

A GEOMETRIC PARTIAL DIFFERENTIAL EQUATION

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Abstract

The Hessian one equation

$$f_{xx}f_{yy} - f_{xy}^2 = \pm 1$$

and its complex resolution provides an important tool in the study of improper affine spheres. Locally, they are given by the graph

$$\psi = \{(x, y, f(x, y)) : (x, y) \in \Omega \subset \mathbb{R}^2\},$$

with constant affine normal

$$\xi = \frac{1}{2} \Delta_h \psi = (0, 0, 1),$$

where

$$h = \kappa^{-\frac{1}{4}} \sigma = f_{xx} dx^2 + f_{yy} dy^2 + 2f_{xy} dx dy$$

is the affine metric, (the $SL(3, \mathbb{R})$ -invariant metric obtained with the Gauss curvature κ and the second fundamental form σ of ψ).

Conversely, the properties of these umbilical affine surfaces of \mathbb{R}^3 play an important role in the development of geometric methods for the study of their PDEs.

We review some results of this good interplay and present our extension of the classical Ribaucour transformations to this subject. In particular, we construct new solutions and families of improper affine spheres, periodic in one variable, with any even number of complete embedded ends and singular set contained in a compact set, see [1].

Also, we compare the Cauchy problem for the elliptic and non-elliptic Hessian equation, with some results about their admissible singularities, mainly, isolated singularities and singular curves with cuspidal edges and swallowtails, see [2].

References

- [1] A. Martínez, F. Milán, Ribaucour type transformations for the Hessian one equation, *Nonlinear Anal.* **112** (2015), 147–155.
- [2] A. Martínez, F. Milán, The geometric Cauchy problem for the hyperbolic Hessian one equation, *Nonlinear Anal.* **125** (2015), 323–333.

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