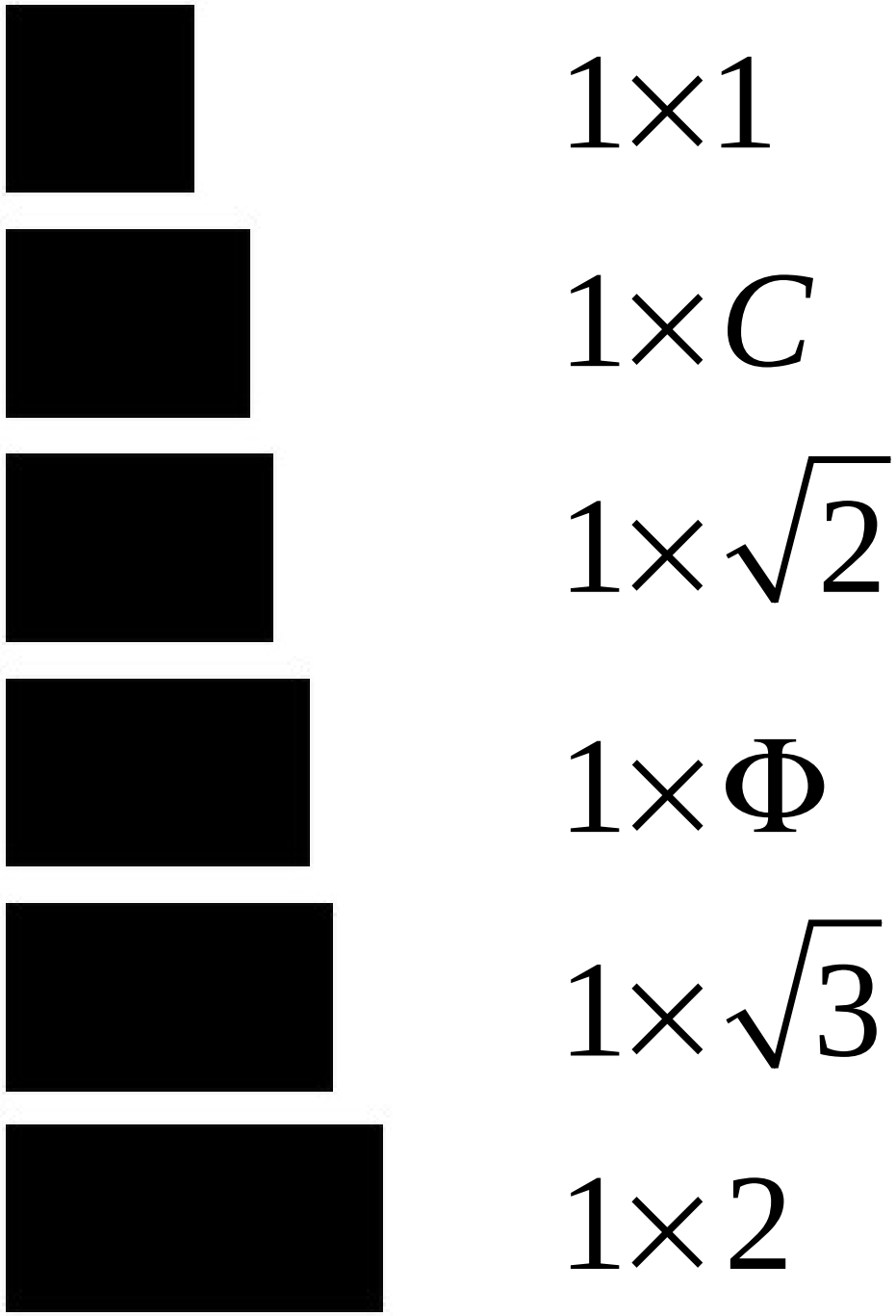
Proporciones básicas en La Alhambra

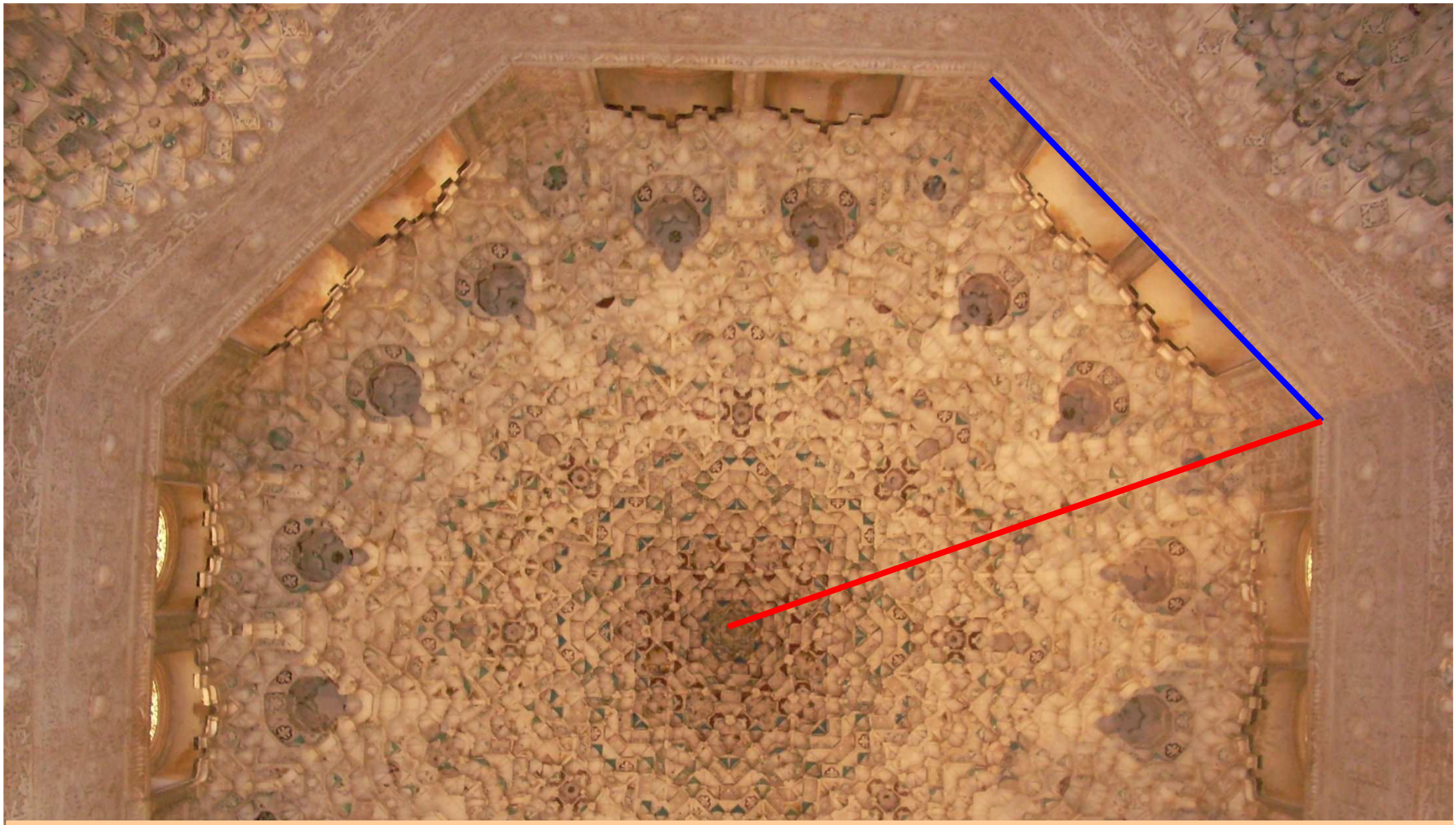



$\sqrt{2}$

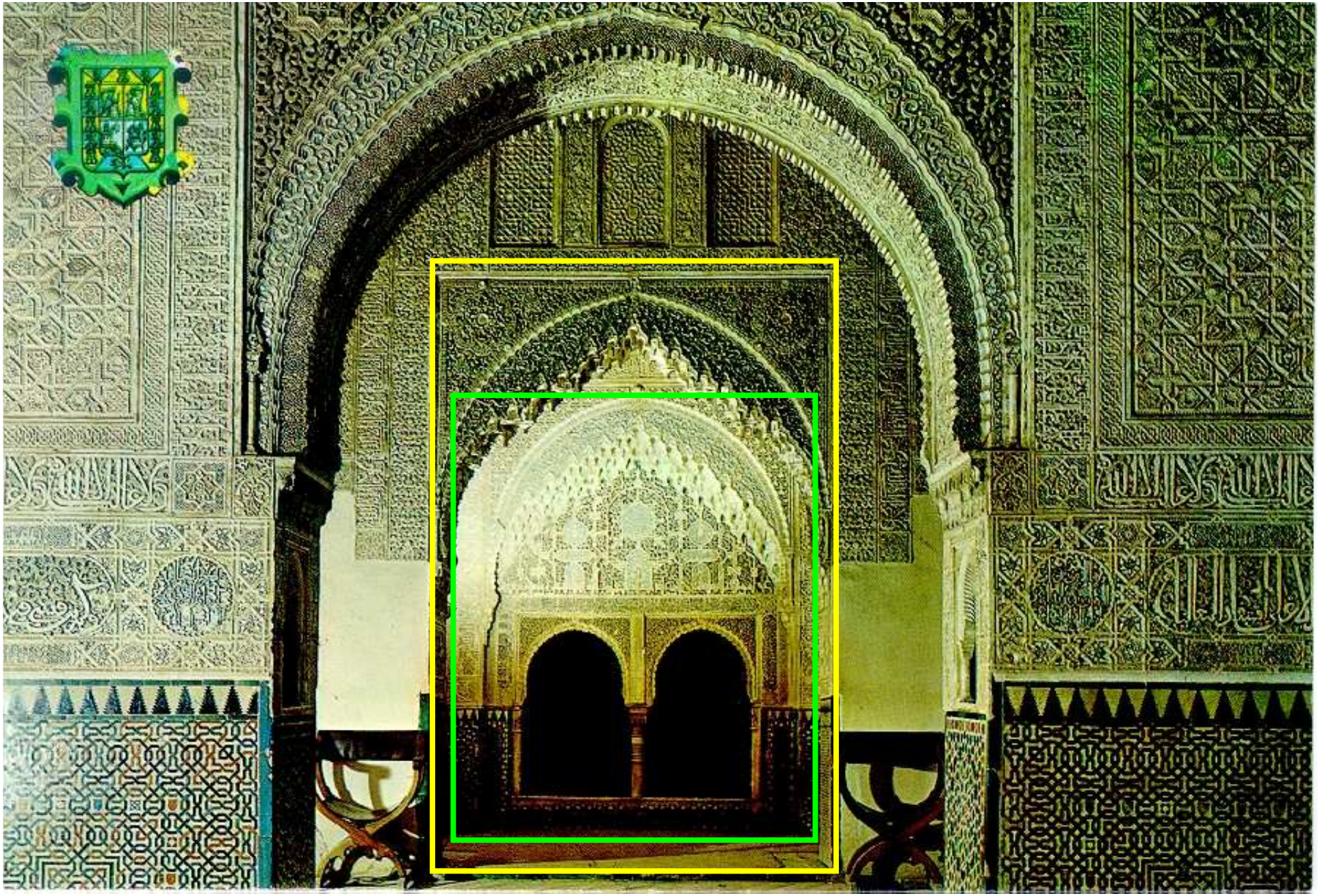

DECORACION DE ESCAYOLA
GROSOR DE LA PIEZA DE MARMOL
PLANCHA DE MADERA
INTRADOS DEL ARCO
RECUBRIMIENTO DE ALICATADO
JAMBA DE MARMOL
SOLERO DE MARMOL
SECCION
M L R
0
5
10 cm


$\sqrt{3}$
$\sqrt{2}$

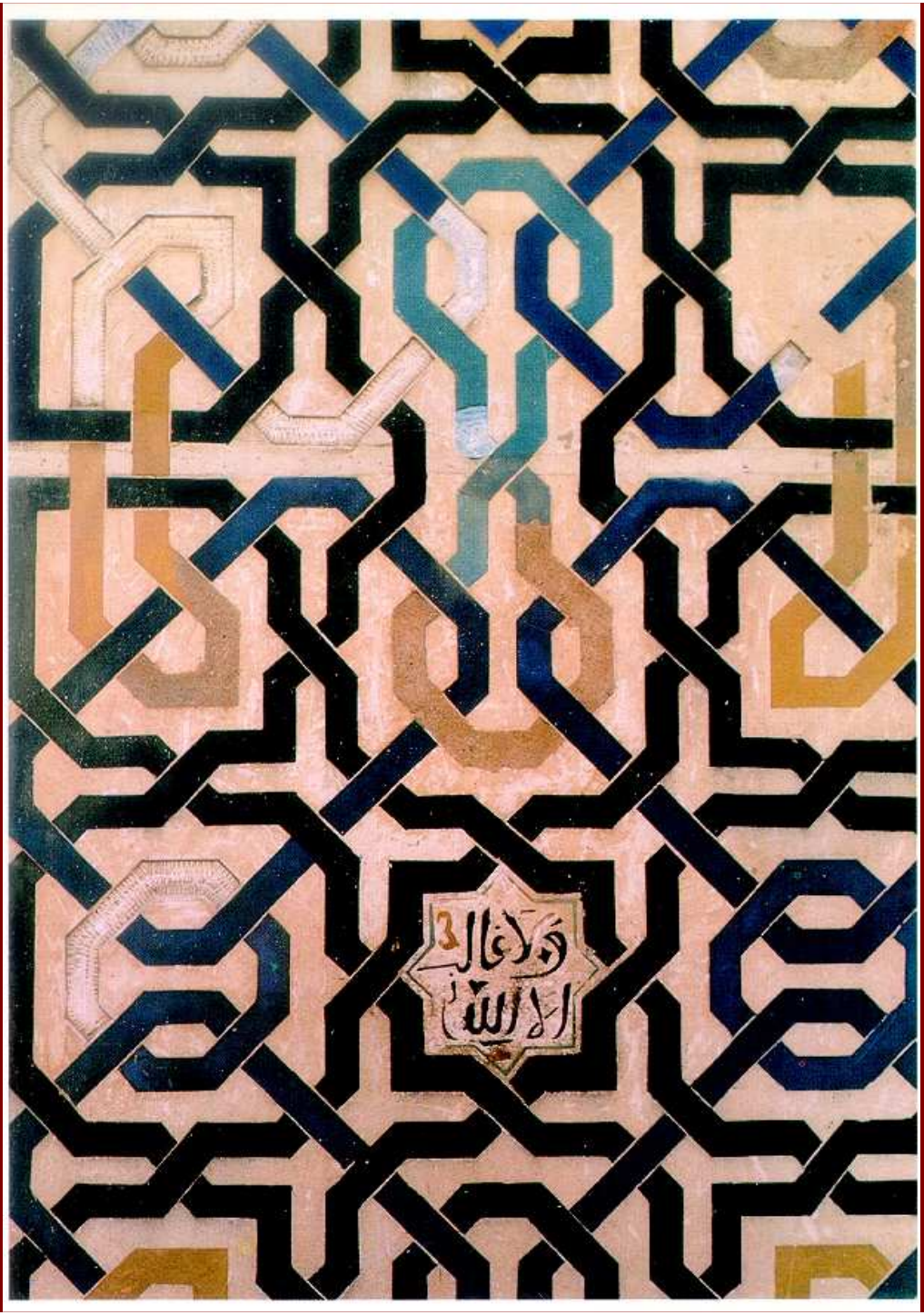

1×1
1×C
1×√2
1×Φ
1×√3
1×2



$$C = \frac{1}{\sqrt{2 - \sqrt{2}}} \approx 1{,}30656296487$$




C
www.sol.com
SpainOnLine


لا غالب
الا الله

$$\Phi := \frac{\ell}{x} = \frac{x}{\ell - x},\ 0 < x < \ell$$

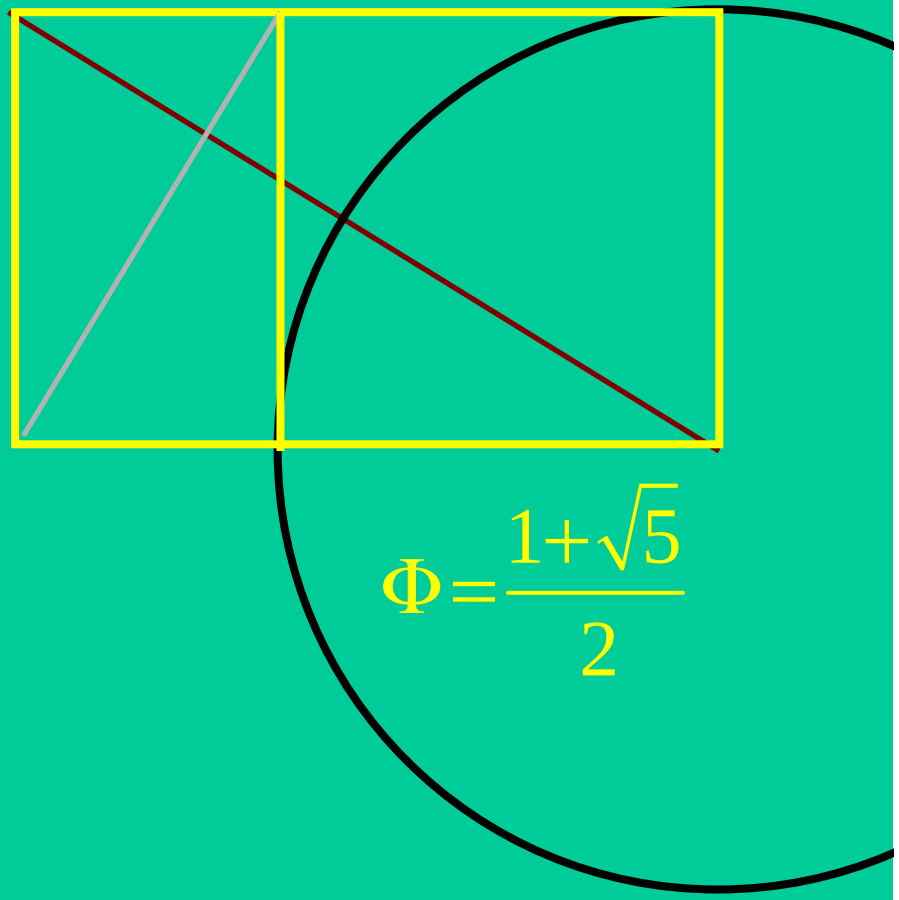
$(\ell = 1)$

$$x^2 = 1 - x \ \Leftrightarrow\ x^2 + x - 1 = 0$$

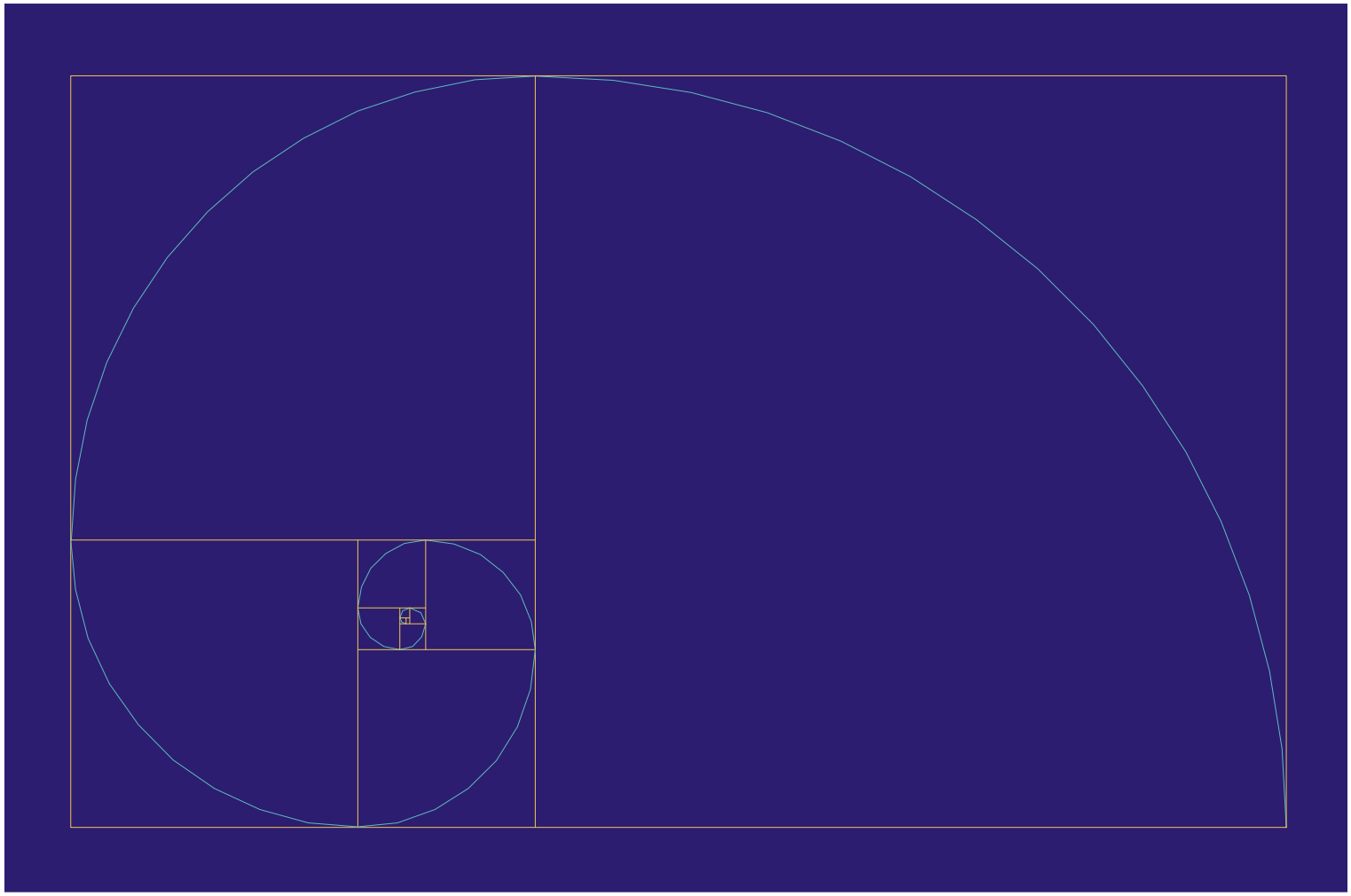
$$x = \frac{-1 \pm \sqrt{1+4}}{2};\ x = \frac{\sqrt{5}-1}{2}$$

