SOLUCIONES RADIALES A. Cañada

$\Delta u(x) = 0$

$u(x) = v(\|x-\xi\|)$, $\|x-\xi\| = r$

$u(x) = v(r)$ $r = \left((x_1-\xi_1)^2 + \dots + (x_n-\xi_n)^2\right)^{1/2} = (r^2)^{1/2}$

$$\frac{\partial u}{\partial x_i} = v'(r)\frac{\partial r}{\partial x_i} = v'(r)\,\frac{1}{\cancel{2}}(r^2)^{-1/2}\,\cancel{2}(x_i-\xi_i) = v'(r)\,\frac{(x_i-\xi_i)}{r}$$

$$\frac{\partial^2 u}{\partial x_i^2} = v''(r)\frac{\partial r}{\partial x_i}\,\frac{(x_i-\xi_i)}{r} + v'(r)\,\frac{r-(x_i-\xi_i)\frac{\partial r}{\partial x_i}}{r^2} =$$

$$= v''(r)\,\frac{(x_i-\xi_i)}{r}\,\frac{(x_i-\xi_i)}{r} + v'(r)\cdot\frac{r-\frac{(x_i-\xi_i)(x_i-\xi_i)}{r}}{r^2} =$$

$$= v''(r)\,\frac{(x_i-\xi_i)^2}{r^2} + v'(r)\cdot\frac{r^2-(x_i-\xi_i)^2}{r^3}$$

$$\Delta u(x) = v''(r)\cdot\frac{r^2}{r^2} + v'(r)\,\frac{nr^2-r^2}{r^3} =$$

$$= v''(r) + v'(r)\,\frac{n-1}{r}$$

$v''(r) + v'(r)\frac{n-1}{r} = 0$; $v'(r) = z(r)$

$z'(r) + z(r)\frac{n-1}{r} = 0$; $\frac{z'(r)}{z(r)} = \frac{1-n}{r}$

$\ln|z(r)| = (1-n)\ln r = \ln r^{1-n}$; $z(r) = r^{1-n}$

$$v'(r) = r^{1-n}\begin{cases} n=2 & v(r) = \ln r \\ n\geq 3 & v(r) = \dfrac{r^{2-n}}{2-n}\end{cases}$$

Base de soluciones de $v''(r) + v'(r)\frac{n-1}{r} = 0$ $\begin{cases} n=2 & \{1, \ln r\} \\ n\geq 3 & \left\{1, \dfrac{r^{2-n}}{2-n}\right\}\end{cases}$

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