CONVERGENCIA
DE SERIES DE FOURIER

1. Introducción

En esta práctica analizaremos diversos aspectos de la convergencia de Series de Fourier. El objetivo es visualizar algunos de los resultados teóricos que hemos visto en clase. También deben obtenerse conclusiones nuevas que luego debemos confirmar (o no) de manera rigurosa.

2. Definición de la función y de sus coeficientes de Fourier

En primer lugar definimos la función que estemos considerando y sus coeficientes de Fourier asociados

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In[85]:=
```
Clear["Global`**"]
f[x_] = x
a0 = (1 / Pi) Integrate[f[x], {x, -Pi, Pi}]
a[n_] = (1 / Pi) Integrate[f[x] * Cos[n * x], {x, -Pi, Pi}]
b[n_] = (1 / Pi) Integrate[f[x] * Sin[n * x], {x, -Pi, Pi}]
```

Out[86]= x

Out[87]= 0

Out[88]= 0

Out[89]= $\frac{-2\, n\, \pi\, \mathrm{Cos}[n\, \pi] + 2\, \mathrm{Sin}[n\, \pi]}{n^2\, \pi}$

3. Definición de las sumas parciales de la serie de Fourier

Aquí definimos las sumas parciales generales y luego obtenemos algunas de ellas

Null

In[90]:= `foursum[x_, n_] = (a0 / 2) + Sum[a[k] Cos[k * x] + b[k] Sin[k * x], {k, 1, n}]`

Out[90]=

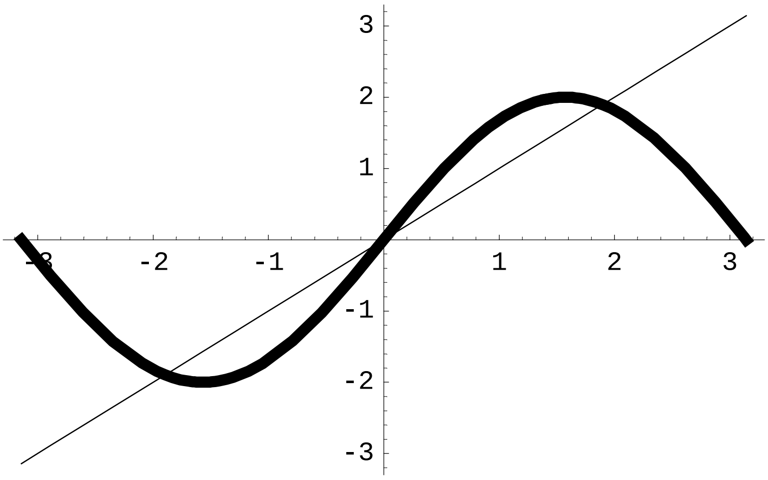
$$\frac{1}{2\pi}\Bigg(-\left(e^{-i(\pi-x)}\right)^{1+n}\text{LerchPhi}[e^{-i(\pi-x)},2,1+n]-$$
$$\left(e^{i(\pi-x)}\right)^{1+n}\text{LerchPhi}[e^{i(\pi-x)},2,1+n]+$$
$$\left(e^{-i(\pi+x)}\right)^{1+n}\text{LerchPhi}[e^{-i(\pi+x)},2,1+n]+$$
$$\left(e^{i(\pi+x)}\right)^{1+n}\text{LerchPhi}[e^{i(\pi+x)},2,1+n]+$$
$$i\pi\left(-\frac{\left(e^{-i(\pi-x)}\right)^{1+n}\text{Hypergeometric2F1}[1+n,1,2+n,e^{-i(\pi-x)}]}{1+n}-\text{Log}[1-e^{-i(\pi-x)}]\right)-$$
$$i\pi\left(-\frac{\left(e^{i(\pi-x)}\right)^{1+n}\text{Hypergeometric2F1}[1+n,1,2+n,e^{i(\pi-x)}]}{1+n}-\text{Log}[1-e^{i(\pi-x)}]\right)-$$
$$i\pi\left(-\frac{\left(e^{-i(\pi+x)}\right)^{1+n}\text{Hypergeometric2F1}[1+n,1,2+n,e^{-i(\pi+x)}]}{1+n}-\text{Log}[1-e^{-i(\pi+x)}]\right)+$$
$$i\pi\left(-\frac{\left(e^{i(\pi+x)}\right)^{1+n}\text{Hypergeometric2F1}[1+n,1,2+n,e^{i(\pi+x)}]}{1+n}-\text{Log}[1-e^{i(\pi+x)}]\right)+$$
$$\text{PolyLog}[2,e^{-i(\pi-x)}]+\text{PolyLog}[2,e^{i(\pi-x)}]-$$
$$\text{PolyLog}[2,e^{-i(\pi+x)}]-\text{PolyLog}[2,e^{i(\pi+x)}]\Bigg)$$

4. Visualización y comparación de las gráficas de la función y de las sumas parciales de su serie de Fourier

Ahora comparamos las gráficas de la función y de diferentes sumas parciales de su serie de Fourier. Puede hacerse con gráficos de diferente grosor (para imprimir en una impresora en blanco y negro) o con colores (para imprimir en una impresora en color).