$$\frac{1}{\Phi} = \frac{\sqrt{5}-1}{2} \qquad \frac{1}{\Phi} = \Phi - 1$$

$$\Phi = \frac{2}{\sqrt{5}-1} = \frac{2(\sqrt{5}+1)}{5-1} = \frac{\sqrt{5}+1}{2}$$



$$\Phi = \frac{1+\sqrt{5}}{2}$$

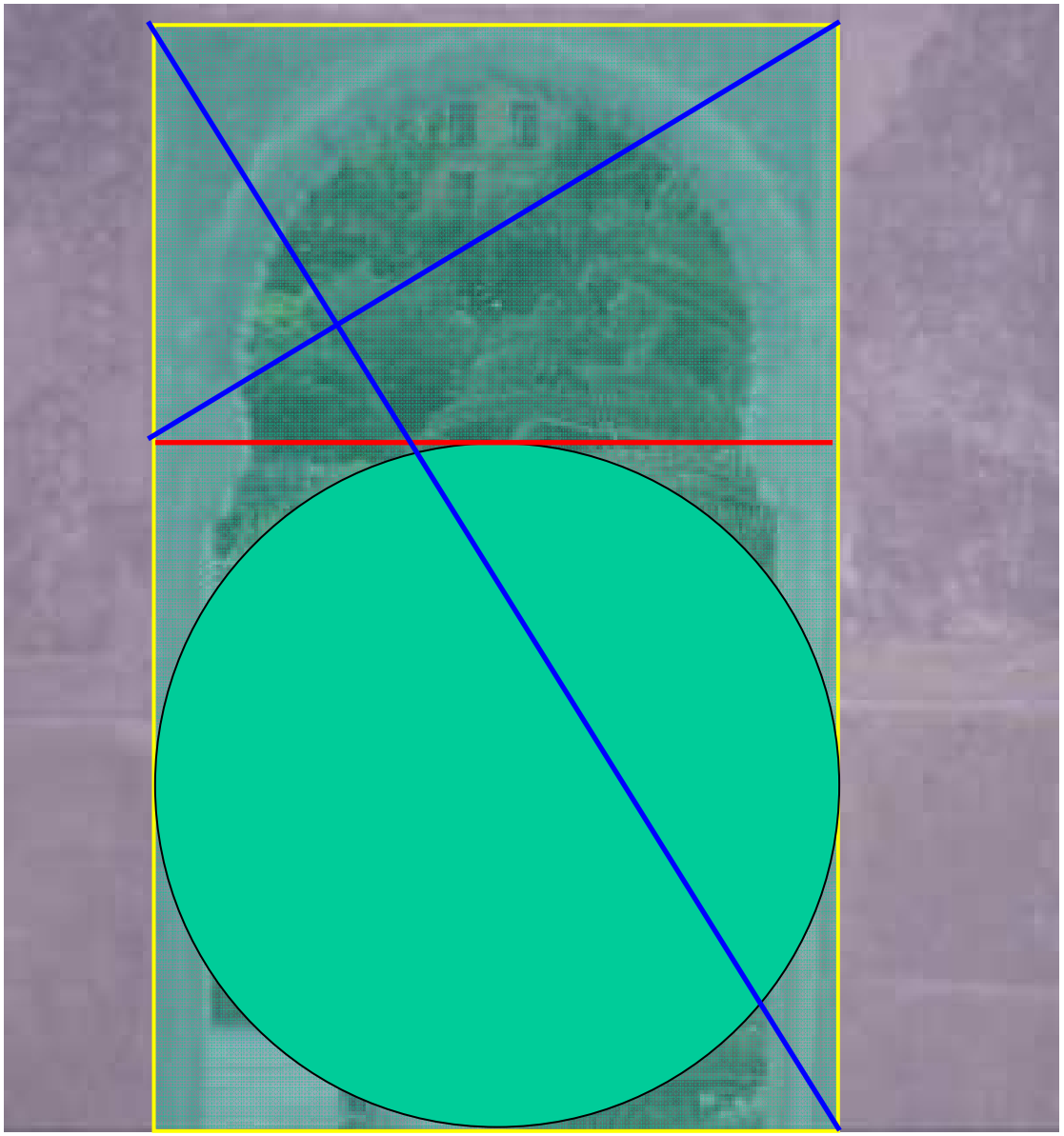


Espiral de Durero

Proporción áurea

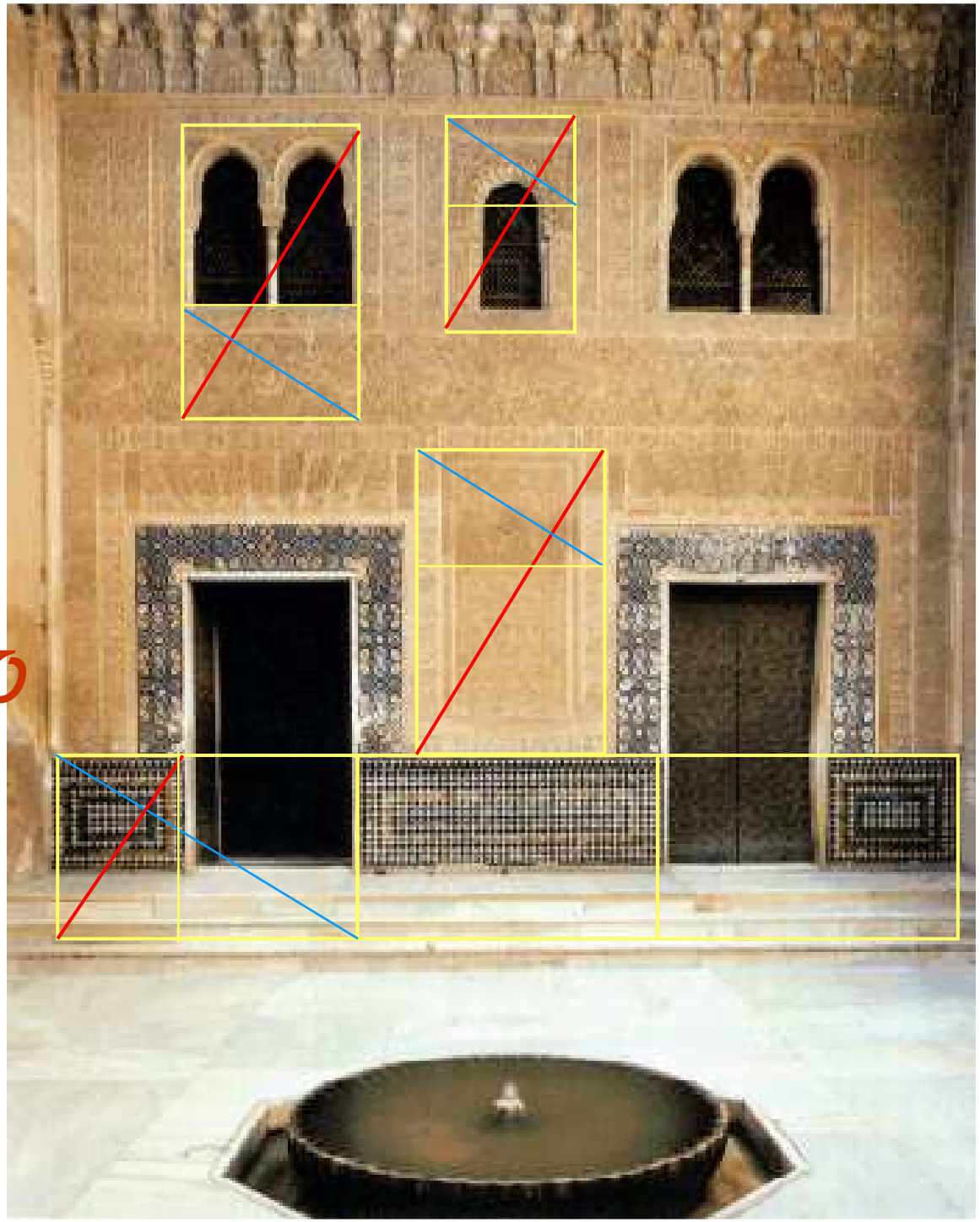
o

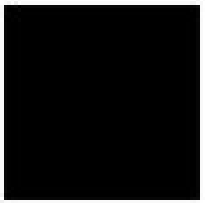
Divina proporción



$$\Phi = \frac{1+\sqrt{5}}{2} \approx 1{,}618033988749894848204$$

Fachada del Palacio de Comares en el Patio del Cuarto Dorado



1×1  $1 \approx 1,00000$

$1 \times C$  $C \approx 1,30656$

$1 \times \sqrt{2}$  $\sqrt{2} \approx 1,41421$

$1 \times \Phi$  $\Phi \approx 1,61803$

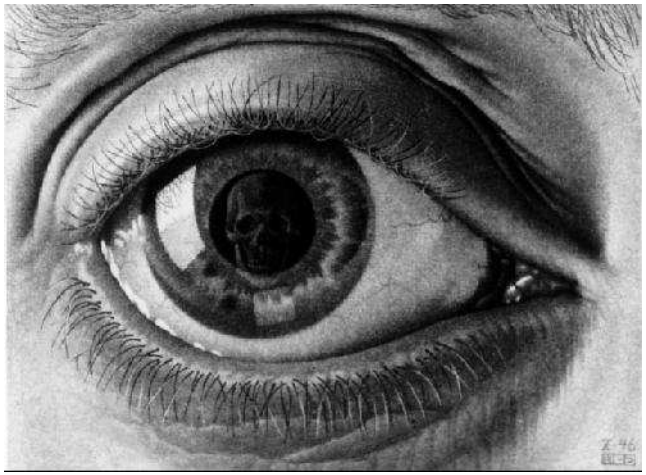
$1 \times \sqrt{3}$  $\sqrt{3} \approx 1,73205$

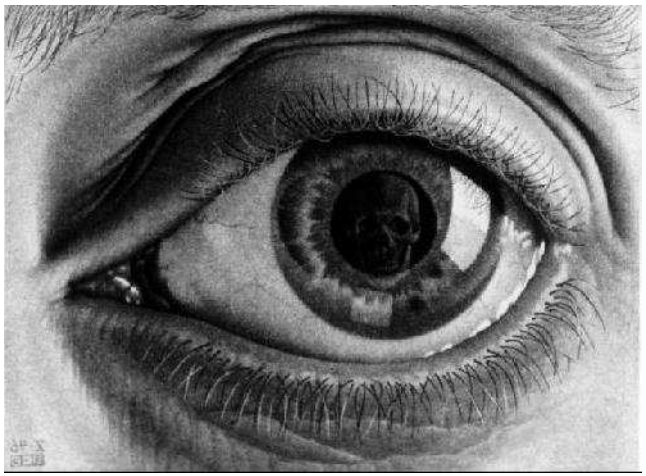
1×2  $2 \approx 1,99999$

Movimientos euclídeos

	Directos	Inversos
Sin puntos fijos	**Translaciones**	**Reflexiones en deslizamiento**
Con puntos fijos	**Rotaciones**	**Reflexiones**

Reflexión o Simetría (axial)





El horizonte está en los ojos y no en la realidad.

Ángel Ganivet

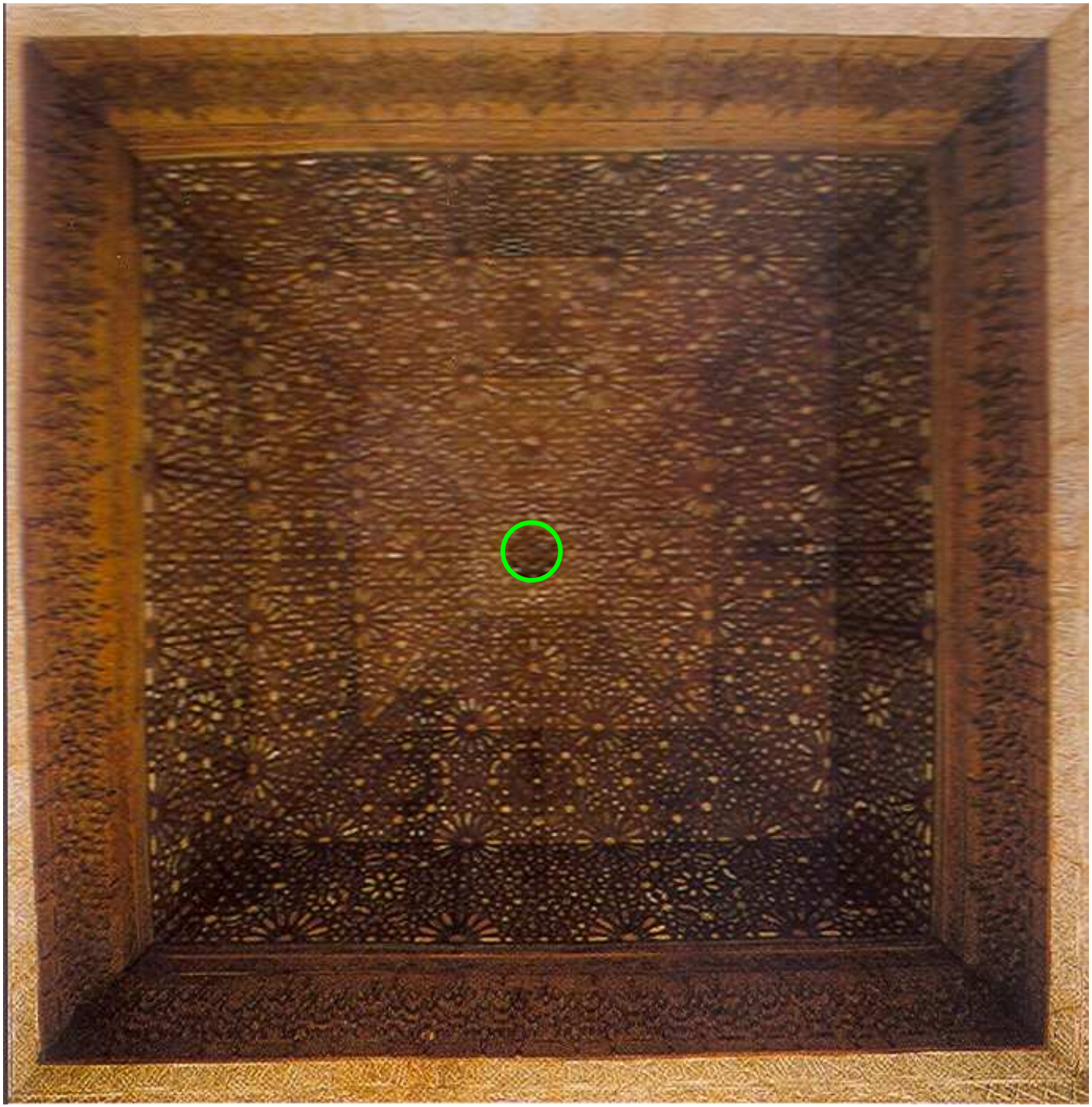
Movimientos euclídeos en La Alhambra

Rosetones o Rosáceas

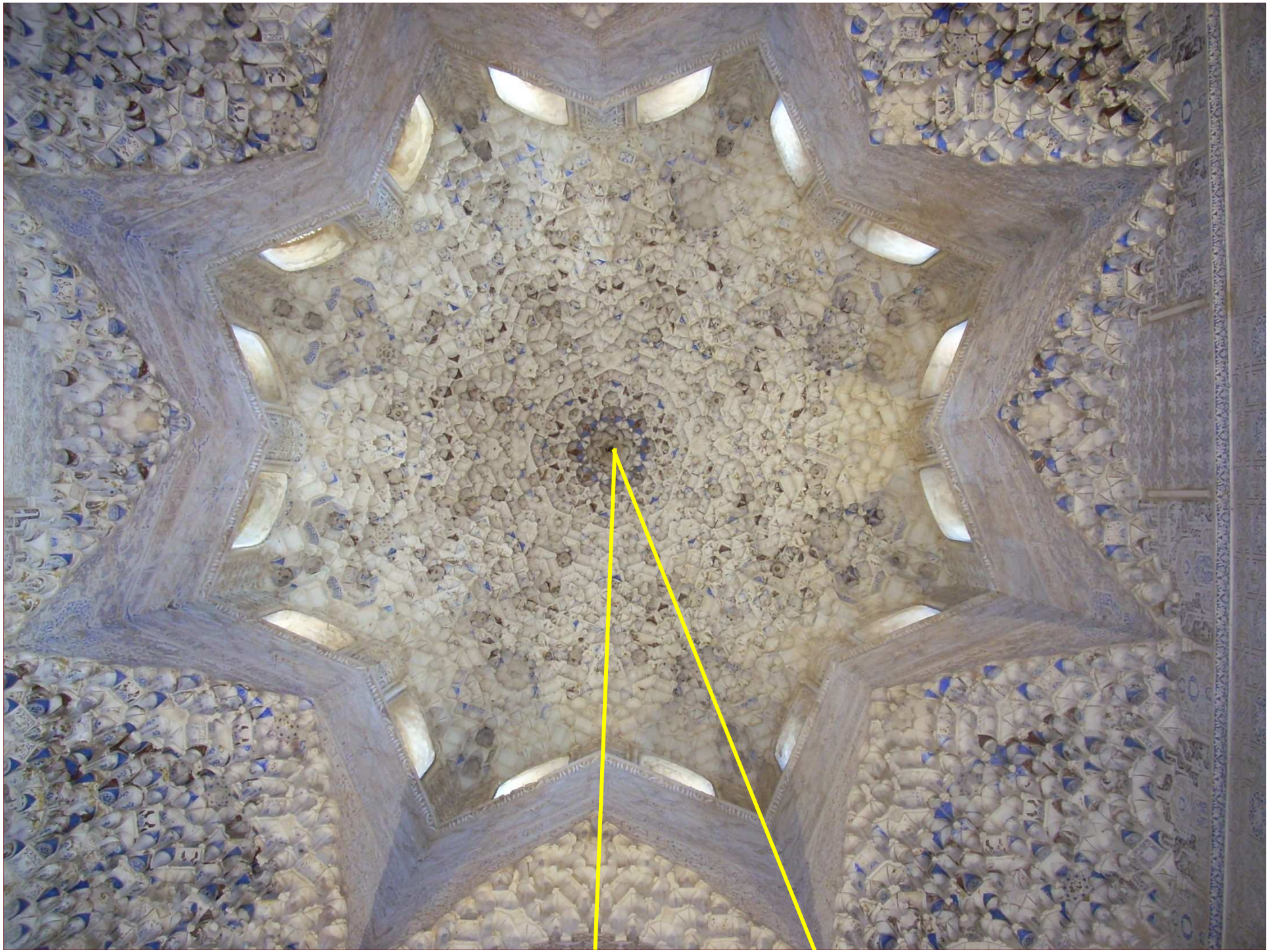
Diseños invariantes por isometrías euclídeas sin translaciones.

