PDEs, relativity & nonlinear waves

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coordinated by
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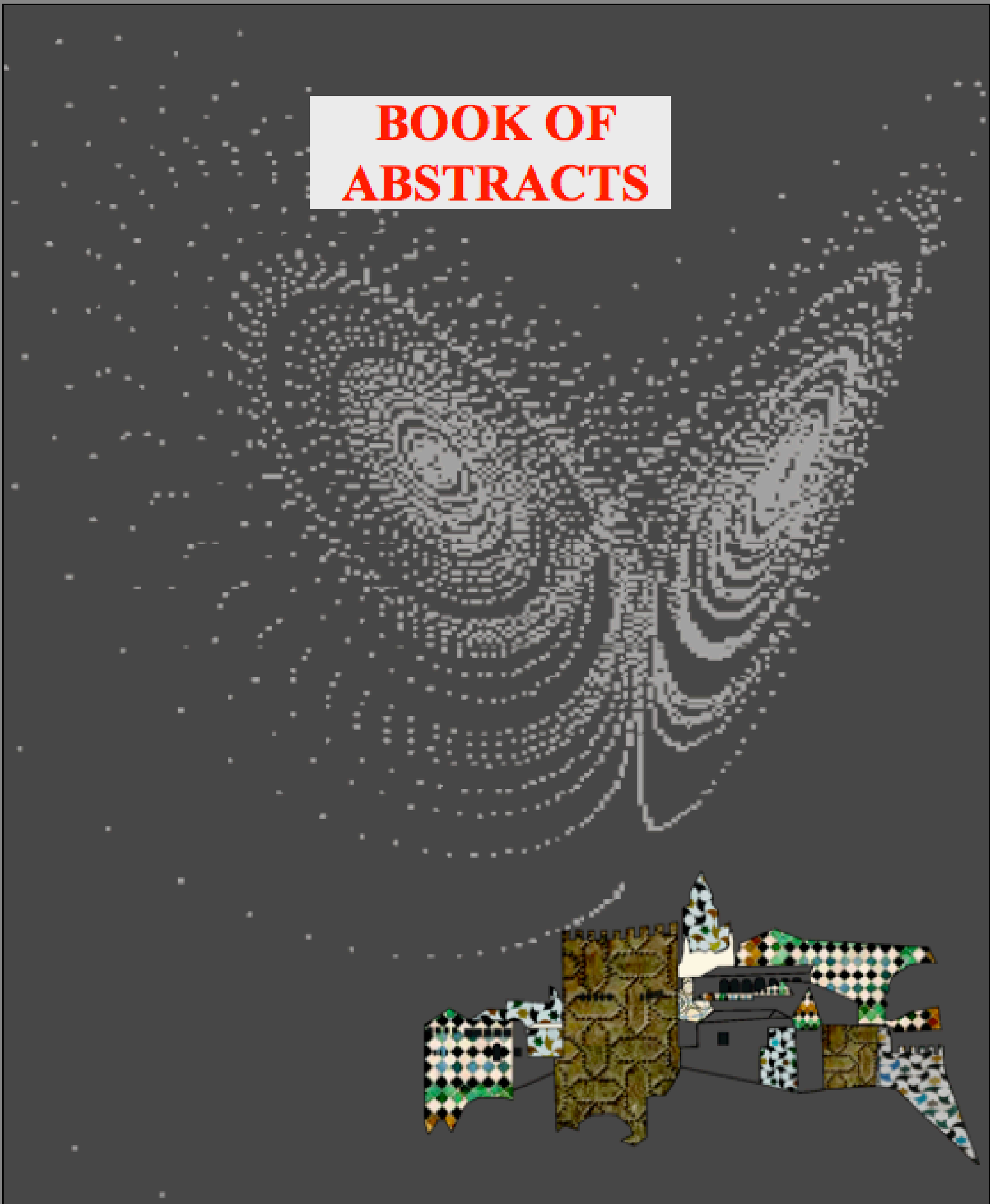
BOOK OF ABSTRACTS



Contents

Posters' abstract

1. Self–gravitating elastic bodies

I will discuss some recent results and open problems concerning self-gravitating elastic bodies in Einstein gravity. Among the topics are constructions of static and rotating bodies, multi-body configurations as well as the dynamics of self-gravitating elastic bodies.

Lars Andersson
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

2. Existence of axially symmteric static solutions of the Einsten–Vlasov system

It is well–known that spherically symmetric static solutions of the Einstein–Vlasov system exist. I will present a result which extends this to include axially symmetric static solutions. This is a joint work with M. Kunze and G. Rein.

Håkan Andreasson
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

3. Hidden symmetries and decay for the wave equation outside a Kerr black hole

The Kerr solutions to Einstein's equations describe rotating black holes. For the wave equation in flat–space and outside the non–rotating, Schwarzschild black holes, one method for proving decay is the vector–field method, which uses the energy–momentum tensor and vector–fields. Outside the Schwarzschild black hole, a key intermediate step in proving decay involved proving a Morawetz estimate using a vector–field which pointed away from the photon sphere, where null geodesics orbit the black hole. Outside the Kerr black hole, the photon orbits have a more complicated structure. By using the hidden symmetry of Kerr, we can replace the Morawetz vector–field by a fifth–order operator which, in an appropriate sense, points away from the photon orbits. This allows us to prove the necessary Morawetz estimate. From this we can prove a decay estimate of almost t^{-1} for fixed r and the corresponding decay rates at the event horizon and null infinity. The major innovation in this result is that, by using the hidden symmetries with the energy–momentum, we can avoid taking Fourier tranforms in time.

This is joint work with Lars Andersson.