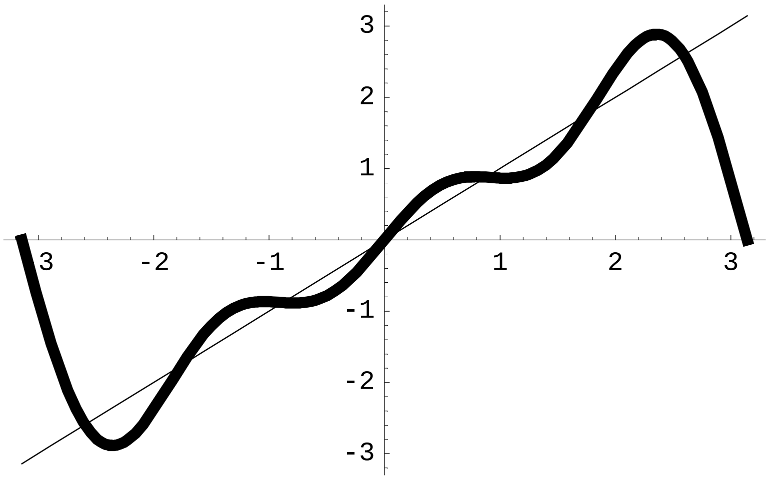
Conviene que recuerdes los resultados teóricos mostrados en clase y pienses lo que ocurre en el intervalo (0,π) y en los estremos del mismo.

In[92]:=
```
Plot[{f[x], foursum[x, 1]}, {x, -Pi, Pi},
  PlotStyle → {{Thickness[0.002]}, {Thickness[0.015]}}]
```



Out[92]= - Graphics -

In[93]:=
```
Plot[{f[x], foursum[x, 3]}, {x, -Pi, Pi},
  PlotStyle → {{Thickness[0.002]}, {Thickness[0.015]}}]
```

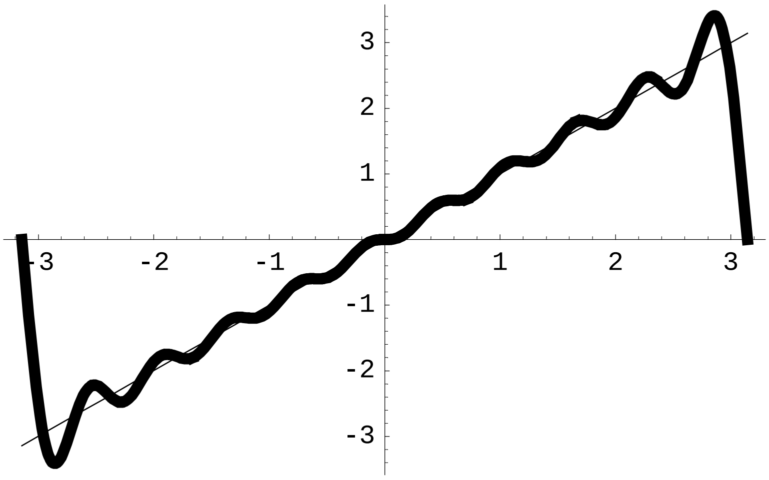


Out[93]= - Graphics -

In[94]:=
```
- Graphics -
Plot[{f[x], foursum[x, 10]}, {x, -Pi, Pi},
 PlotStyle → {{Thickness[0.002]}, {Thickness[0.015]}}]
```

```
Syntax::noinfo : Input expression contains
   insufficient information to interpret result. More…
```

Out[94]= - Graphics -

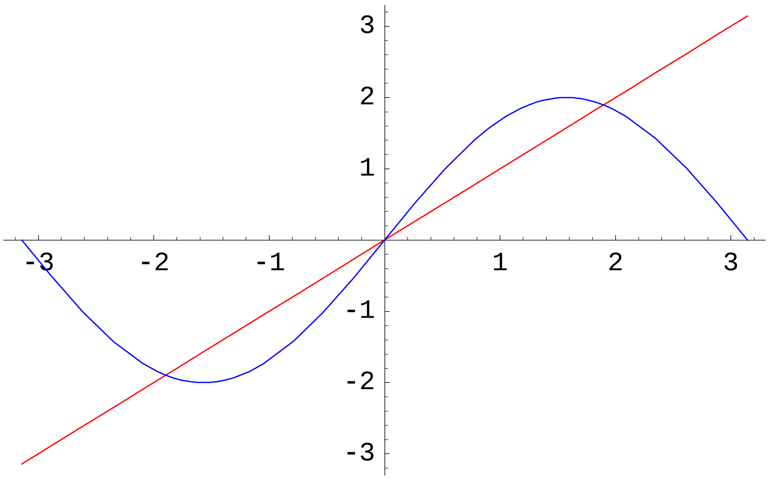


Out[95]= - Graphics -

In[96]:=
```
- Graphics -
Plot[{f[x], foursum[x, 1]}, {x, -Pi, Pi},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]
```

```
Syntax::noinfo : Input expression contains
   insufficient information to interpret result. More…
```