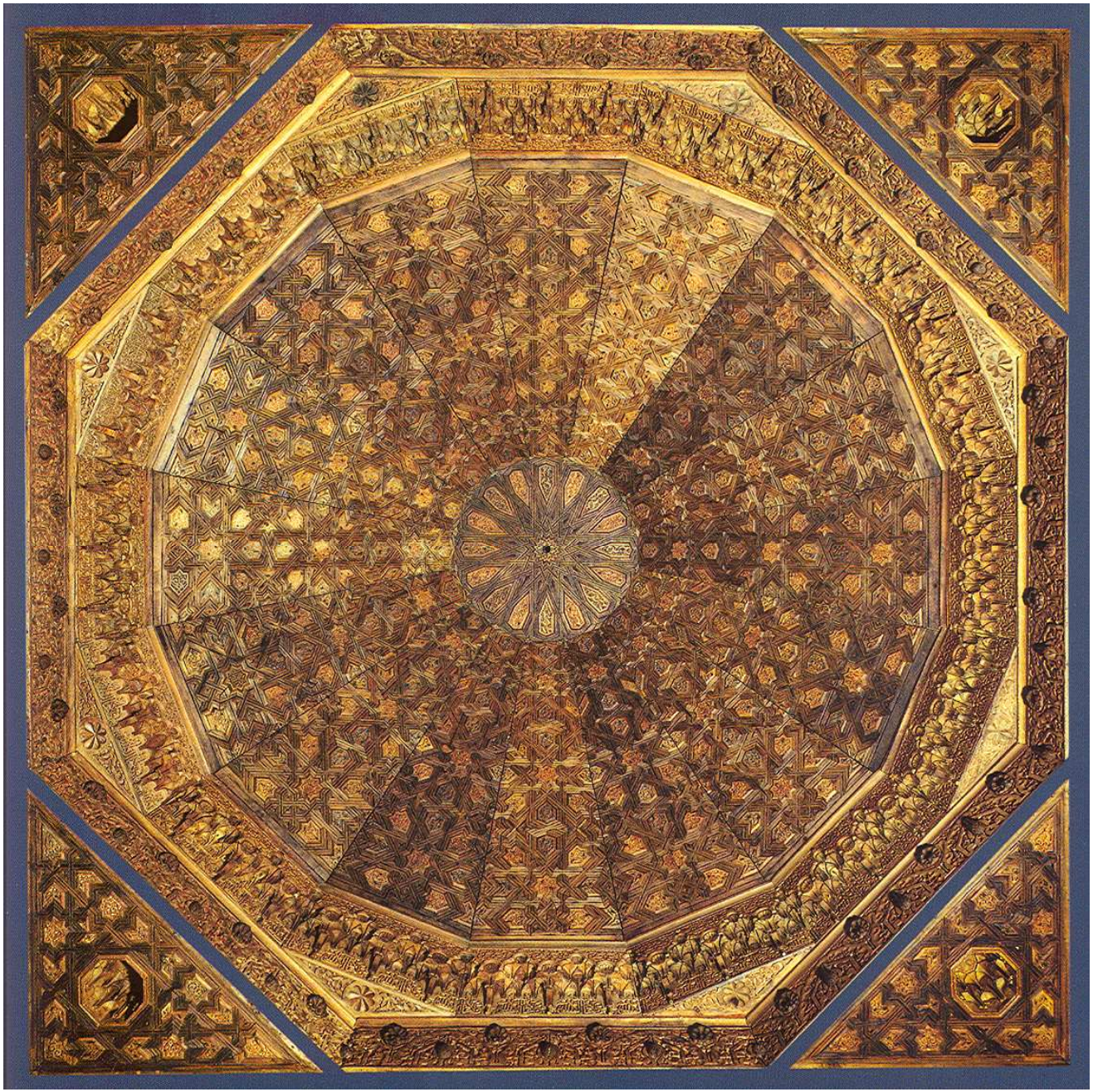
Dos familias infinitas de grupos:

- ***Cn, cíclico de orden n,***
- ***Dn, diédrico de orden n.***




epsilon





Techo de la Torre de las Damas

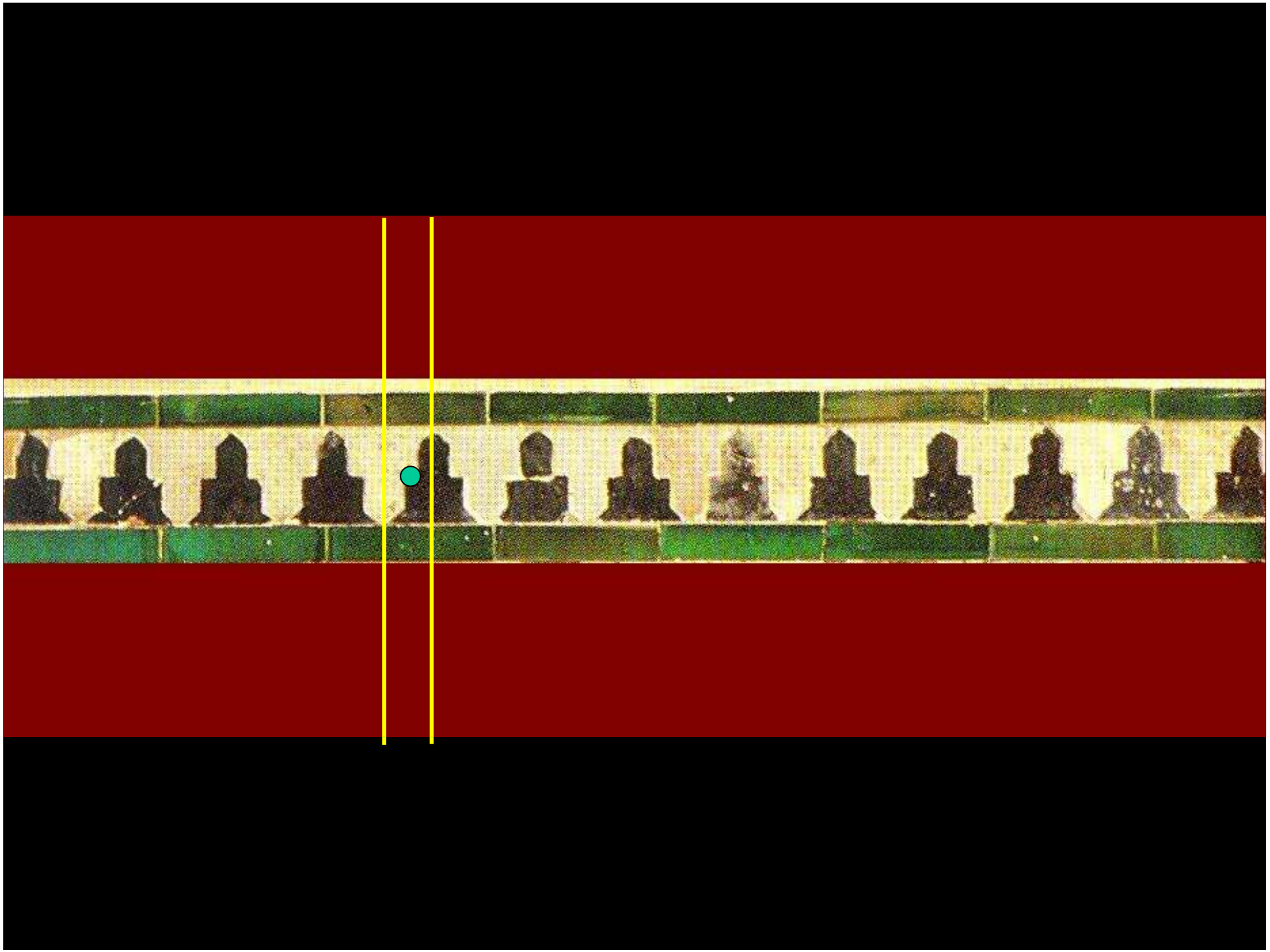
(Museo de Pérgamo, Berlín)

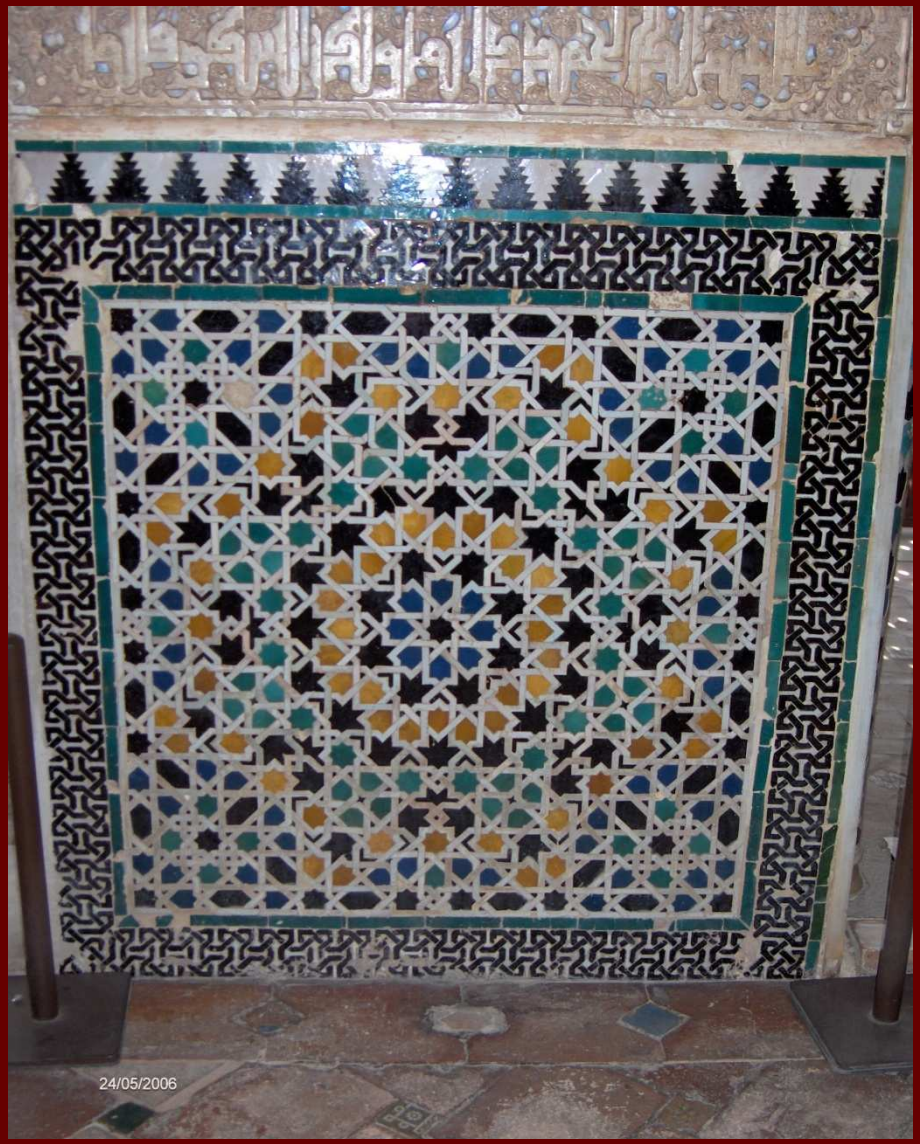

16/05/2007 17:06

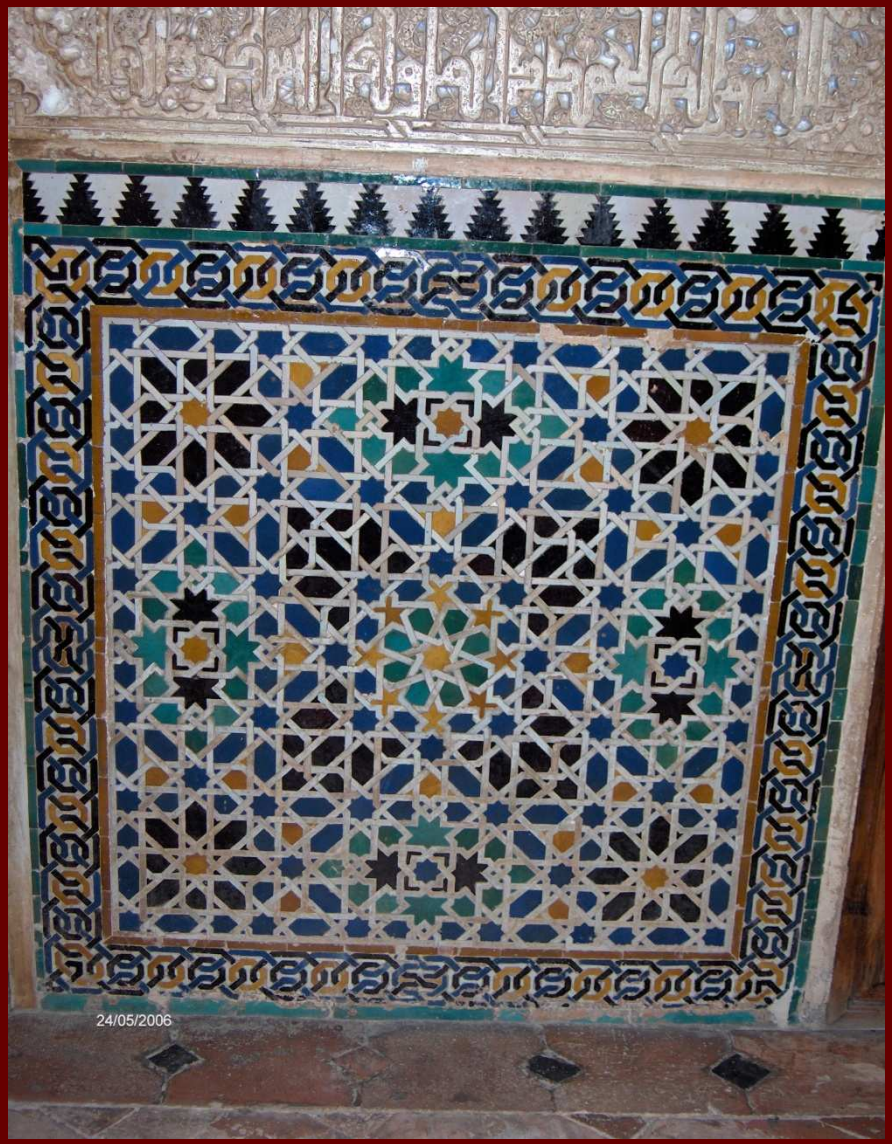
Frisos, Cenefas o Grecas

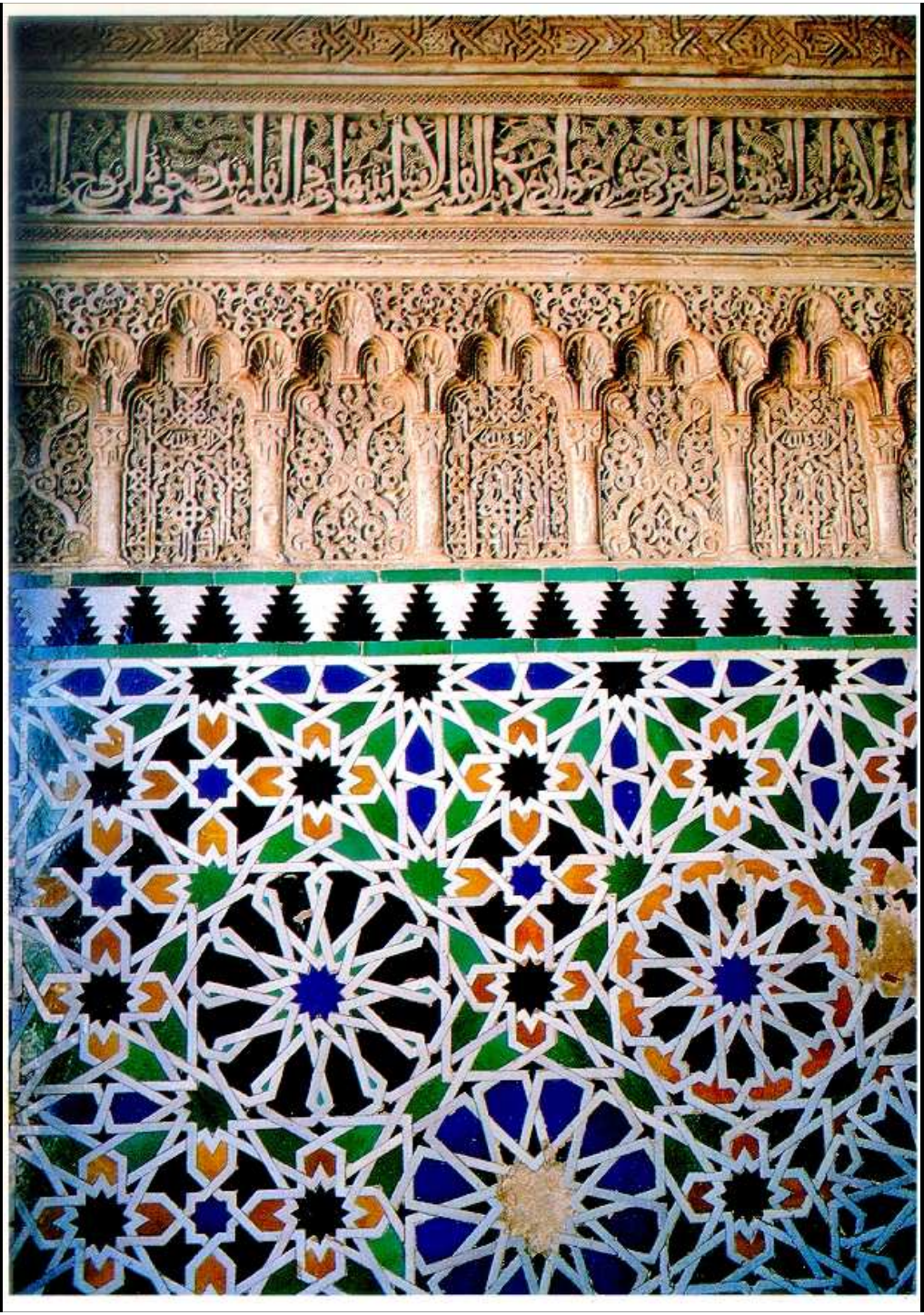
Diseños invariantes por isometrías euclídeas con todas las traslaciones en la misma dirección:

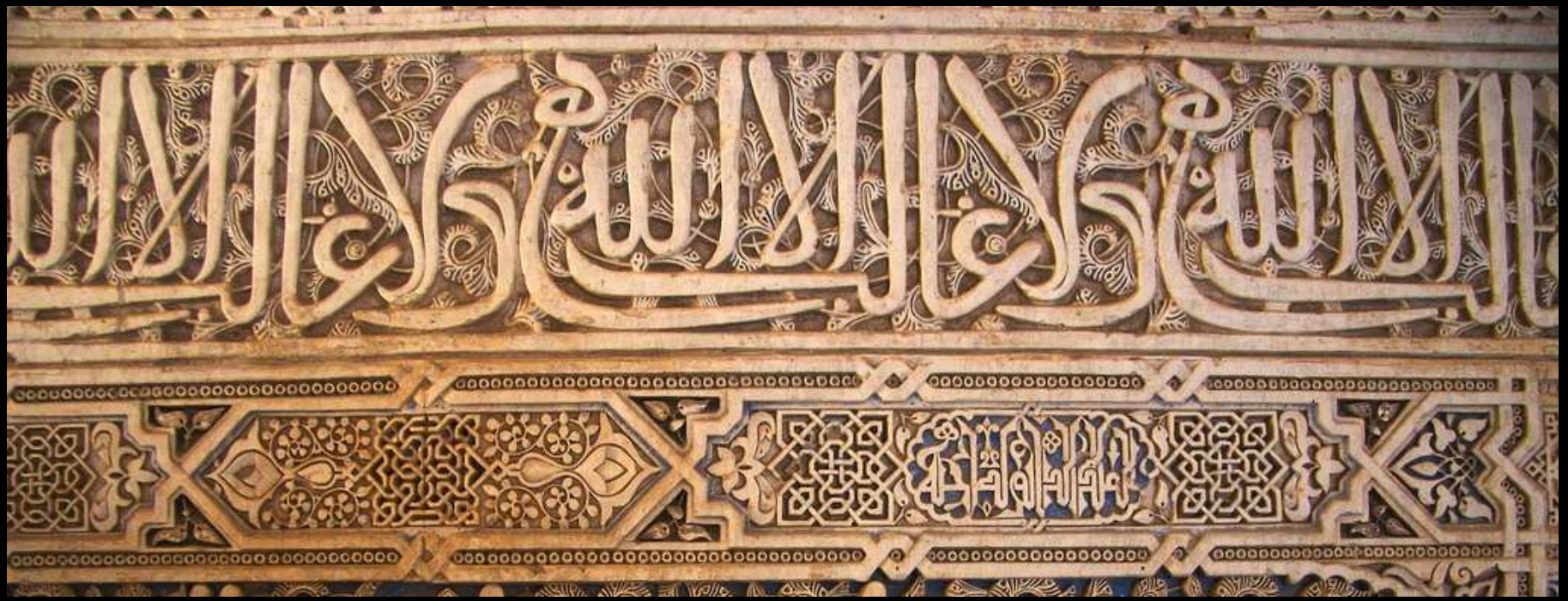
7 clases afines de grupos.




