Cañada, Antonio; **Villegas, Salvador**

A variational approach to Lyapunov type inequalities. From ODEs to PDEs. (English) Zbl 06492237

SpringerBriefs in Mathematics. Cham: Springer (ISBN 978-3-319-25287-2/pbk; 978-3-319-25289-6/ebook). xviii, 120 p. (2015).

In this brief monograph, the authors present their recent results related to the Lyapunov-type inequalities and some applications for the stability of linear periodic equations, the sign of the eigenvalues of eigenvalue problems and nonlinear resonant problems. The book contains five chapters, each of them with a list of references, and an index.

Chapter 1 deals with some historical motivations for the study of Lyapunov-type inequalities, and three topics are shortly presented, namely: the stability properties of the Hill's equation, the study of sign of the eigenvalues of certain eigenvalue problems and the analysis of nonlinear resolvent problems.

Chapter 2 is focused on the definition and main properties of the L_p Lyapunov constant, $1 \leq p \leq \infty$, for scalar ordinary differential equations with various boundary conditions, in a given interval $(0, L)$, which include resonant problems at the first eigenvalue and nonresonant problems. The authors present a variational characterization of Lyapunov constant as a minimum for some minimization problem defined in appropriate subsets X_p of the Sobolev space $H^1(0, L)$, and also an explicit expression of this constant. In the case of resonant problems, they obtain constrained minimization problems with a restriction to the definition of space X_p, $1 \leq p \leq \infty$, which is not necessary in the nonresonant case. The relation between Neumann boundary conditions and disfocality is also presented. For the nonlinear equations, they use the Schauder fixed point theorem and the obtained results for linear equations.

Chapter 3 is devoted to the study of L_1 Lyapunov-type inequalities for various boundary conditions at higher eigenvalues. The authors investigate the number and the distributions of zeros of nontrivial solutions and their first derivatives, by considering suitable minimization problems. As in Lyapunov's classical theorem, the L_1 best constant at higher eigenvalues is not attained. They establish new conditions for the stability of linear periodic equations, and for the existence and uniqueness of solutions for resonant nonlinear problems at higher eigenvalues.

Chapter 4 is concerned with the study of L_p Lyapunov-type inequalities ($1 \leq p \leq \infty$) for linear partial differential equations. The case of Neumann boundary conditions on bounded and regular domains in $\mathbb{R}^N$ is investigated. By using some appropriate minimizating sequences, and the analysis of the number and distribution of the zeros of radial nontrivial solutions, the authors also study the case of higher eigenvalues in the radial case. They obtain new conditions for the existence and uniqueness of solutions for resonant nonlinear problems.

Chapter 5 deals with L_p Lyapunov-type inequalities for linear systems of ordinary differential equations with different boundary conditions, such as Neumann, Dirichlet, periodic and antiperiodic boundary conditions, for any constant $p \geq 1$. Resonant nonlinear problems and the stable boundedness of linear periodic conservative systems are also investigated.

The book will be useful to graduate students and researchers interested in Lyapunov-type inequalities and stability problems for differential equations.

Reviewer: Rodica Luca (Iaşi)

MSC:

34-02 Research monographs (ordinary differential equations)
35-02 Research monographs (partial differential equations)
34B05 Linear boundary value problems for ODE
34B15 Nonlinear boundary value problems for ODE
35A23 Inequalities involving derivatives etc. (PDE)

Keywords:

Lyapunov-type inequalities; stability; variational methods; eigenvalues; ordinary differential equations; partial differential equations; Dirichlet boundary conditions; Neumann boundary conditions; periodic boundary conditions; antiperiodic boundary conditions

Full Text: DOI