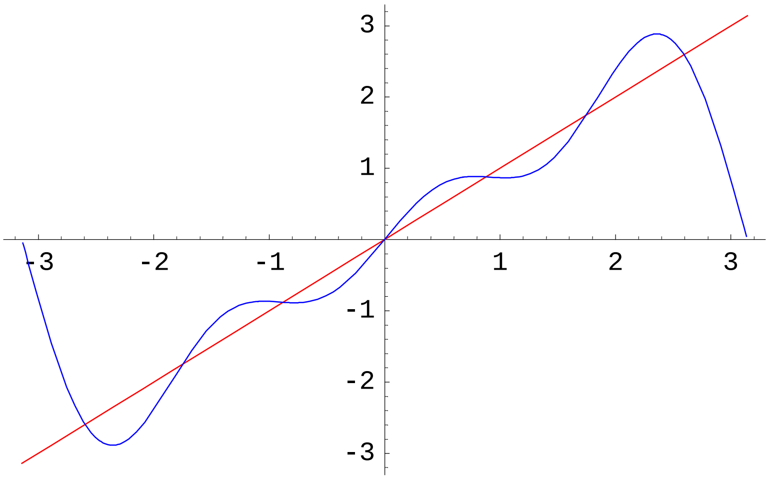
Out[96]= - Graphics -



Out[97]= - Graphics -

In[152]:=
```
- Graphics -
Plot[{f[x], foursum[x, 3]}, {x, -Pi, Pi},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]
```

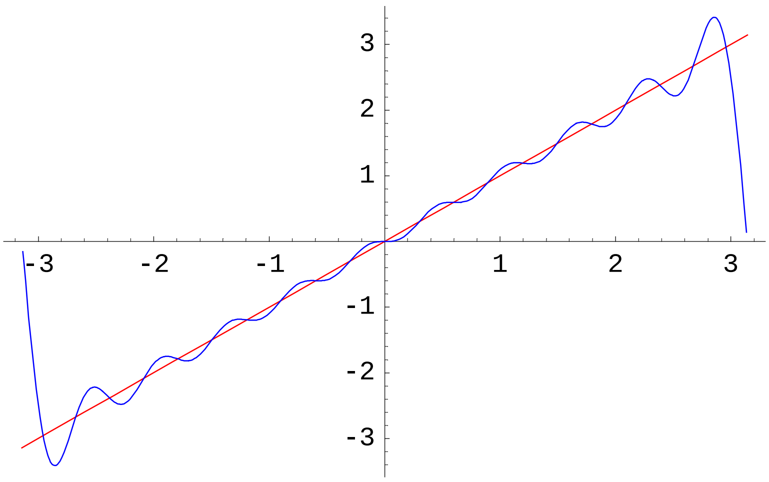
Out[152]= - Graphics -



Out[153]= - Graphics -

In[150]:=
```
- Graphics -
Plot[{f[x], foursum[x, 10]}, {x, -Pi, Pi},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]
Null
```