24/05/2006


24/05/2006





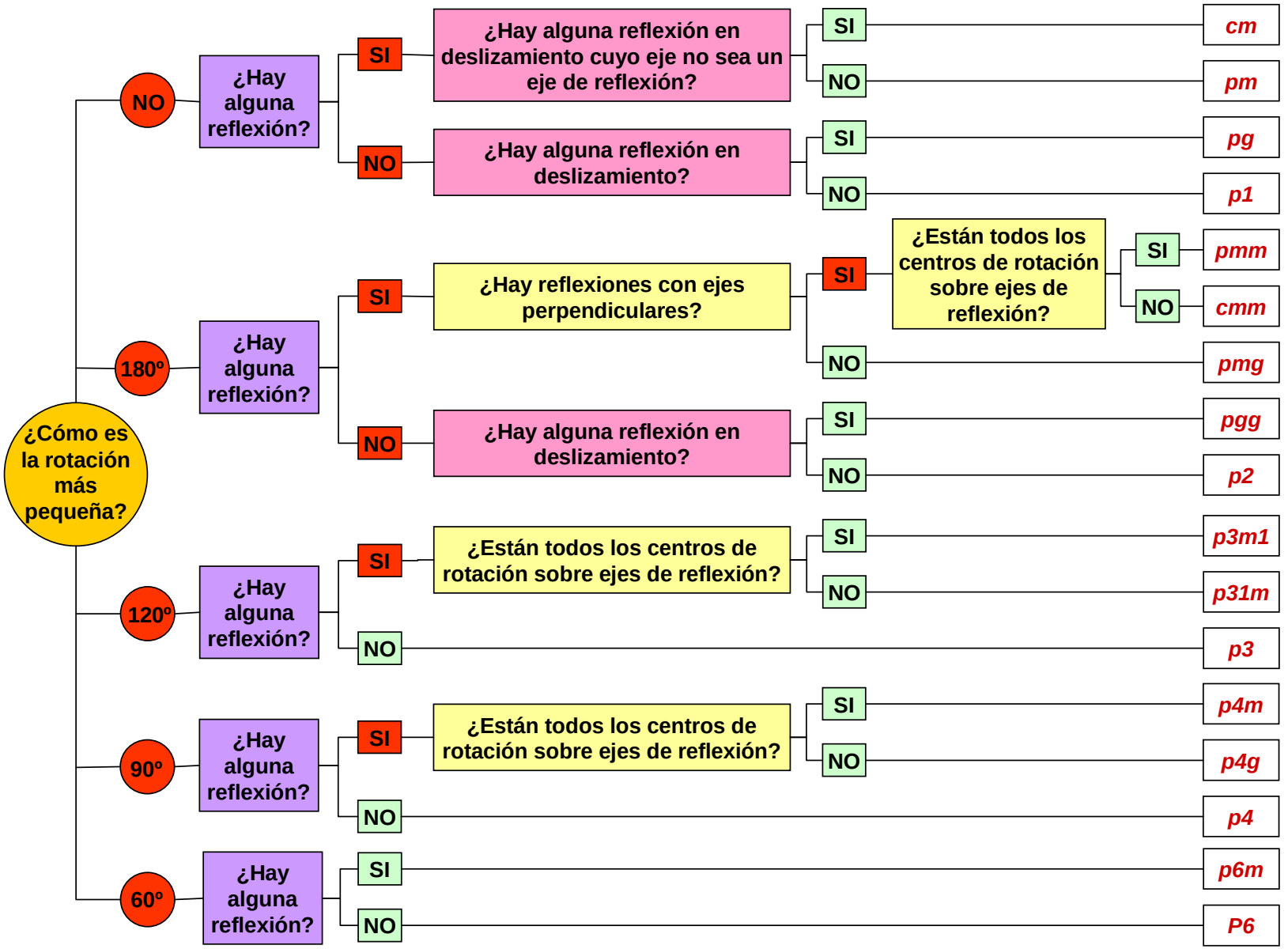
Mosaicos, Embaldosados, Alicatados, Arabescos, Teselados y Kaleidoscopios.

Diseños periódicos invariantes por isometrías euclídeas con translaciones en direcciones independientes:

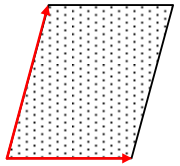
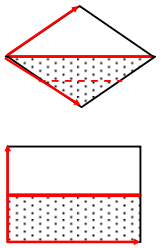
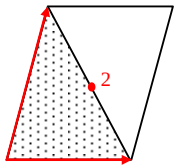
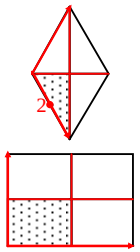
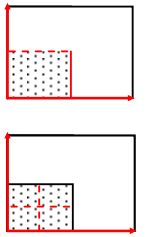
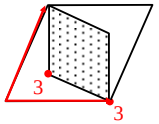
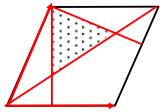
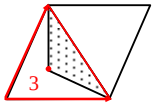
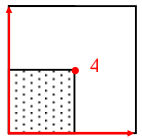
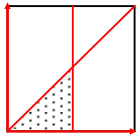
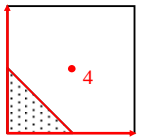
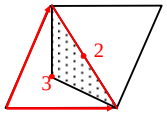
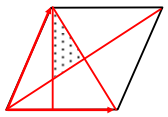
17 clases afines de grupos.


p1

Algoritmo de Reconocimiento de Grupos Cristalográficos Planos



Clasificación de Grupos Cristalográficos Planos

Sin isometrías inversas		Con isometrías inversas			
p1 (p111) t_a, t_b		***cm (c1m1)*** σ_L, $\sigma_{L'a/2}$, a$\|\|$L, L' $\neq$ L ***pm (p1m1)*** t_a, t_b, σ_L, a$\|\|$L, b $\perp$ L		***pg (p1g1)*** t_b, $\sigma_{L,a/2}$, a $\perp$ b	
p2 (p211) t_a, t_b, $r_{C,\pi}$		***cmm (c2mm)*** $r_{C,\pi}$, σ_L, σ_M, L $\perp$ M, C $\notin$ L, C $\notin$ M ***pmm (p2mm)*** t_a, t_b, σ_L, σ_M, a$\|\|$L $\perp$ M$\|\|$b		***pmg (p2mg)*** t_a, σ_L, $\sigma_{M,b/2}$, a$\|\|$L $\perp$ M$\|\|$b ***pgg (p2gg)*** $\sigma_{L,a/2}$, $\sigma_{M,b/2}$, L $\perp$ M	
p3 (p311) $r_{C,2\pi/3}$,$r_{C',2\pi/3}$		***p3m1 (p3m1)*** σ_L, σ_M, σ_N, $\angle$ (L, M) = $\angle$ (M,N) = $\angle$ (L, N) = π/3 L $\cap$ M $\cap$ N = $\varnothing$		***p31m (p31m)*** $r_{C,2\pi/3}$, σ_L, C $\notin$ L	
p4 (p411) t_a, $r_{C,\pi/2}$		***p4m (p4mm)*** σ_L, σ_M, σ_N, L$\perp$M, $\angle$ (L, N) = $\angle$ (N,M) = π/4 L $\cap$ M $\cap$ N = $\varnothing$		***p4g (p4gm)*** $r_{C,\pi/2}$, σ_L, C $\notin$ L	
p6 (p611) $r_{C,\pi/3}$, $r_{C',\pi}$		***p6m (p6mm)*** σ_L, σ_M, σ_N, L$\perp$M, $\angle$ (L, N) = π/3, $\angle$ (N,M) = π/6		***notación internacional abreviada (ampliada)*** generadores	región fundamental

Notaciones equivalentes de Grupos Cristalográficos Planos

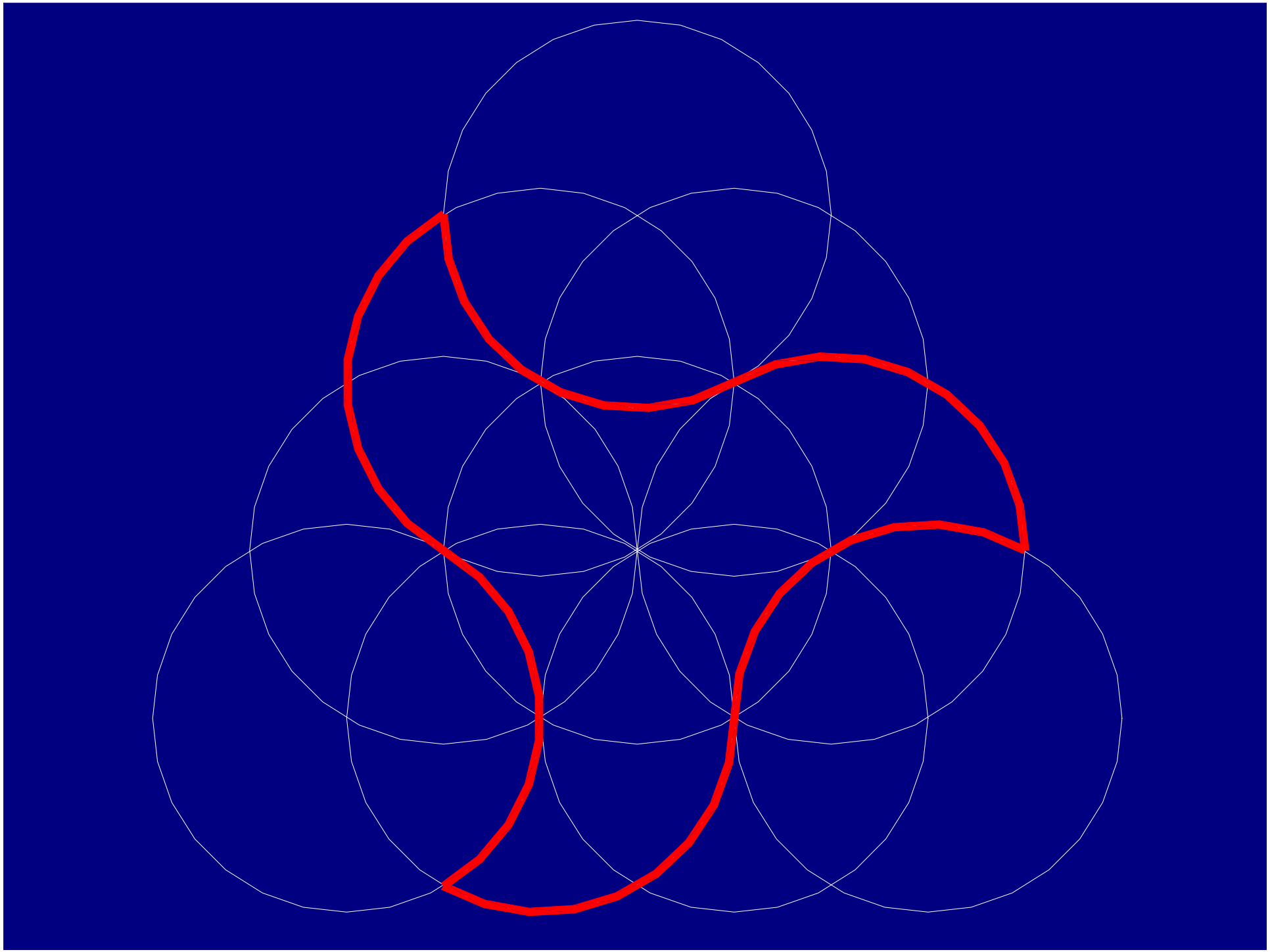
Internacional		Topológica		Bossard	Pólya	Niggli Speiser	Conway	Fejes-Thot	Shubinov Koptsik	Wells Bell Flech
amplia	corta	Γ	(Γ^{∞})							
p111	p1	T	0	p1	C_1	C_1^{I}	o	W_1	(b:a):1	1
p211	p2	S2222	(C_2)	p2	C_2	C_2^{II}	2222	W_2	(b:a):2	2
p1m1	pm	A	(D_2)	p1m	D_1KK	C_S^{I}	**	W_1^{2}	(b:a):m	3
p1g1	pg	K	(D_2)	p1g	D_1gg	C_S^{II}	xx	W_1^{3}	(b:a):b	4
c1m1	cm	M	(D_2)	pm1	D_1Kg	C_S^{III}	x *	W_1^{1}	(a:a):m	8
p2mm	pmm	D 2222	(D_4)	p2m	D_2KKKK	C_{2V}^{I}	*2222	W_2^{2}	(b:a):2.m	5
p2mg	pmg	D22	(D_4)	p2g	D_2KKgg	C_{2V}^{III}	22*	W_2^{3}	(b:a):m: a	6
p2gg	pgg	P22	(D_4)	pg2	D_2gggg	C_{2V}^{II}	22x	W_2^{4}	(b:a): b : a	7
c2mm	cmm	D2 22	(D_4)	pm2	D_2KgKg	C_{2V}^{IV}	2*22	W_2^{1}	(a:a):2.m	9
p411	p4	S442	(C_4)	p4	C_4	C_4^{I}	442	W_4	(a:a):4	10
p4mm	p4m	D 442	(D_8)	p4m	D_4*	C_{4V}^{I}	*442	W_4^{1}	(a:a):4.m	11
p4gm	p4g	D4 2	(D_8)	pm4	D_4°	C_{4V}^{II}	4*2	W_4^{2}	(a:a):4. a	12
p311	p3	S333	(C_3)	p3	C_3	C_3^{I}	333	W_3	(a:a):3	13
p3m1	p3m1	D 333	(D_6)	p3m	D_3*	C_{3V}^{I}	*333	W_3^{1}	(a:a):m.3	15
p31m	p31m	D3 3	(D_6)	pm3	D_3°	C_{3V}^{II}	3*3	W_3^{2}	(a:a):3.m	14
p611	p6	S632	(C_6)	p6	C_6	C_6^{I}	632	W_6	(a:a):6	16
p6mm	p6m	D632	(D_{12})	p6m	D_6	C_{6V}^{I}	*632	W_6^{1}	(a:a):m.6	17

