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How to Use MATHEMATICA^{TM} to Find Cyclic Surfaces of Constant Curvature in Lorentz-Minkowski Space

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ABSTRACT. The development in recent years of computer tools make it possible for geometers to approach old and new subjects with different methods than some decades ago. Recent advances in symbolic computation as well as in computer graphics open new perspectives in research in differential geometry. Using the software MATHEMATICA, we investigate a new family of maximal surfaces in Lorentz-Minkowski space. A maximal surface is a spacelike surface with zero mean curvature. The consequent visualisation of these surfaces goes hand-in-hand with the understanding of the geometry behind the computer construction. An advantage that MATHEMATICA offers is that runs on standard PC and Workstation, without the necessity to use big computers, which are only available for a small number of researchers. Gray's book **[2]** is a good example of the potentialities that MATHEMATICA offers in this sense.

1. Statement of results

In 1744 Euler successfully applied the calculus of variations to the problem of the determination of the minimum area surface bounded by two parallel coaxial circles. He proved that the catenoid is the only surface of revolution with the minimum area property. In the 1770s, Meusnier gave the geometric interpretation of this result: the catenoid is the only non-planar surface of revolution in Euclidean space with zero mean curvature H. In the 1860s Riemann found all minimal (non-rotational) surfaces foliated by circles in parallel planes **[11]**. Each one of such surfaces is an embedded surface invariant by a group of translations. This surface has planar ends with the exception of a discrete set of straight lines a discrete set of heights.

A cyclic surface M in Euclidean three-dimensional space $\mathbf{E}^3$ is a surface foliated by a smooth one-parameter family of circular arcs. This means that there is a one-parameter family of planes that intersect the surface M in circular arcs. Surfaces of revolution are the best known examples of cyclic surfaces. Enneper showed that in a cyclic minimal surface, the planes of the foliation must be parallel. Thus, the catenoid and Riemann minimal examples are the only cyclic surfaces in $\mathbf{E}^3$ with zero mean curvature.

In 1989 Nitsche considered non-zero constant mean curvature cyclic surfaces **[10]**. He proved that all cyclic surfaces with non-zero constant mean curvature

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