Out[150]= - Graphics -

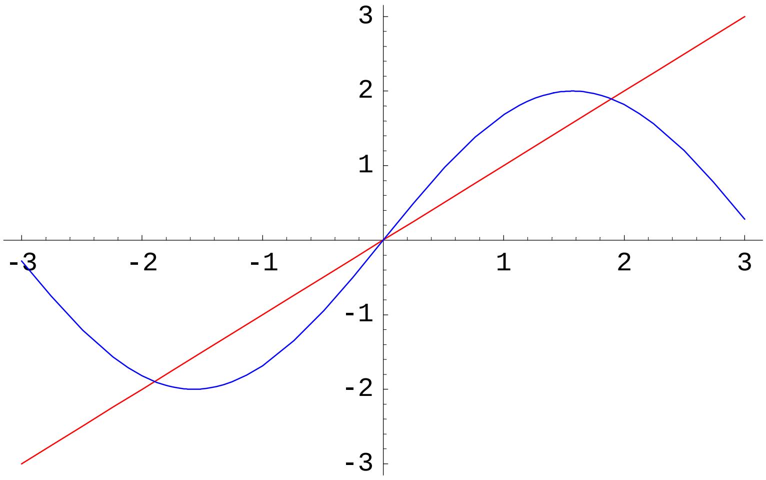


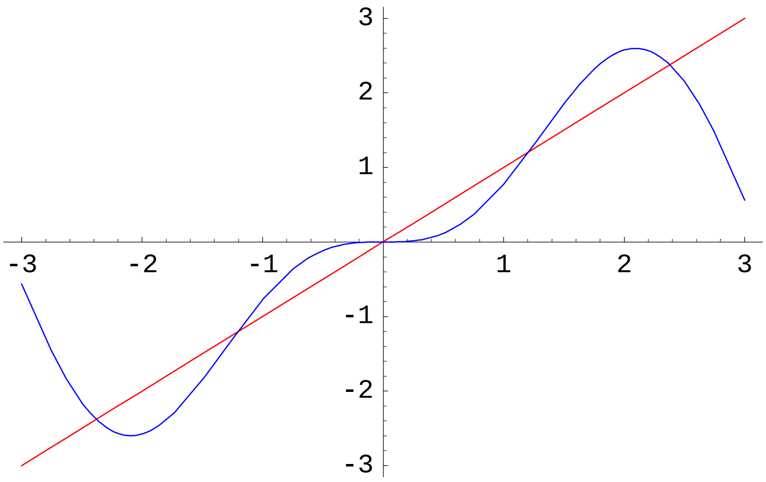
Out[151]= - Graphics -

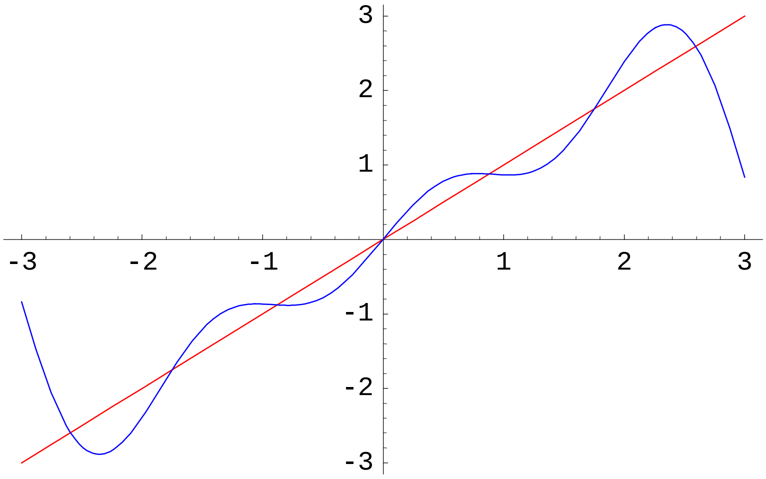
5. Una animación

Podemos hacer una visualización "animada" de lo anterior.

In[98]:=
```
For[n = 1, n ≤ 20, n = n + 1, Plot[{f[x], foursum[x, n]},
  {x, -3, 3}, PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]]
```