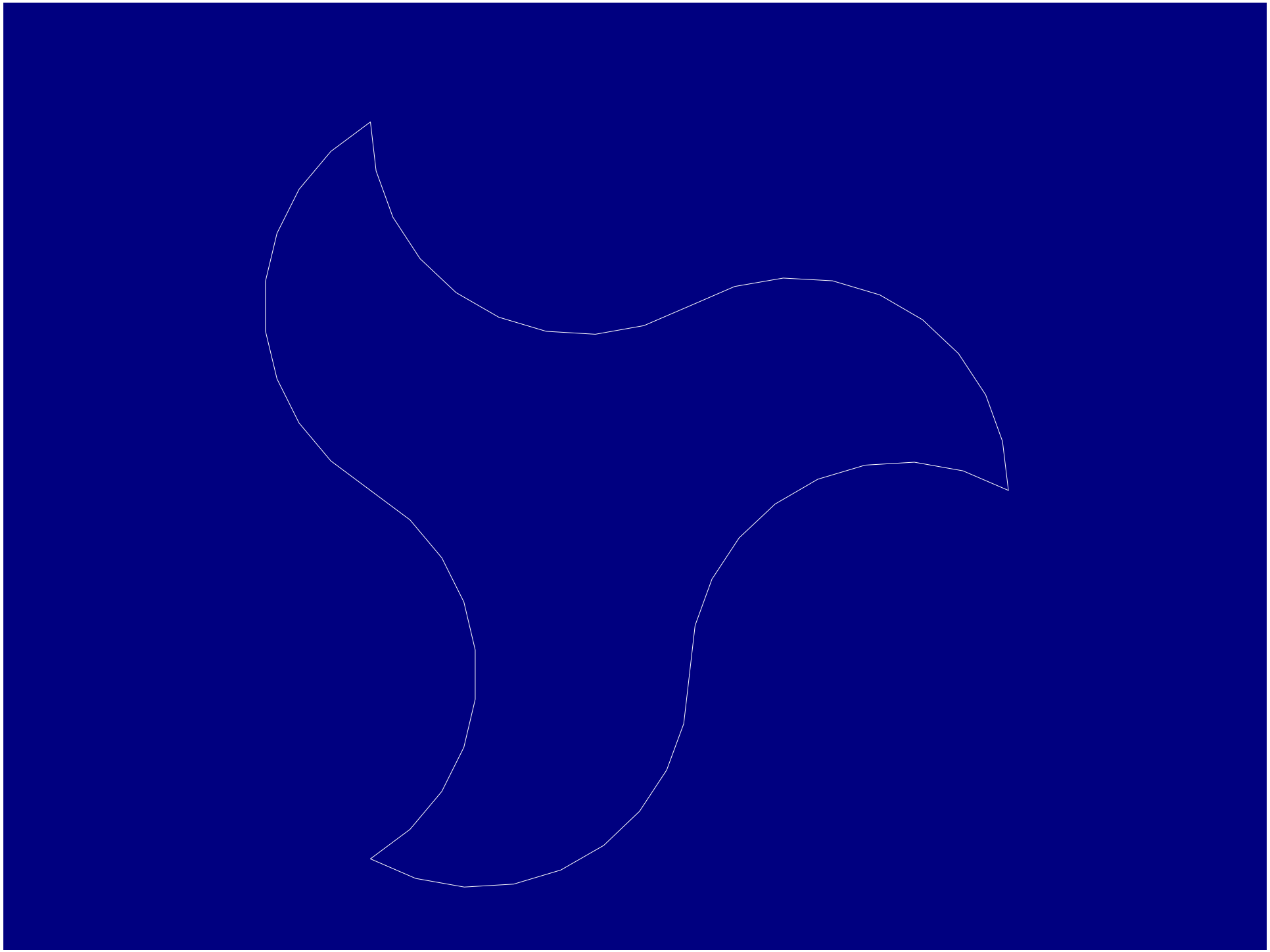
Teorema de la Alhambra

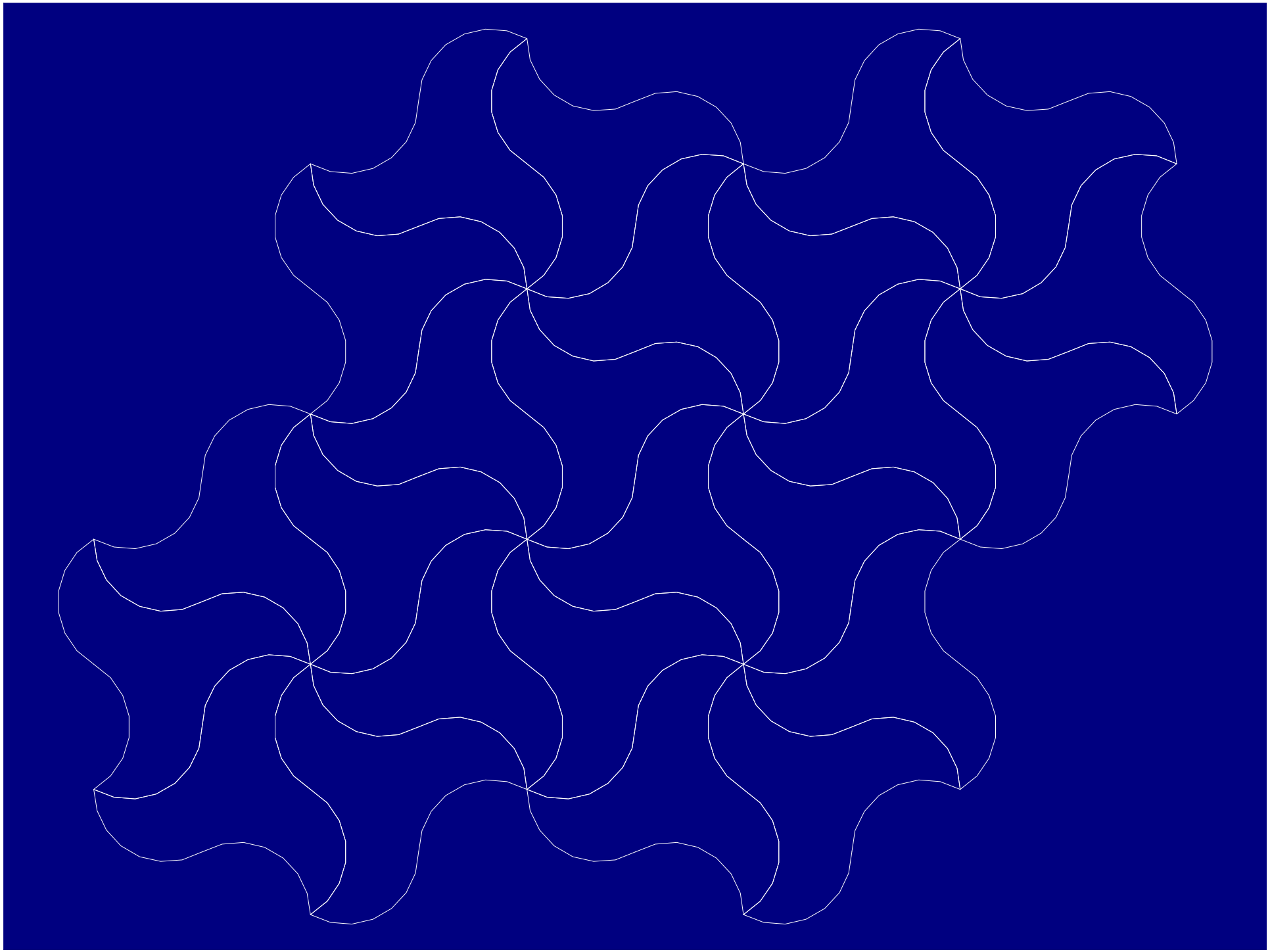
La Alhambra es el único monumento de la antigüedad (anterior al siglo XIX) donde se han hecho dibujos o diseños periódicos con los 17 grupos cristalográficos planos.

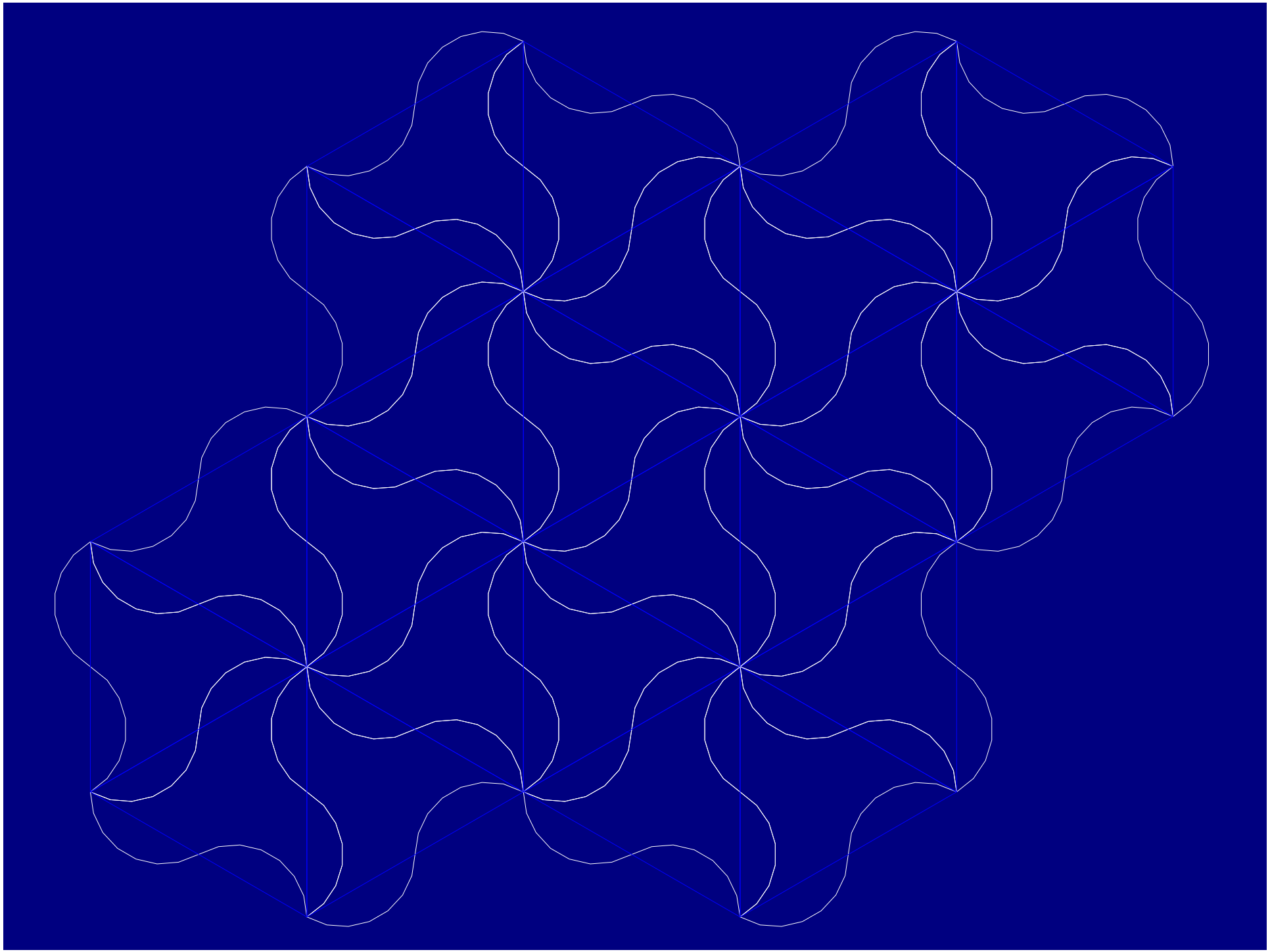
5 grupos cristalográficos que sólo tienen movimientos directos:

