Soluciones del examen del 25/5/2018

1. a) $u(x_1,t) = \sqrt{3}\cos(x_1)e^{-t} - \pi \operatorname{sen}(8x_1)e^{-64t} + e^3\cos(\sqrt[4]{5}\,x_1)e^{-\sqrt{5}\,t}$

b) $u(x_1,x_2,t) = \frac{1}{2} + \frac{1}{2}\cos(x_1+x_2)e^{-2t}$

c) $u(x_1,\dots,x_n,t) = \operatorname{sen}(x_1+\dots+x_n)e^{-nt}$

2. a) La función dada u no es solución de (1), puesto que $\nexists\ \frac{\partial u(\pi/3,t)}{\partial x}$, ya que

$$\frac{\partial^+ u(\pi/3,t)}{\partial x} = -3e^{-9t}, \quad \frac{\partial^- u(\pi/3,t)}{\partial x} = 0$$

b) $\sum_{n=1}^{\infty} f_n \operatorname{sen}(nx)e^{-n^2t}$, $f_n = \frac{2}{\pi}\int_0^{\pi} f(x)\operatorname{sen}(nx)\,dx$

c) $f_3 \operatorname{sen}(3x)e^{-9t}$, $f_3 = \frac{2}{\pi}\int_{\pi/3}^{\pi} \operatorname{sen}^2(3x)\,dx$

$f_5 \operatorname{sen}(5x)e^{-25t}$, $f_5 = \frac{2}{\pi}\int_{\pi/3}^{\pi} \operatorname{sen}(3x)\operatorname{sen}(5x)\,dx$

3. Hecho en clase.

4. Si $v(r) = \frac{r^7}{49} - \frac{2^7}{49}$, entonces $u(x) = v(\|x\|)$