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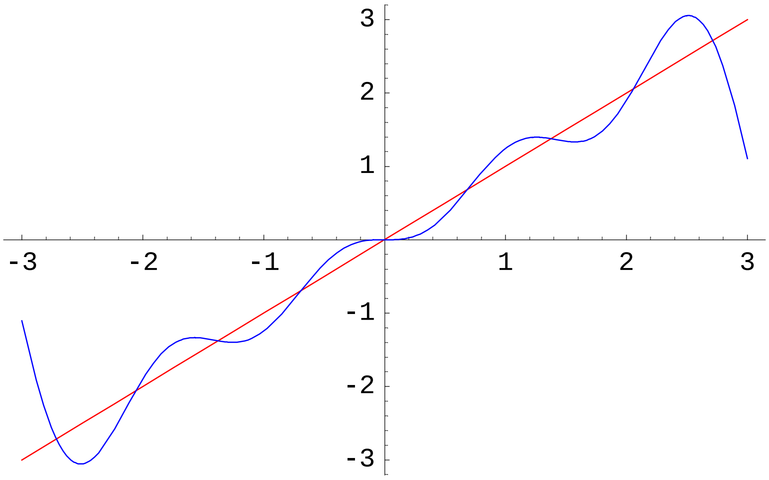
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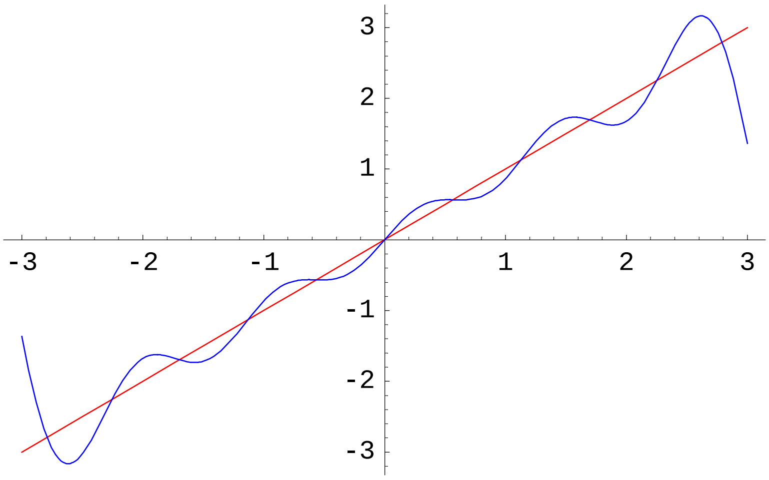
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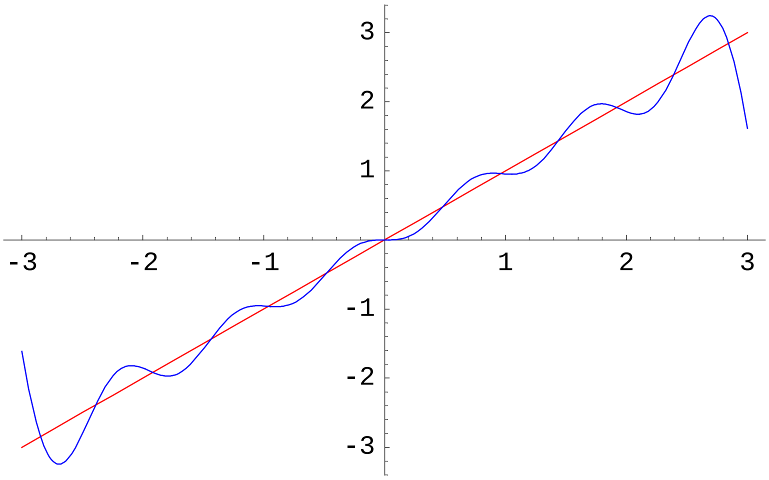
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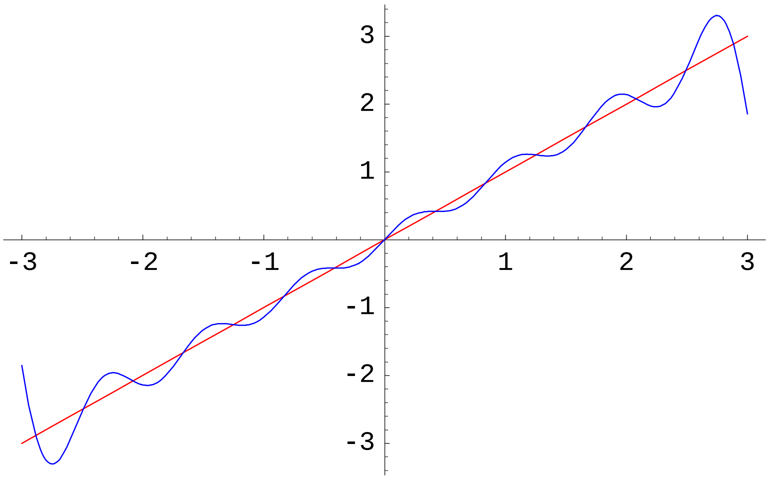
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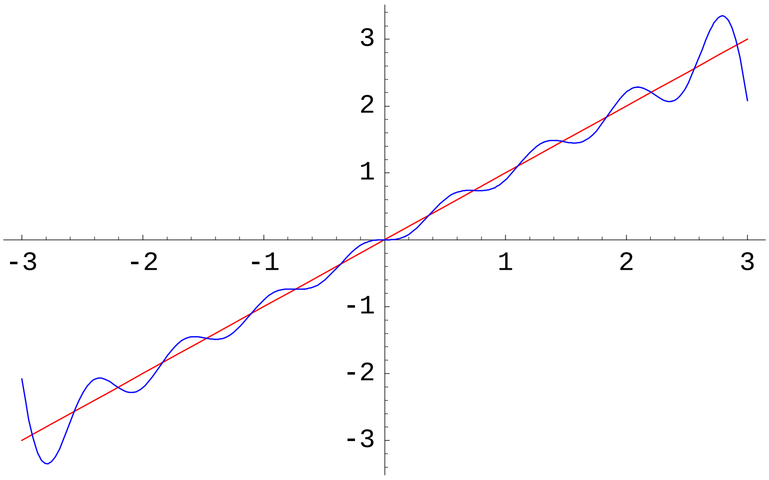
