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provided by the Geometry group of the Williams College SMALL Undergraduate Research Project 1990. SMALL's 1993 Geometry Group examined a similar problem. We sought the optimal "double Wulff cluster" enclosing two regions of area while minimizing a generalized perimeter length derived from a norm on unit tangent vectors. We considered the rectilinear norm, so that the optimal shape enclosing a single region of area, or the "Wulff shape", is a square. I will discuss similar research which seeks the norm-minimizing double cluster when the Wulff shape is a regular hexagon, explaining results and techniques. Further, I will compare the results on hexagonal double Wulff clusters to the Geometry Group results. (Received April 19, 1995)

*902-49-46 **Joseph D. Masters**, University of Texas at Austin, Austin, TX 78712. *The double bubble on S^2.* Preliminary report.

We show that the standard double bubble on S^2 uniquely minimizes perimeter with respect to areas enclosed. Some preliminary results for the double bubble on the flat torus may also be discussed. (Received April 20, 1995)

*902-49-61 **Rafael López**, Departamento de Geometría y Topología, Universidad de Granada, 18071 Granada -Spain-. *Constant Mean Curvature Surfaces With Boundary.*

We are concerned with the shape of a constant mean curvature compact (=cmc) surface with prescribed boundary when the algebraic volume is small. Exactly, when the boundary Γ is a convex planar closed curve, we prove that *there exists a positive constant V_Γ depending only on the curve that any cmc surface with boundary Γ and volume less than V_Γ must be a graph.* We also proved that $2\pi/3$ is the critical volume where the cmc surface is a topological disc and the boundary is a circle. Moreover we obtain certain results about the isoperimetric regions and cmc surfaces when the boundary is convex and planar.

The proofs come from a height estimate for cmc surfaces with planar boundary and we allow to prove a new existence theorem for cmc graphs and planar boundary, which it improves, in some sense, the Serrin classical result. In the case of embedded cmc surfaces and convex planar boundary, we show that they lie in one of the half spaces bounded by the planes containing the boundary, provided they are *small* in some sense. (Received April 24, 1995)

*902-49-93 **Joel D. Shore**, Physics Department, McGill University, Montréal, QC Canada H3A 2T8, and **Dirk Jan Bukman**, Department of Chemistry, Cornell University, Ithaca, NY 14853. *Conical points on equilibrium crystal shapes.*

Given an expression for the surface tension as a function of surface orientation, the celebrated Wulff construction tells us how to determine the resulting equilibrium crystal shape, which is the shape minimizing the total free energy for a given volume of crystal. However, we need guidance from statistical mechanical models in order to learn the form of the surface tension and the resulting crystal shapes which can actually occur for real materials. Toward this end, we have recently investigated the exactly-solved six-vertex model which can be mapped onto the crystal shape of a face-centered-cubic crystal with simple atomic interactions. At the point where an edge, separating two flat facets, meets a rounded part of the crystal shape, we find an interesting "conical point" where a whole one parameter family of surface orientations come together at a single point on the crystal surface, forming a cone-shaped singularity. Since the crystal shape and projected surface tension are Legendre transforms of one another, these points are the converse of the cusps in the surface tension which yield facets on the crystal shape: Here, instead, we have cusps in the crystal shape, and they are associated with flat regions in the (projected) surface tension, representing "coexistence regions" for surfaces of varying orientation. (Received April 30, 1995)