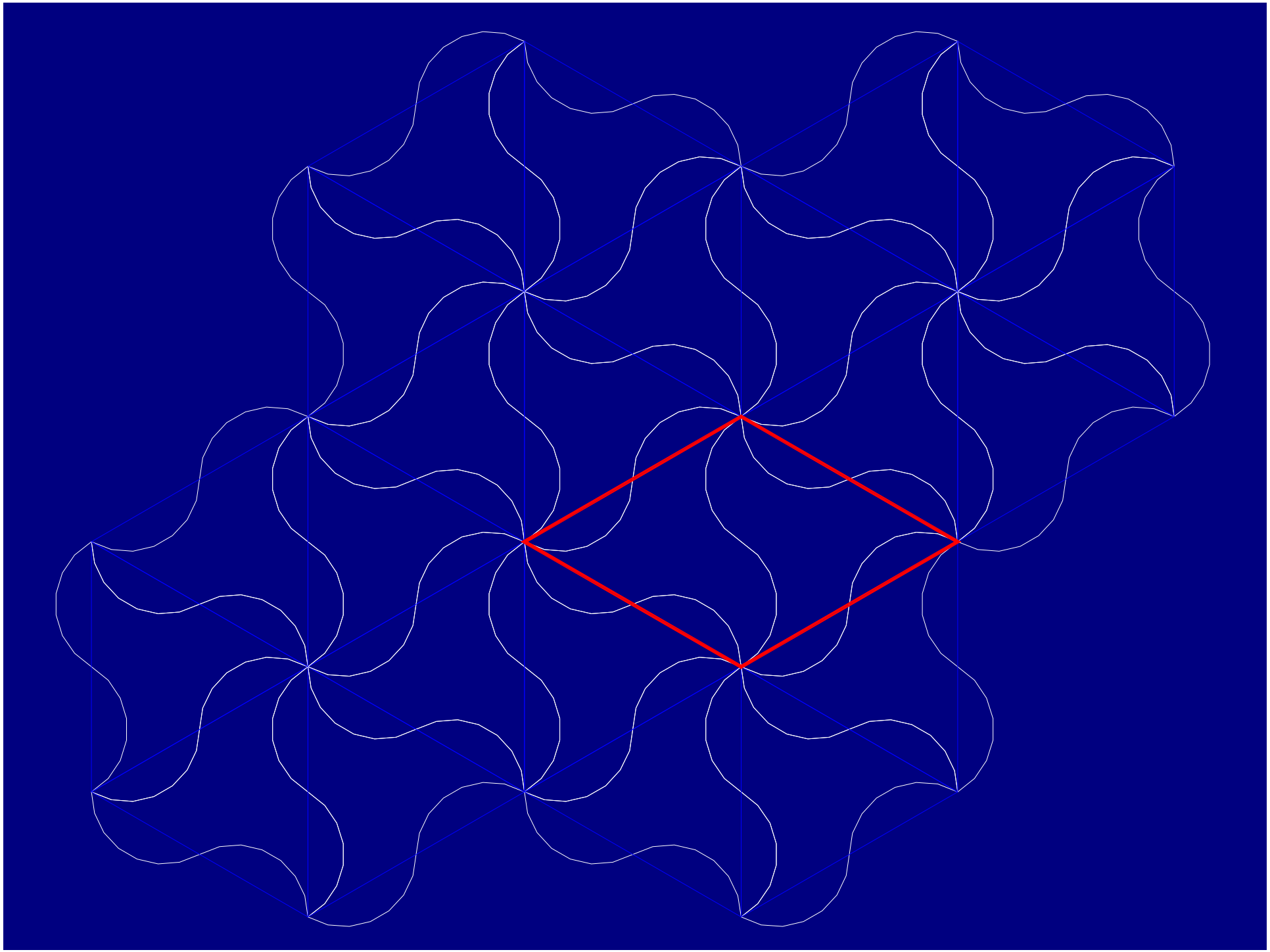
p1, p2, p4, p3, p6

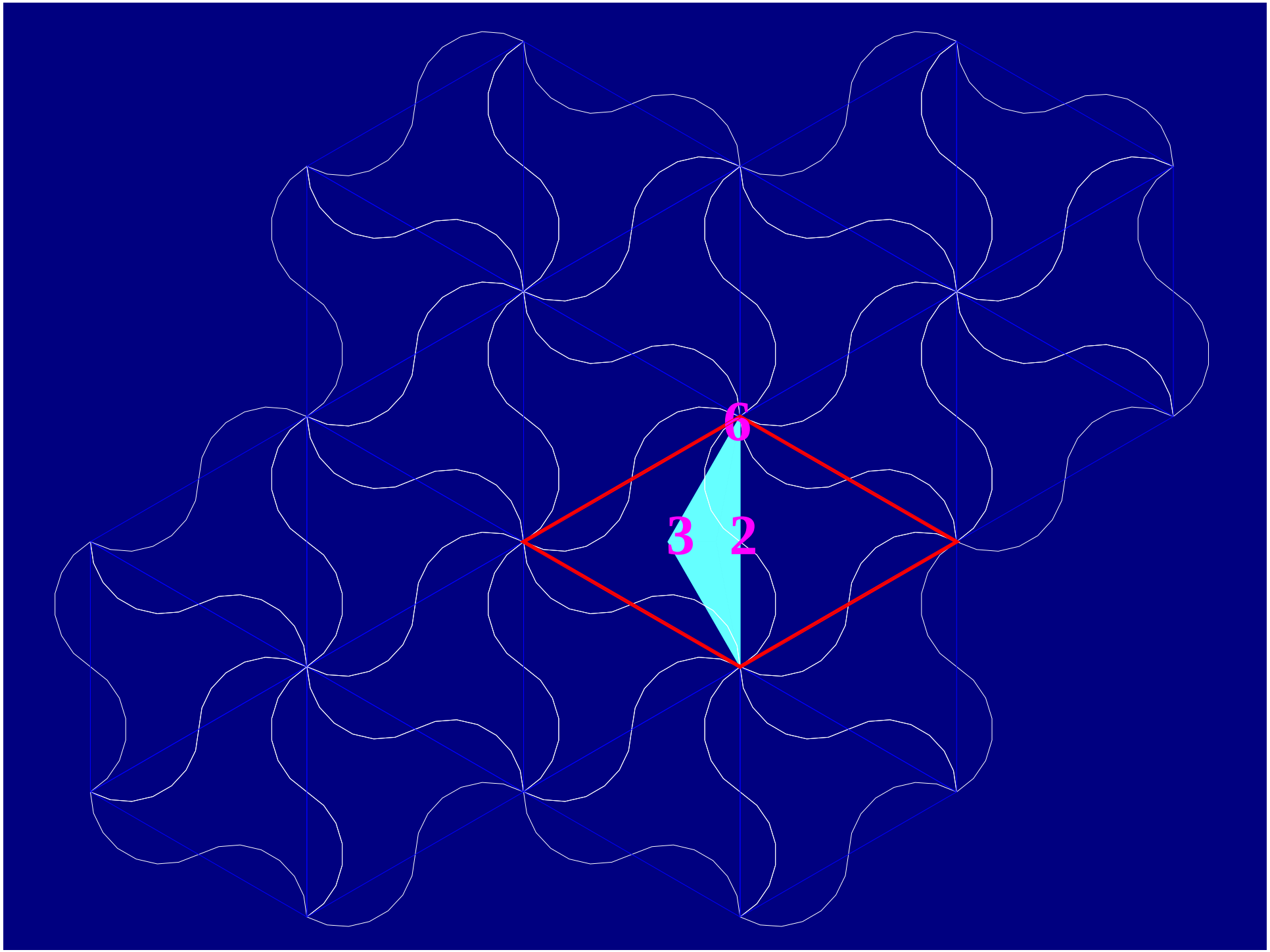






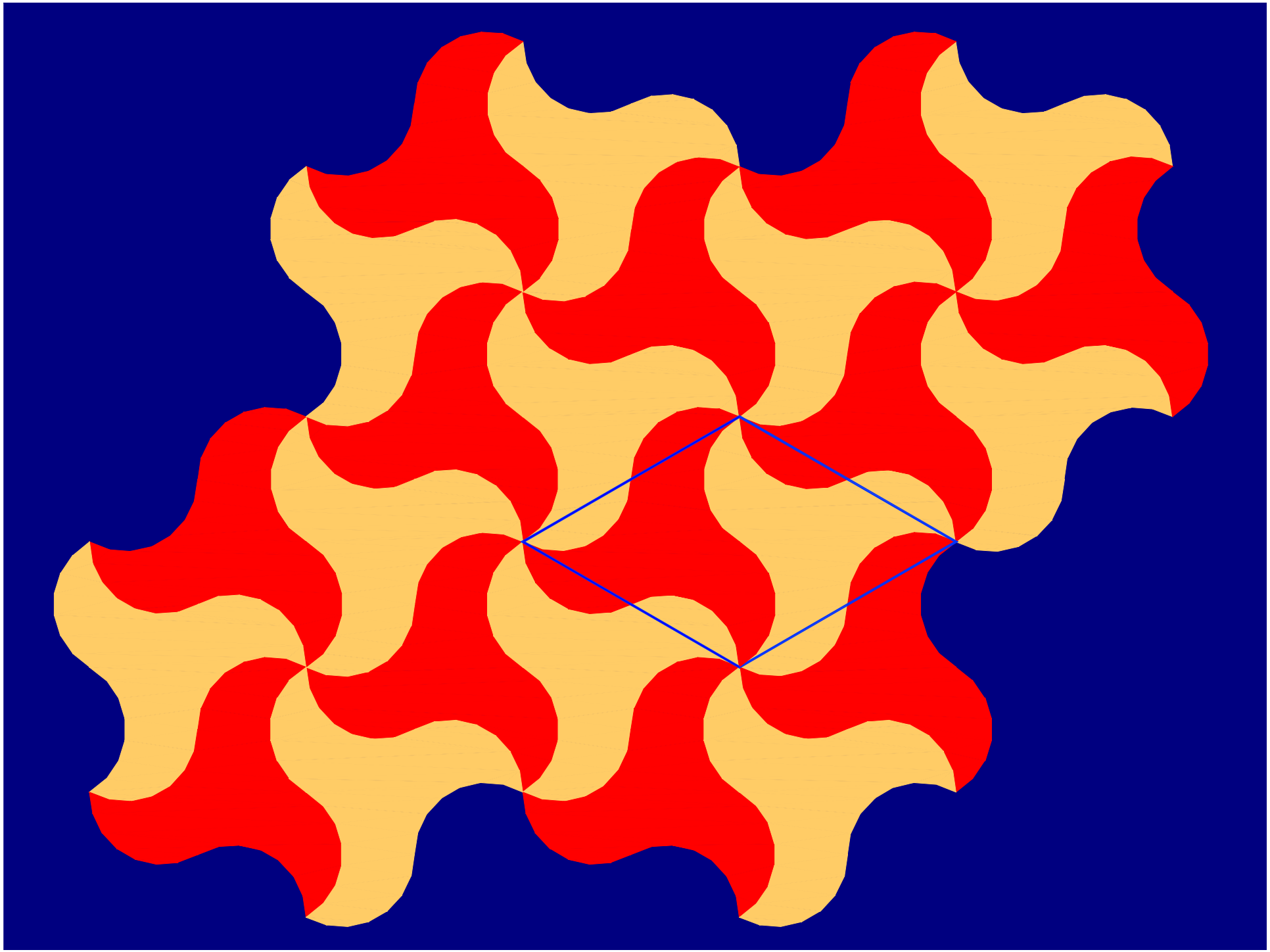


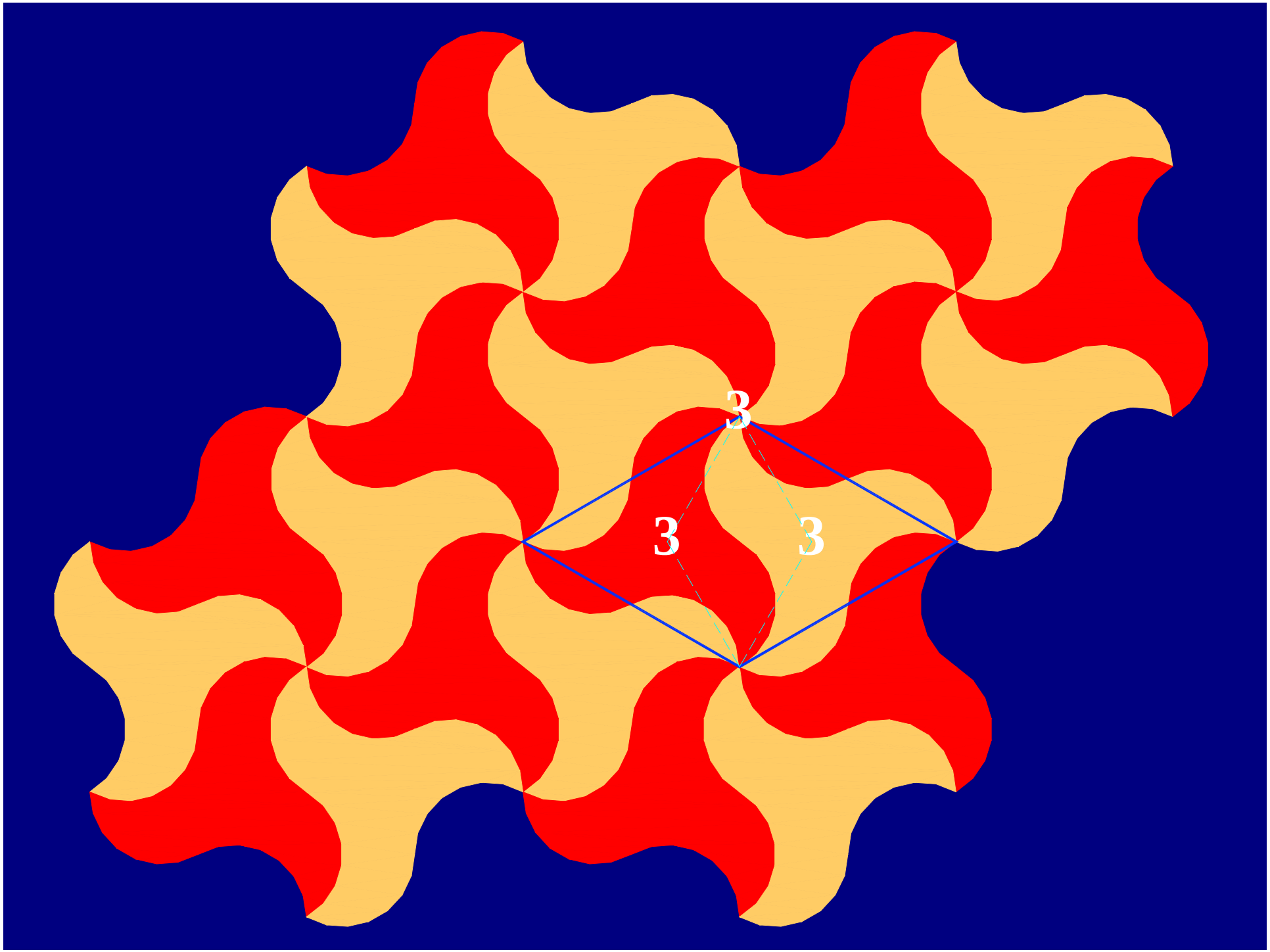


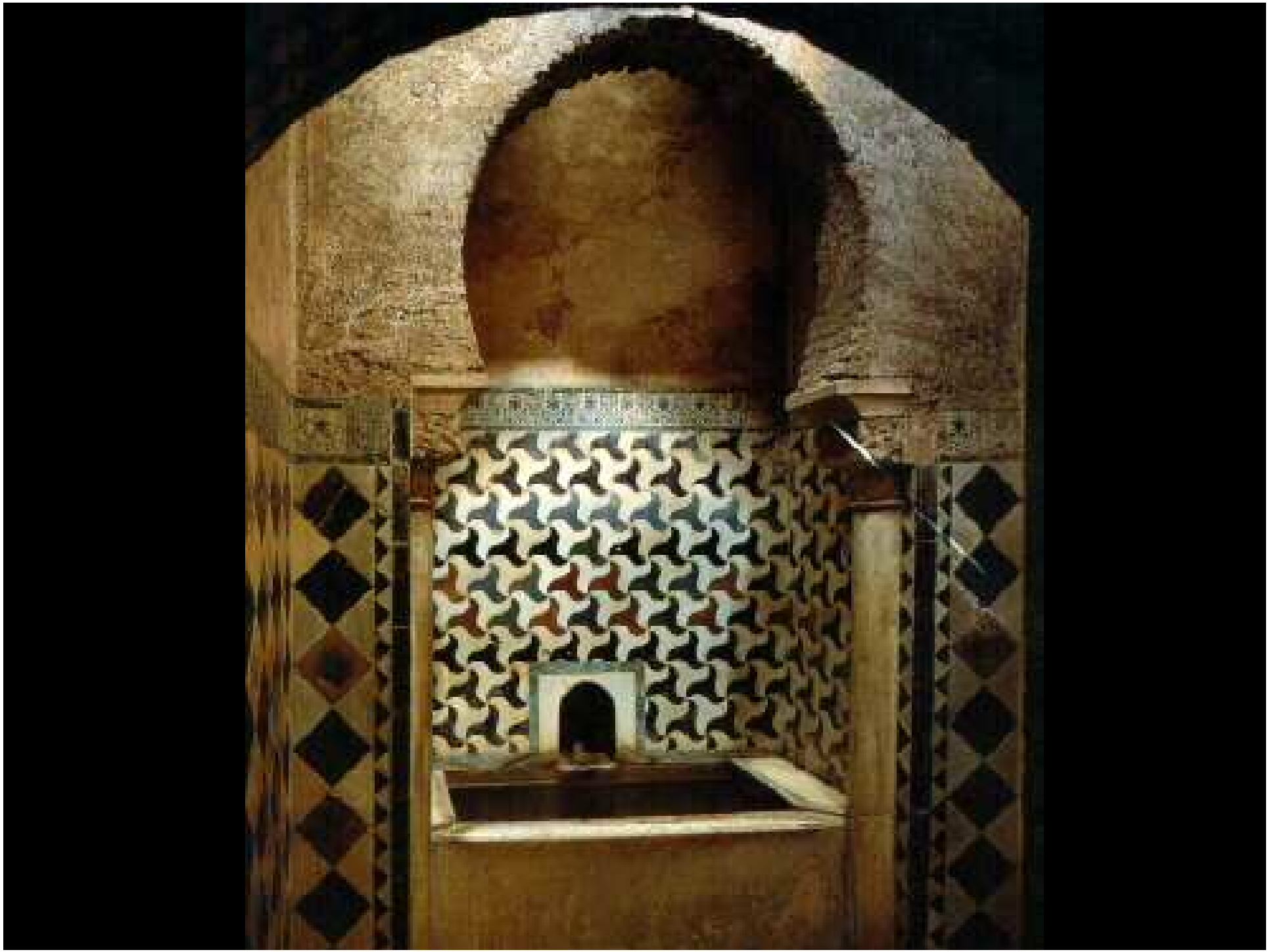

6
3
2

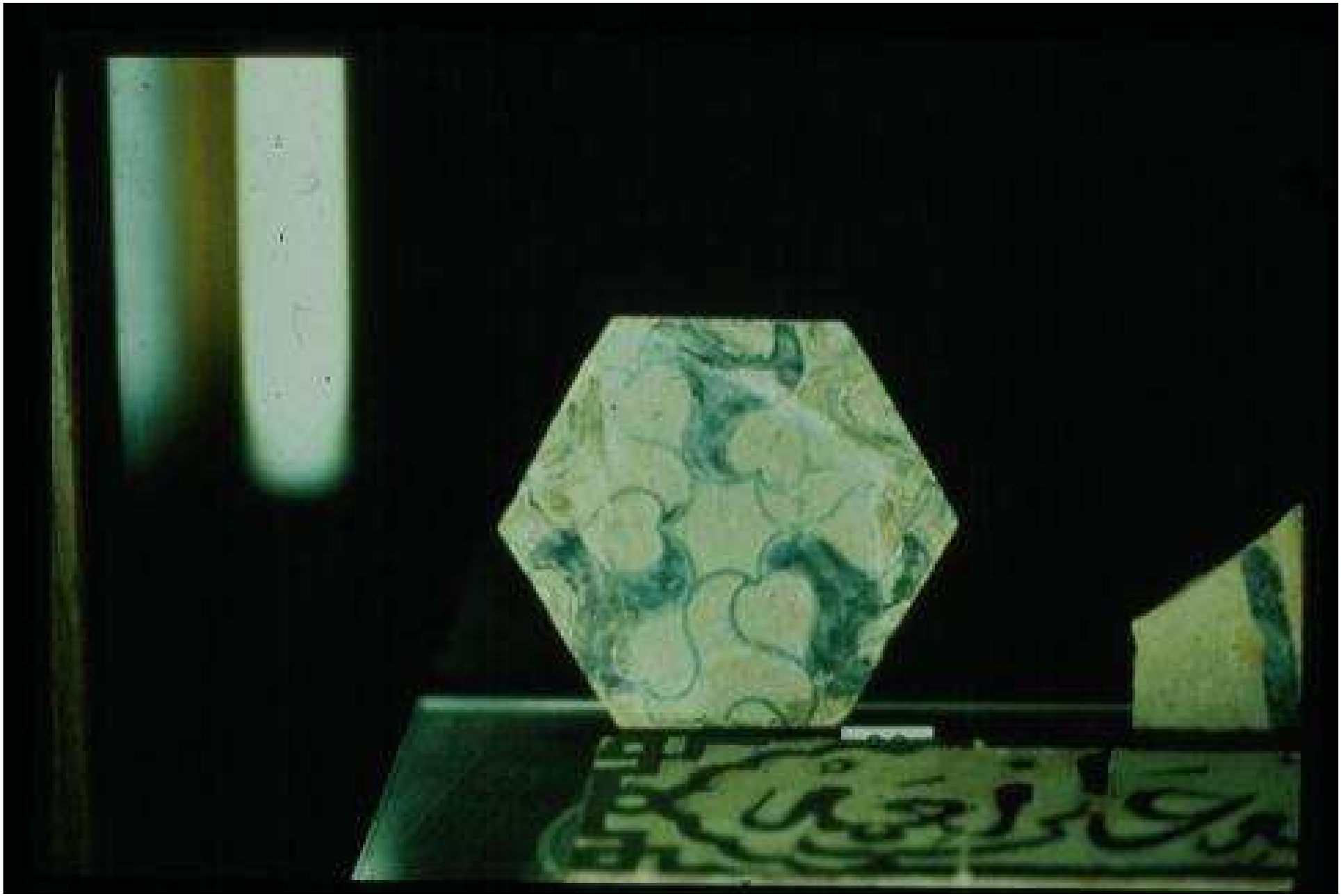
p6

Museo de
La Alhambra

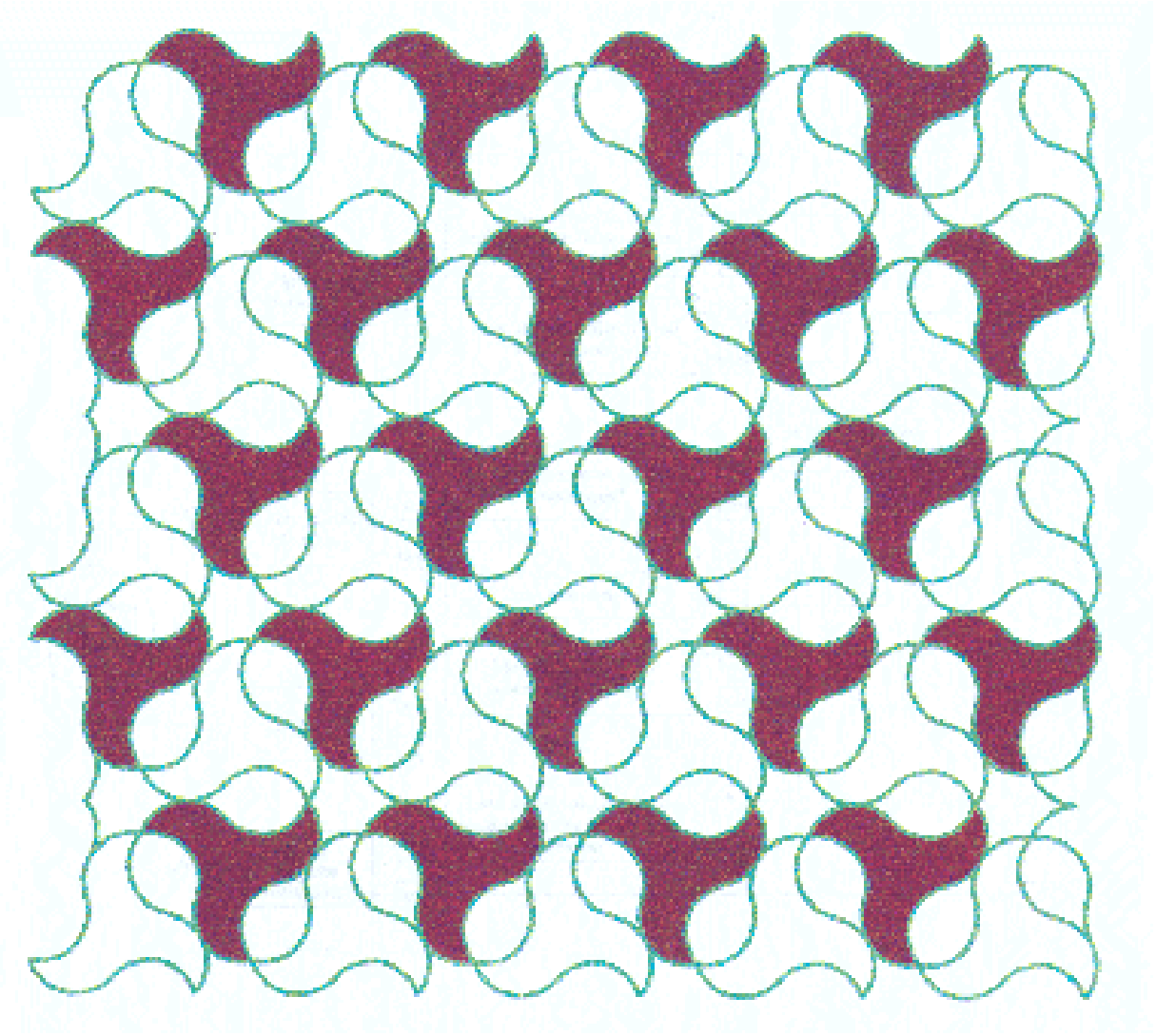



3
3
3



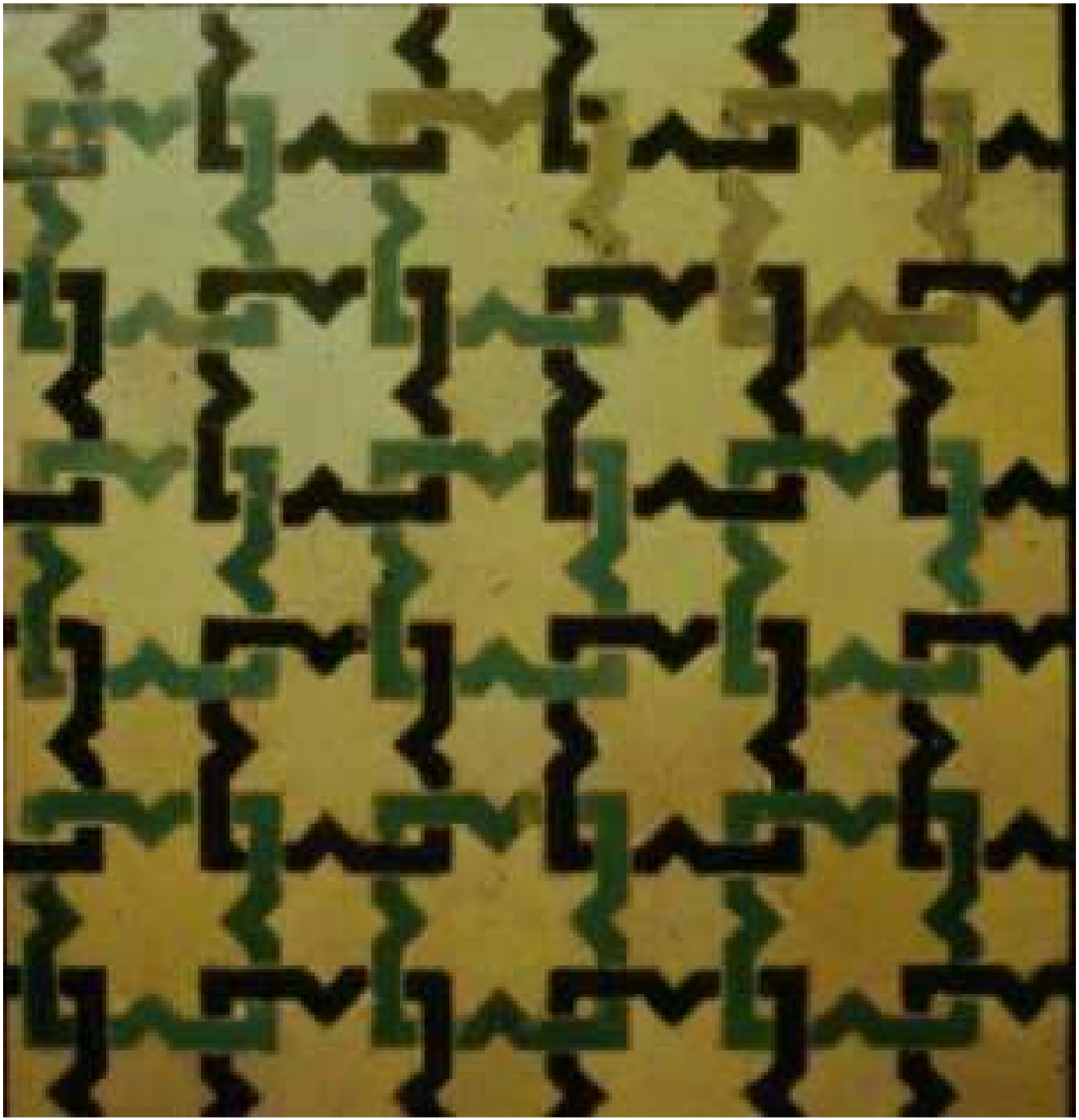


p3




p2

p4



12 grupos cristalográficos que tienen movimientos directos y movimientos inversos:

p1: cm, pm, pg

p2: cmm, pmm, pmg, pgg

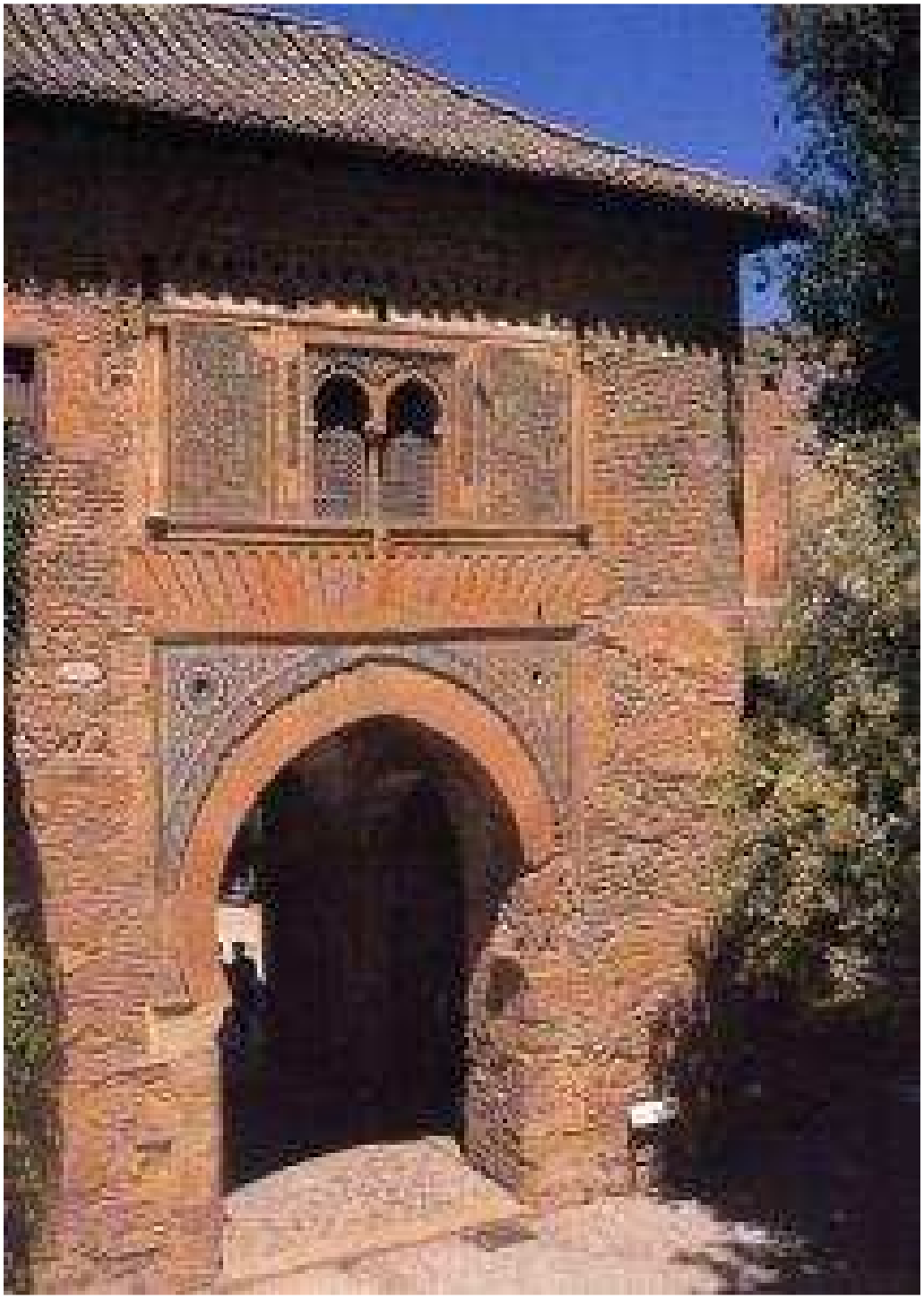
p3: p3m1, p31m

p4: p4m, p4g

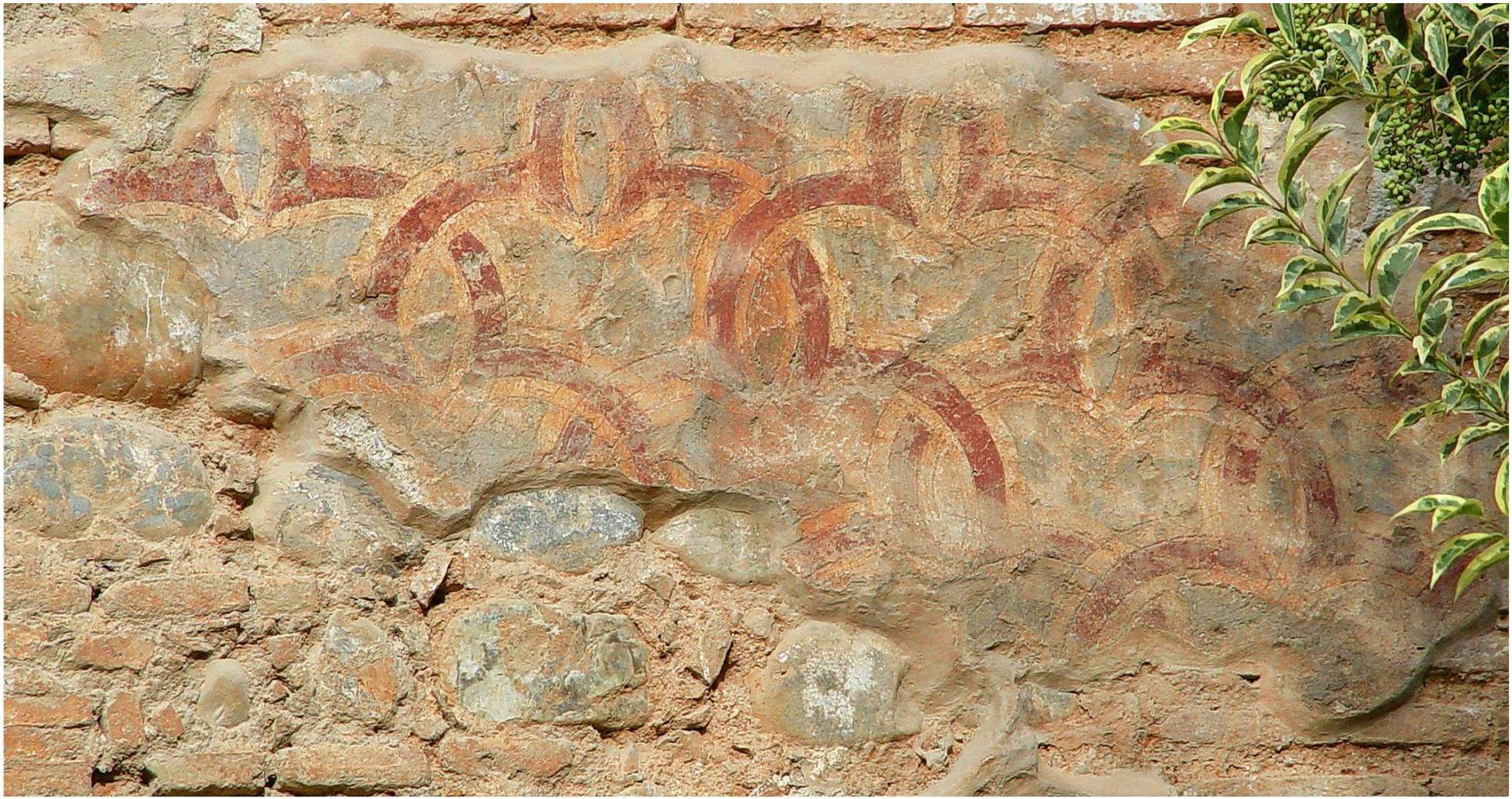
p6: p6m

cm

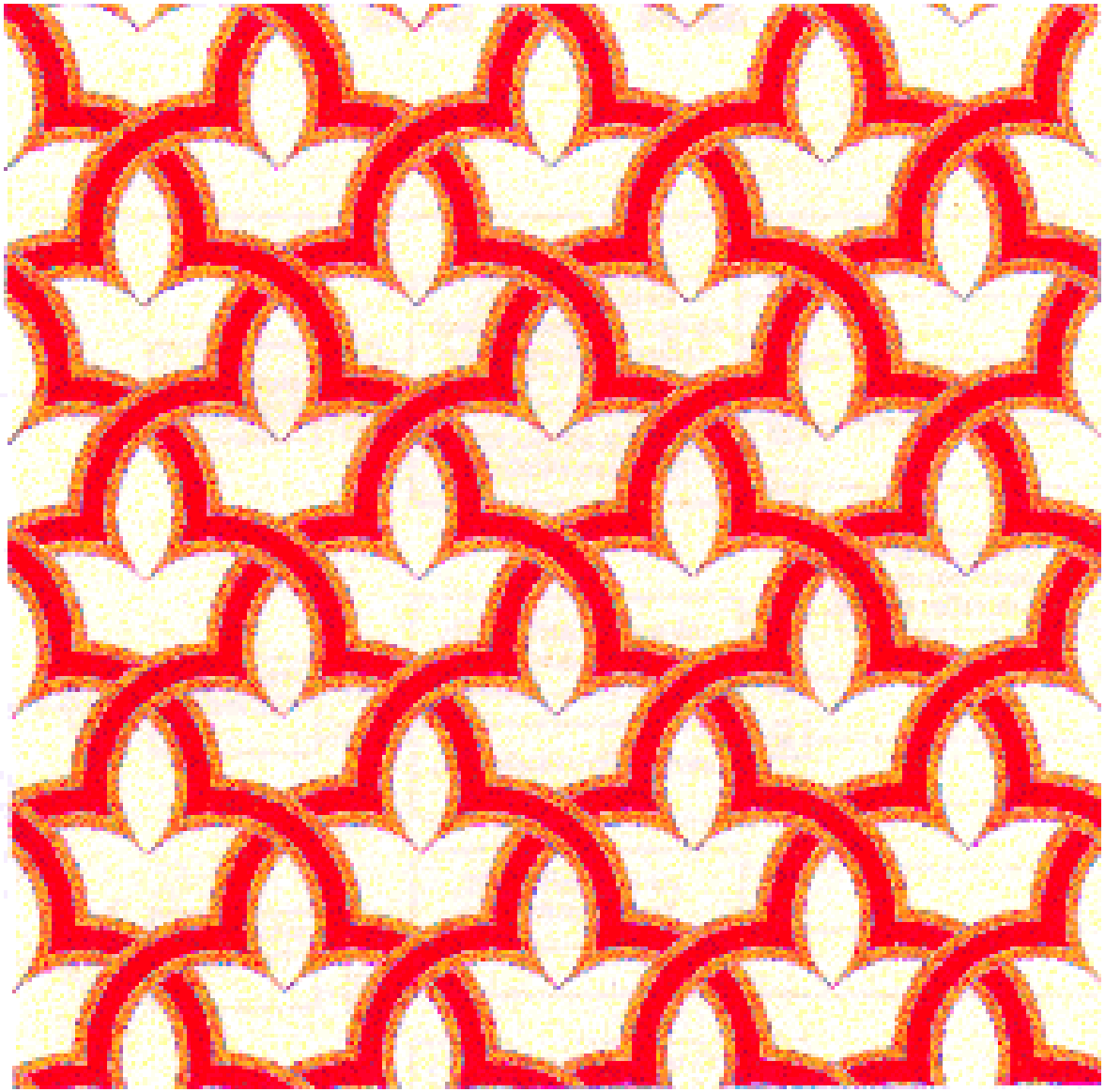
pm



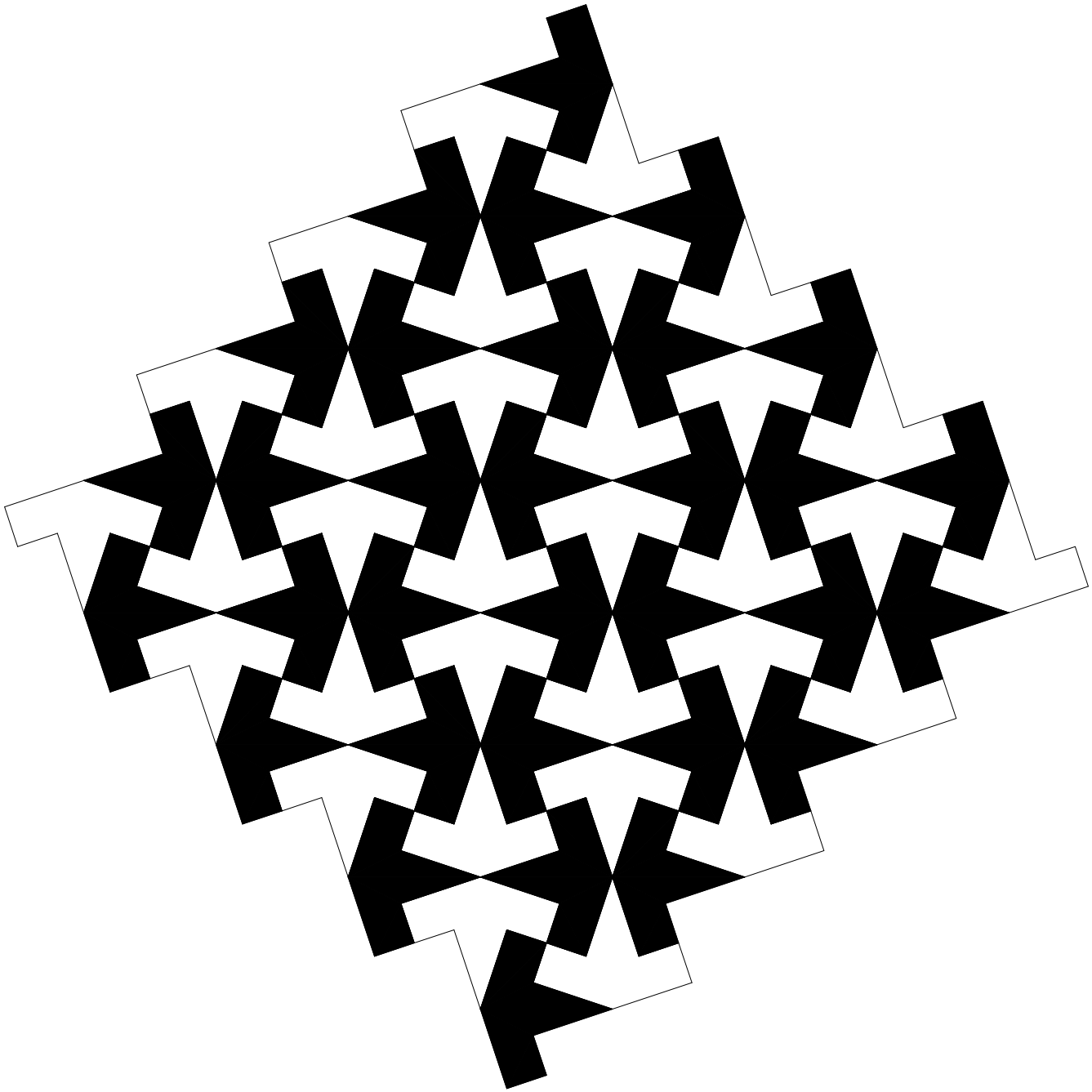
Puerta del Vino

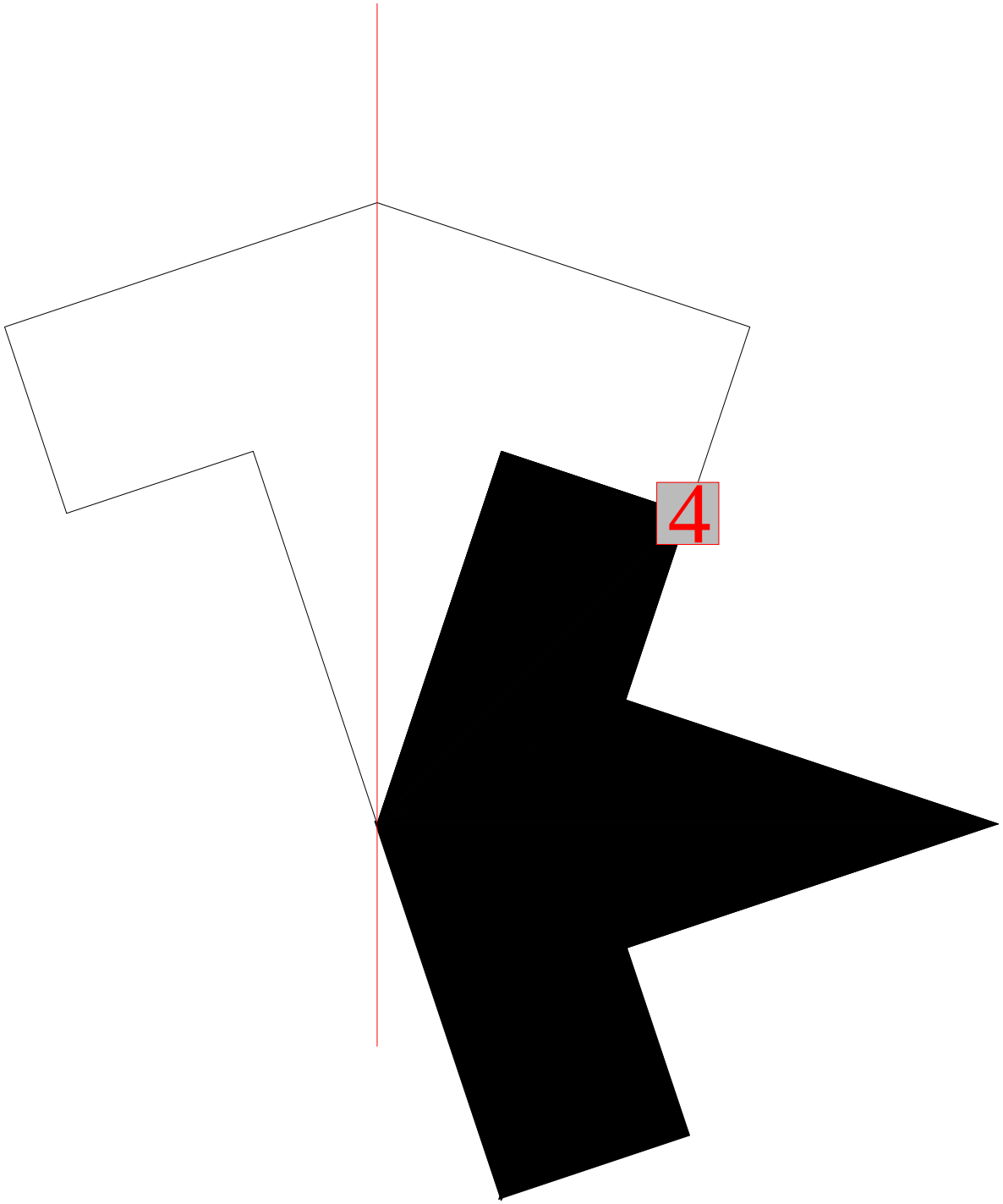


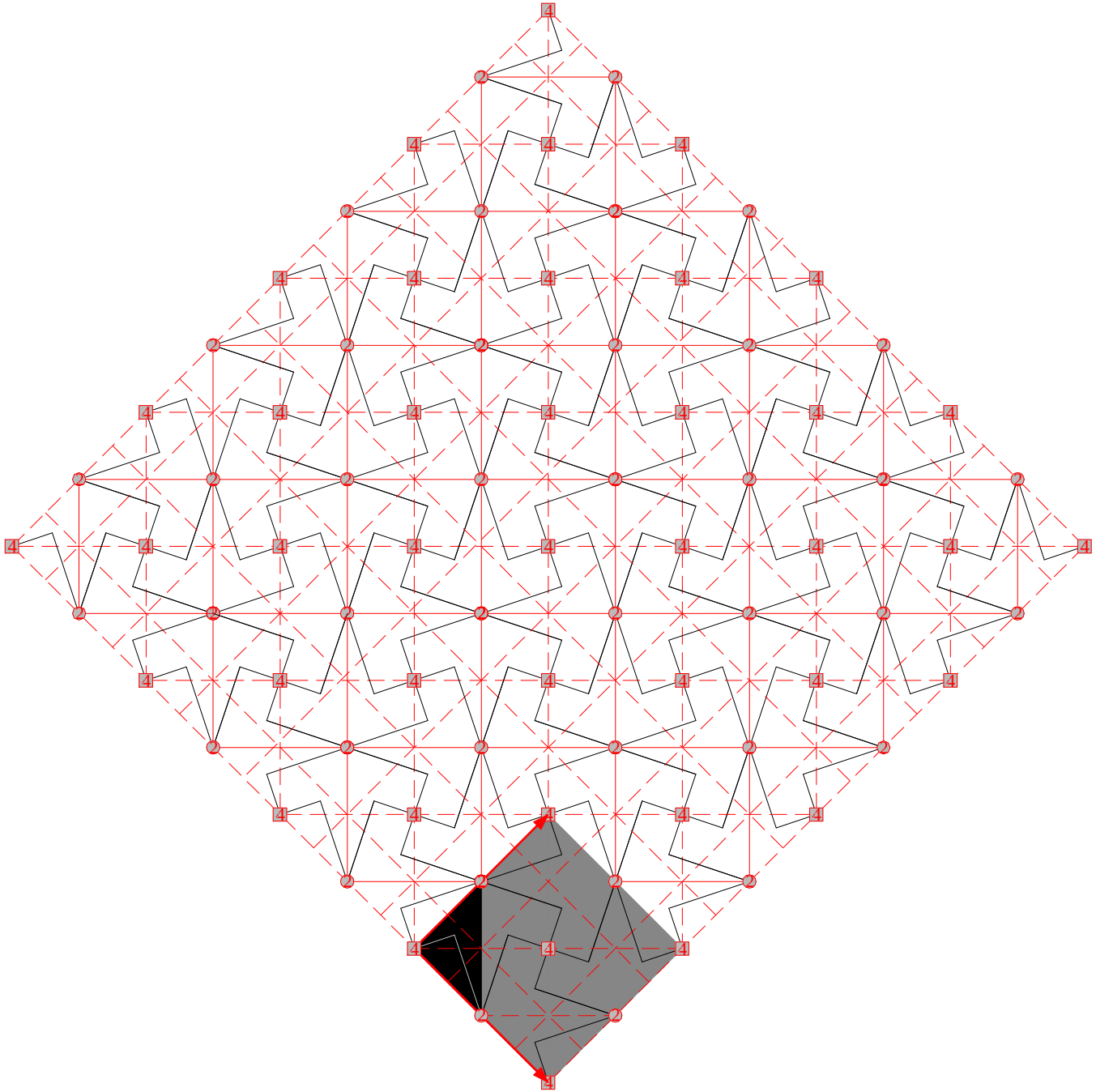
pg

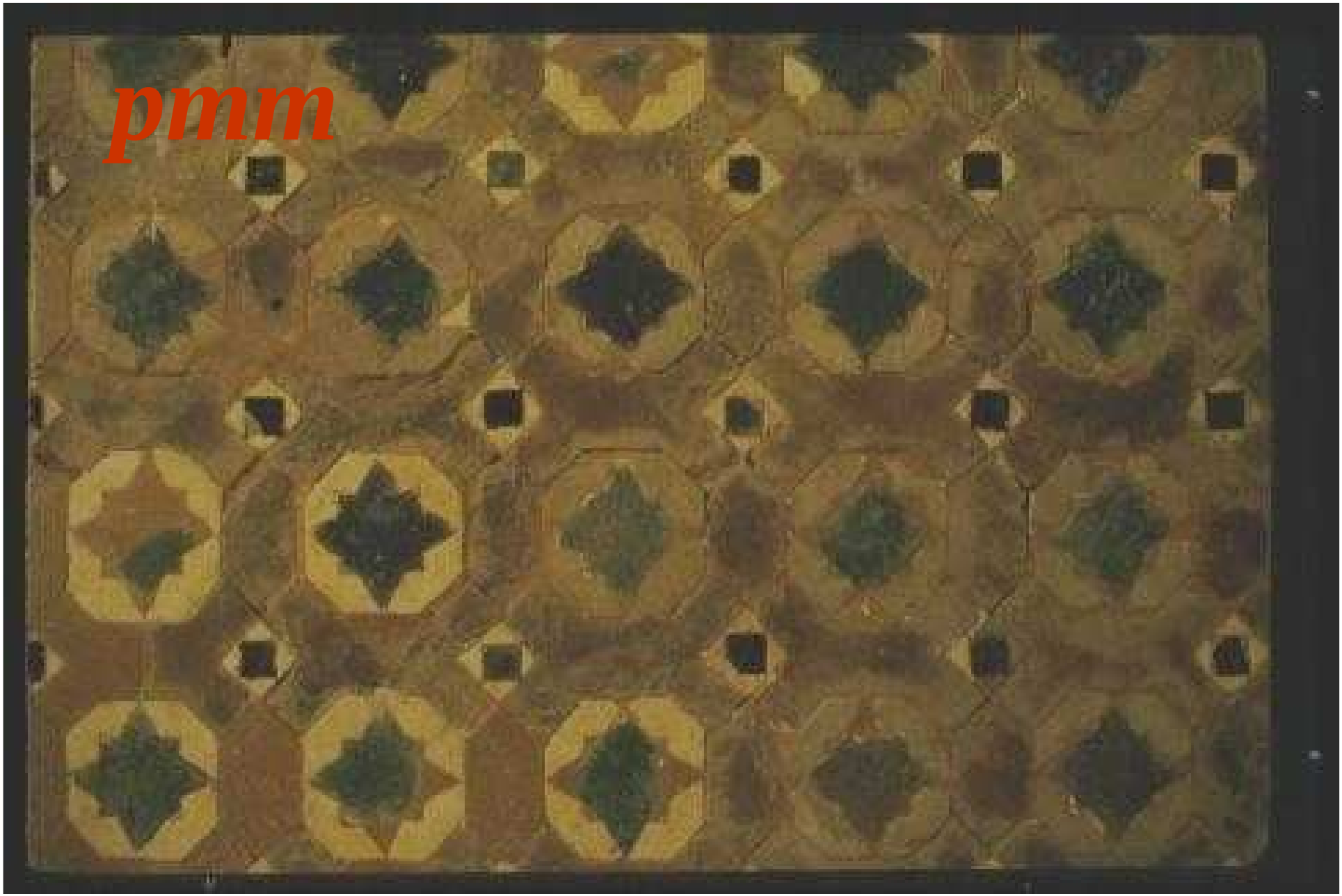