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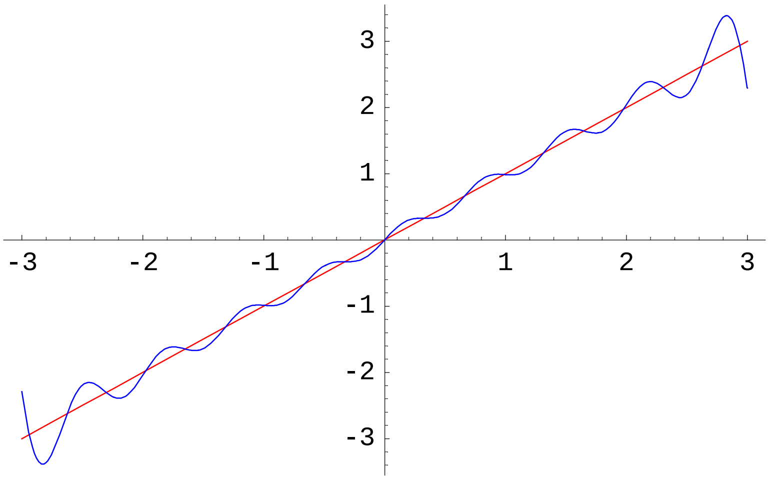
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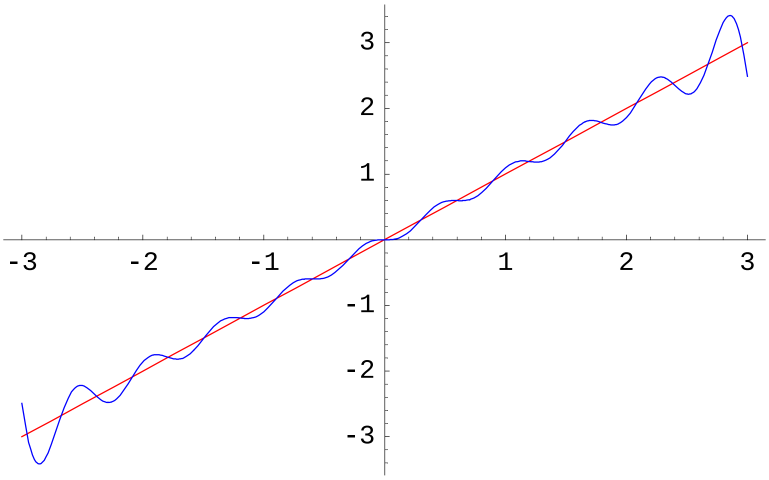
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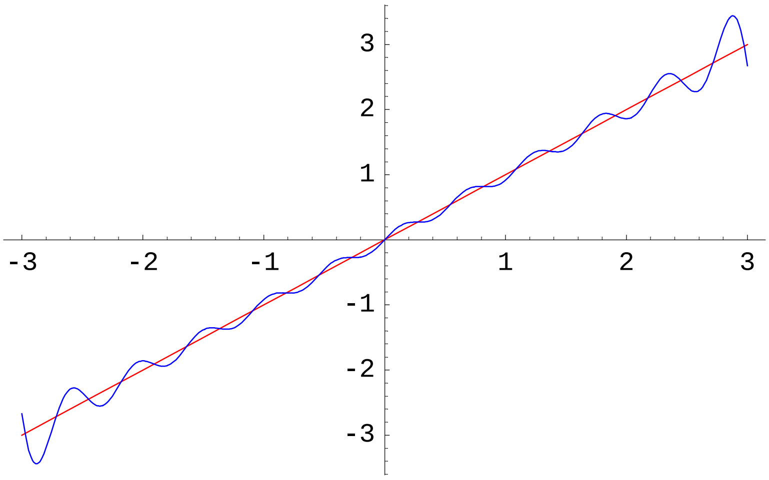
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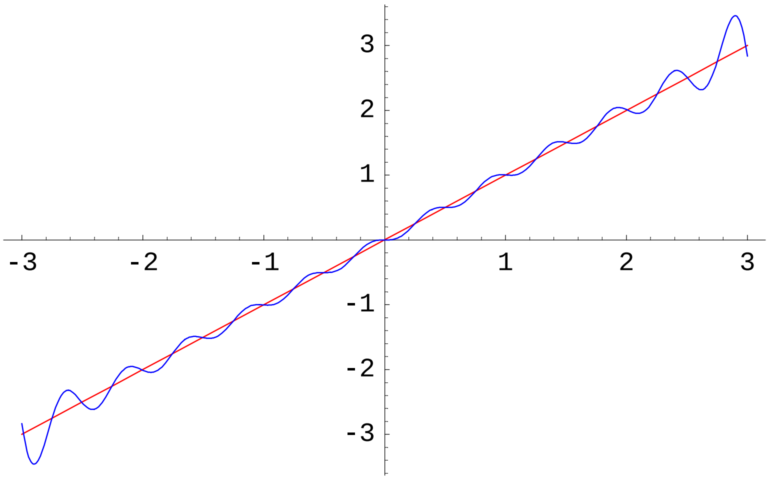

