We study holomorphic immersions of open Riemann surfaces into $\mathbb{C}^n$ whose derivative lies in a conical algebraic subvariety $A$ of $\mathbb{C}^n$ that is smooth away from the origin. Classical examples of such $A$-immersions include null curves in $\mathbb{C}^3$ which are closely related to minimal surfaces in $\mathbb{R}^3$, and null curves in $SL_2(\mathbb{C})$ that are related to Bryant surfaces. We establish a basic structure theorem for the set of all $A$-immersions of a bordered Riemann surface, and we prove several approximation and desingularization theorems. Assuming that $A$ is irreducible and is not contained in any hyperplane, we show that every $A$-immersion can be approximated by $A$-embeddings; this holds in particular for null curves in $\mathbb{C}^3$. If in addition $A \setminus \{0\}$ is an Oka manifold, then $A$-immersions are shown to satisfy the Oka principle, including the Runge and the Mergelyan approximation theorems. Another version of the Oka principle holds when $A$ admits a smooth Oka hyperplane section. This lets us prove in particular that every open Riemann surface is biholomorphic to a properly embedded null curve in $\mathbb{C}^4$. 