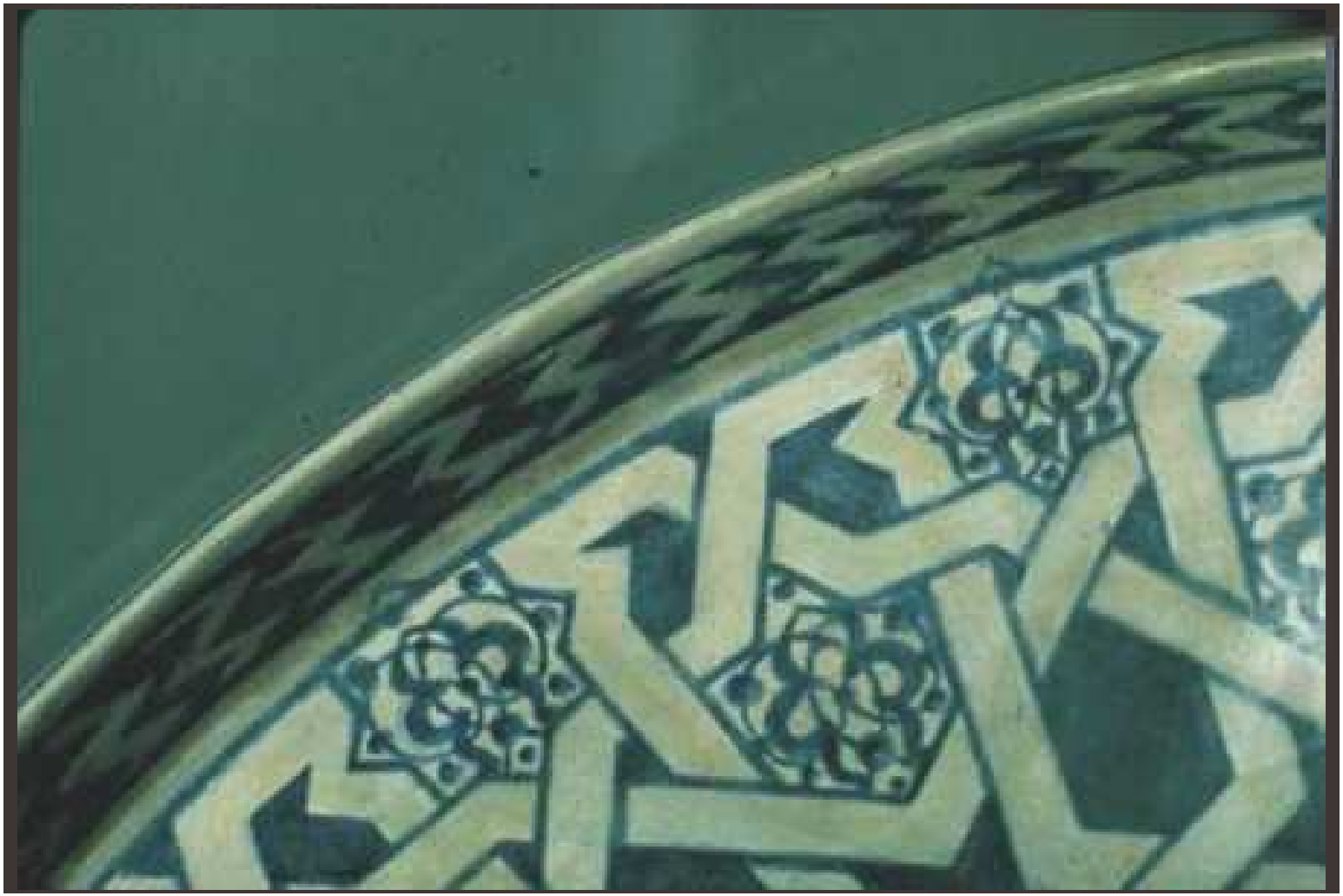

cmm



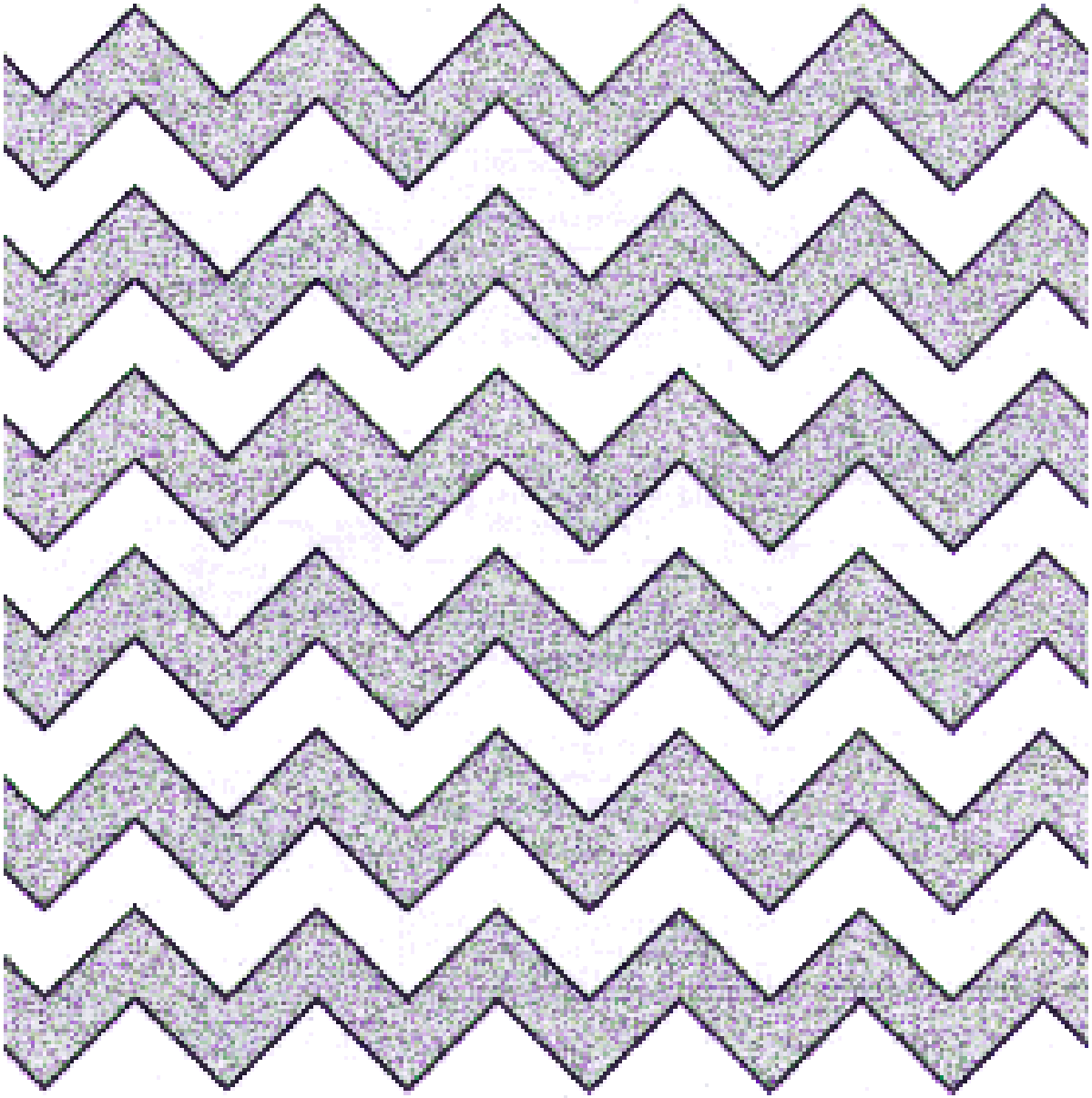

4


4
2


pmm



pmg



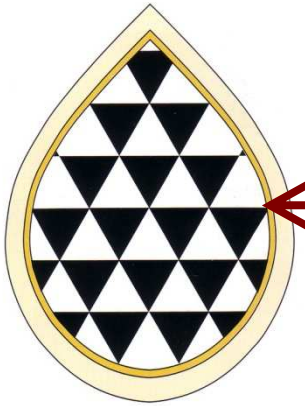
pgg

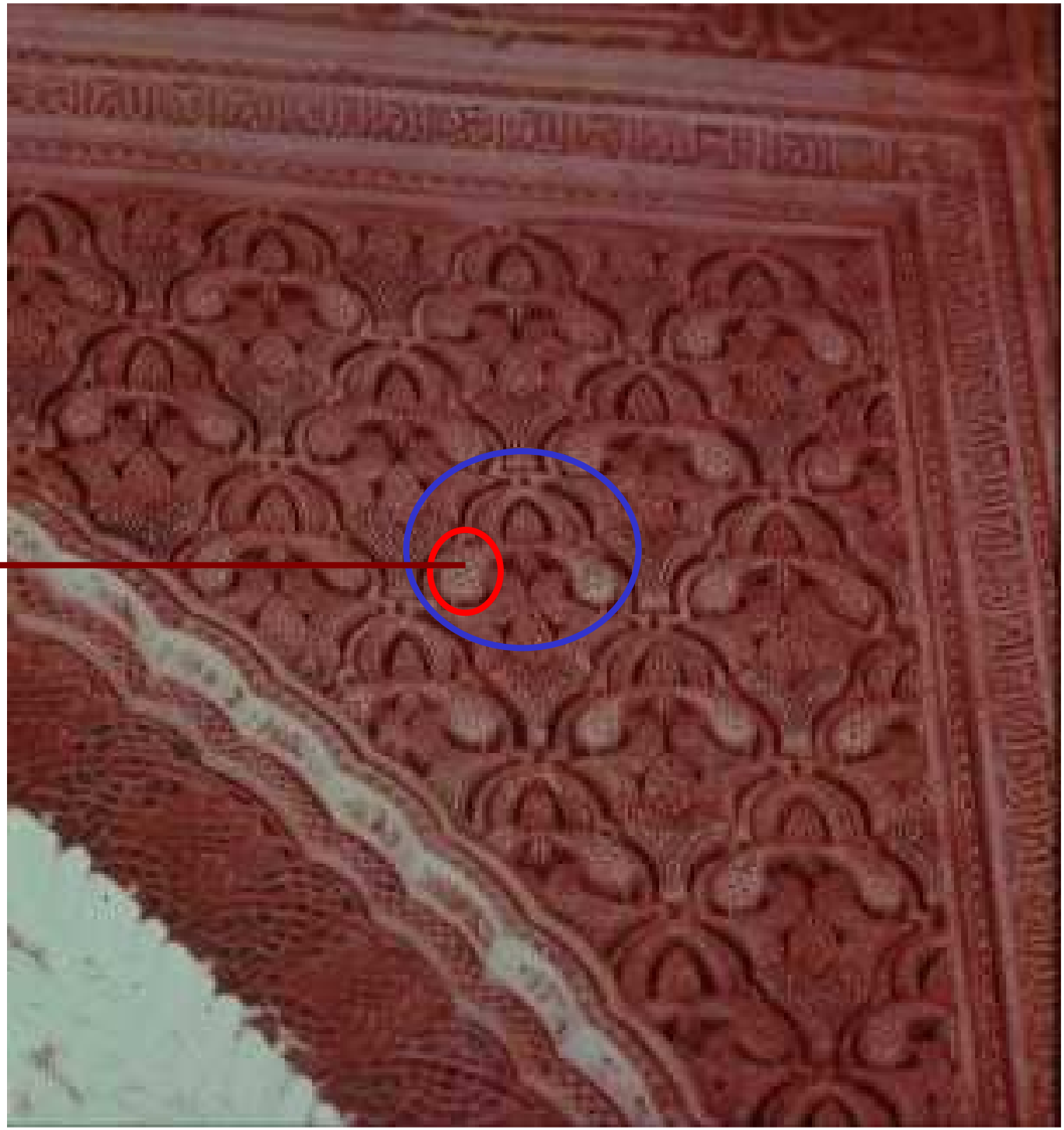

p4m


p4g


16/05/2007 16:47

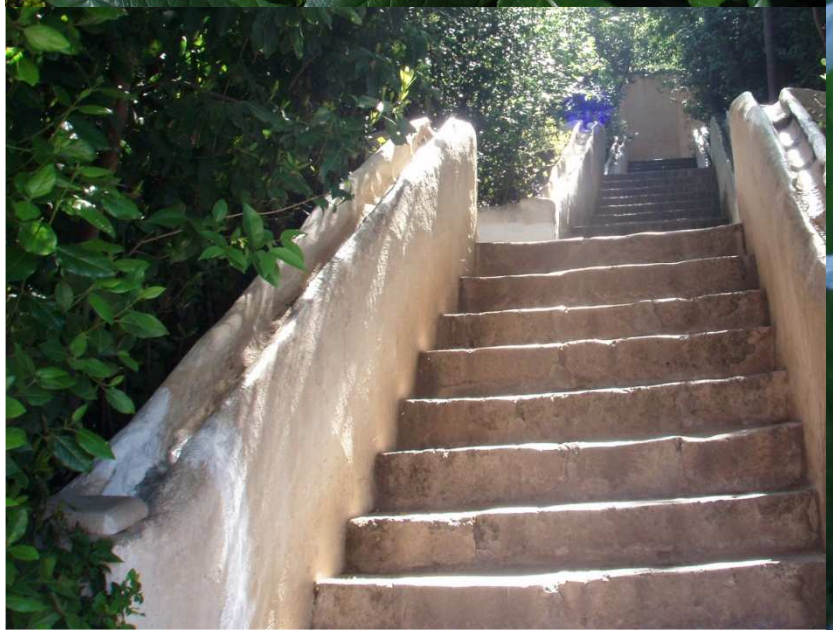
p3m1






p31m
16/05/2007 16:23

p6m



Ejercicios para casa


16/05/2007 16:35


¿ p4g ?