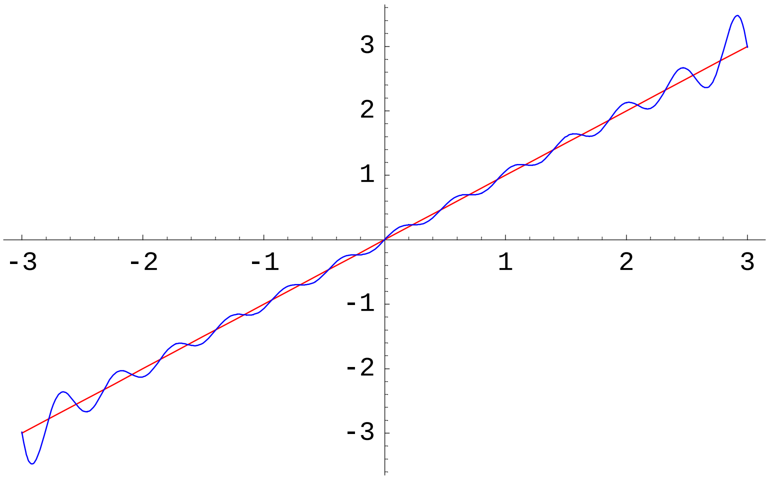

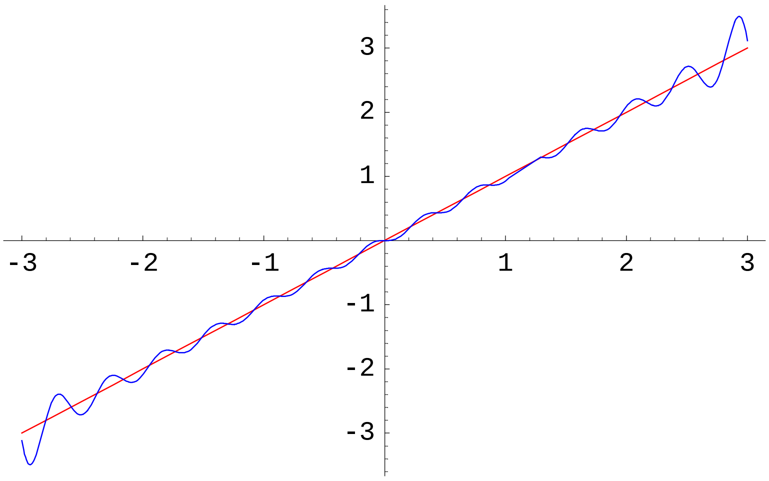

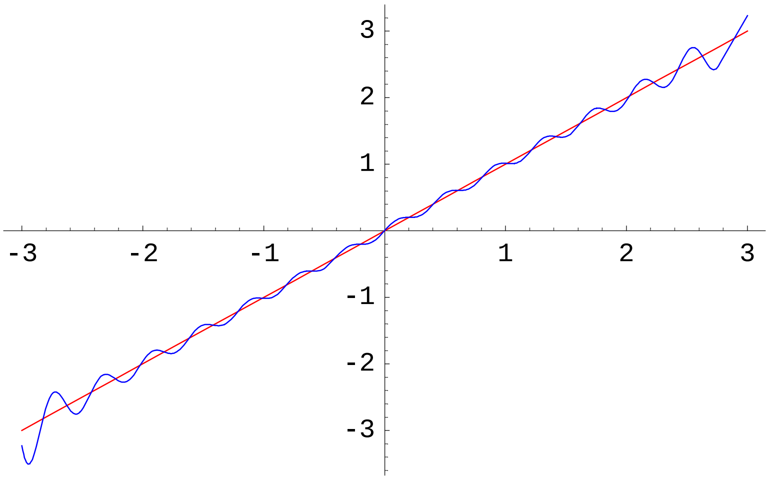

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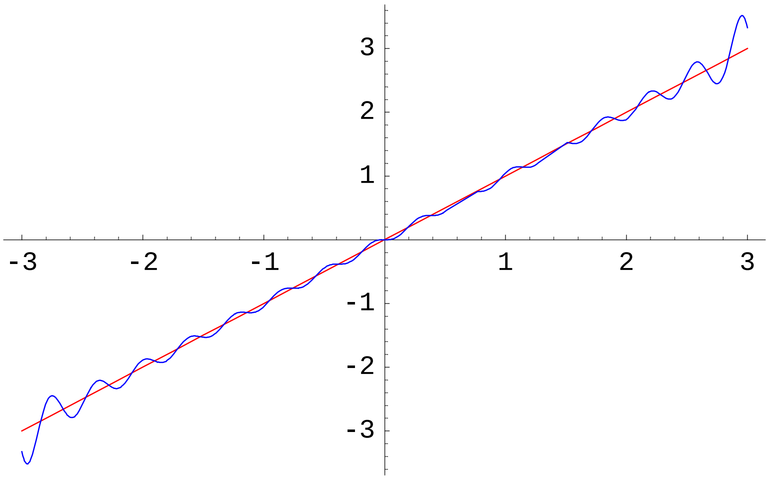

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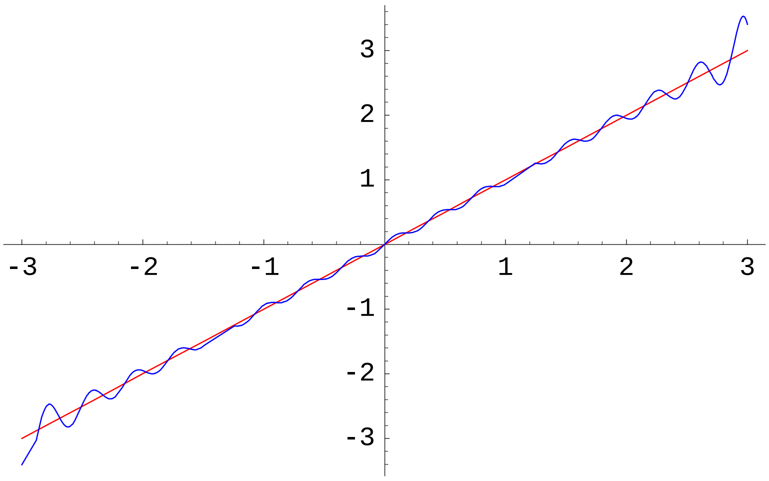

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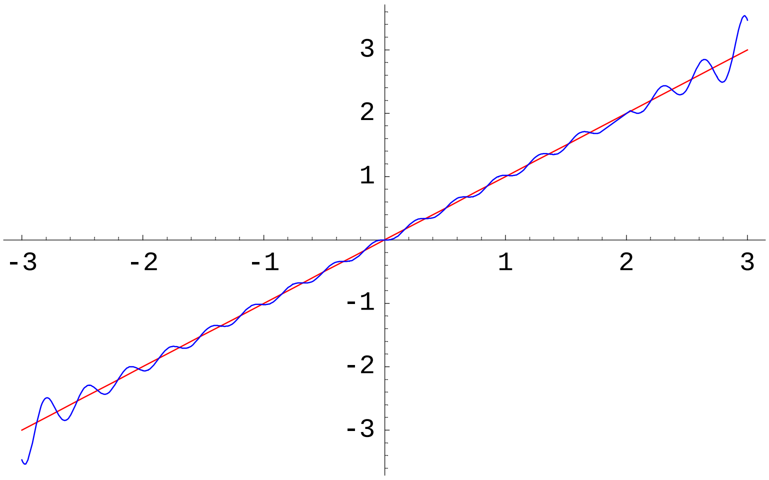
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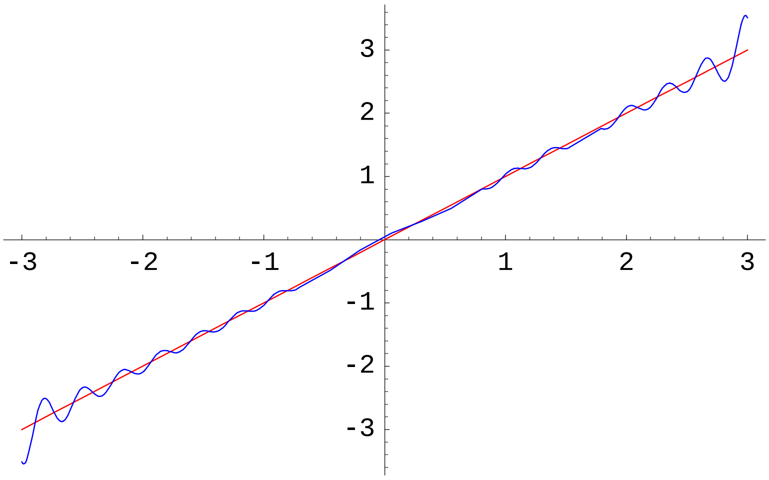
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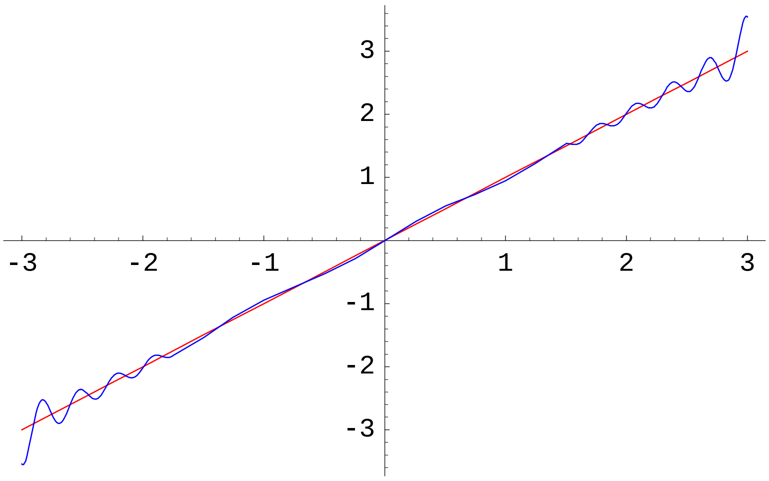
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