Pieter Blue
University of Edinburgh
United Kingdom

4. Dispersion properties in gravitational dynamics

The topic of this talk is the dynamics of a many–particle self–gravitating system, modelled with either classical or relativistic kinetic theory. As a way to gain insight into the relativistic models, we will review the examples avalaible in the classical case and discuss several concepts of dispersion in this setting. Then we present some preliminary results concrening dispersive behaviour for the relativistic models, and necessary conditions for the existence of steady states as well.

Juan Calvo
University of Granada
Spain

5. T.B.A.

Mihalis Dafermos
University of Cambridge
United Kingdom

6. On well-posedness, linear perturbations and mass conservation for axisymmetric Einstein equations

For axially symmetric solutions of Einstein equations there exists a gauge which has the remarkable property that the total mass can be written as a conserved, positive definite, integral on the spacelike slices. The mass integral provides a nonlinear control of the variables along the whole evolution. In this gauge, Einstein equations reduce to a coupled hyperbolic-elliptic system which is formally singular at the axis. As a first step in analyzing this system of equations in this talk I will discuss linear perturbations on flat background. I will prove that the linear equations reduce to a very simple system of equations which provide, thought the mass formula, useful insight into the structure of the full system. However, the singular behavior of the coefficients at the axis makes the study of this linear system difficult from the analytical point of view. In order to understand the behavior of the solutions, we study the numerical evolution of them. We provide strong numerical evidence that the system is well–posed and that its solutions have the expected behavior. This linear system allows us to formulate a model problem which is physically interesting in itself, since it is connected with the linear stability of black hole solutions in axial symmetry. This model can contribute significantly to solve the nonlinear problem and at the same time it appears to be tractable. I will also discuss initial data for this system of equations which are close to the extreme Kerr initial data.

Sergio Dain
Universidad Nacional de Córdoba
Argentina

7. On conformal structures of static vacuum data

We discuss possibilities to characterize the conformal structures of three-dimensional, asymptotically flat, static vacuum data in terms of conditions on the metric field at space-like infinity.

Helmut Friedrich
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

8. Collisionless matter in Bianchi cosmology

In this talk we analyze cosmological solutions of Bianchi class A of the Einstein-Vlasov equations, which represent ensembles of collisionless particles that interact gravitatively through Einstein's equations. We prove that generic solutions exhibit an oscillatory approach toward the initial singularity (the 'big bang'), which is in contrast to the behavior of Einstein-vacuum or Einstein-Euler solutions. This talk is based on joint work with Simone Calogero.

Mark Heinzle
University of Vienna
Austria

9. Non-existence of stationary two-black-hole configurations

We resume former discussions of the question, whether the spin–spin repulsion and the gravitational attraction of two aligned rotating black holes can balance each other. To answer the question we formulate a boundary value problem for two separate (Killing-) horizons in axisymmetry and stationarity and apply the inverse scattering method to solve it. Finally, we use a universal inequality for sub–extremal black holes and the positive mass theorem to show that two (sub-extremal or degenerate) black holes cannot be in equilibrium.

Jörg Hennig
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

10. Theorems about black holes in higher dimensions

The purpose of the talk is to review recent progress in the classification problem for higher dimensional black holes. This will include 1) Generalizations of the classical uniqueness theorems for e.g. vacuum black holes in $D \geq 5$ dimensions. These results typically assume the presence of certain sufficiently large abelian isometry group and state that there can be at most one solution for given values of the mass and angular momenta and and various globally defined "moduli" and "winding numbers".

2) Classifications of possible near horizon geometries for extremal black holes. 3) Theorems classifying the possible spacetime- and horizon topologies.

Stefan Hollands
Cardiff University
United Kingdom

11. Einstein spacetimes with distributional curvature

I will discuss the Einstein's field equations of general relativity when weak regularity only is assumed on the initial data set, and the curvature must be understood in the distributional sense. In particular, I will establish the existence of a class of spacetimes with Gowdy symmetry, containing compressible fluids described by the Euler equations. The proposed framework includes impulsive gravitational waves propagating at the speed of light and shock waves propagating at about the sound speed. Vacuum spacetimes with $T2$ symmetry will also be discussed. This lecture is based on several joint papers written with A.D. Rendall, J. Smulevici, and J.M. Stewart.

[1] A.P. Barnes, P.G. LeFloch, B.G. Schmidt, and J.M. Stewart, The Glimm scheme for perfect fluids on plane-symmetric Gowdy spacetimes, Classical Quantum Gravity 21 (2004), 5043–5074.

[2] P.G. LeFloch and C. Mardare, Definition and weak stability of spacetimes with distributional curvature, Port. Math. 64 (2007), 535–573.

[3] P.G. LeFloch and A.D. Rendall, A global foliation of Einstein-Euler spacetimes with Gowdy-symmetric on T3, Class. Quantum Gravity, submitted.

[4] P.G. LeFloch and J. Smulevici, Global geometry of future expanding T2–symmetric spacetimes with weak regularity, in preparation.

[5] P.G. LeFloch and J.M. Stewart, Shock waves and gravitational waves in matter spacetimes with Gowdy symmetry, Portugal. Math. 62 (2005), 349–370.

[6] P.G. LeFloch and J.M. Stewart, The characteristic initial value problem for plane-symmetric spacetimes with weak regularity, Comm. Math Phys., submitted.

Philippe G. LeFloch
University Paris 6 and CNRS
France

12. On the Penrose inequality in static initial data sets

Bray and Khuri have recently put forward an interesting new approach to address the Penrose inequality for arbitrary initial data sets. I will discuss this approach in the particular case of static initial data sets, in order to describe some of the issues that need to be addressed for the method to be successful. I will show, in particular, that the version of the Penrose inequality in terms of so–called "generalized apparent horizon" fails for suitably chosen slices of the Kruskal spacetime.

MARC MARS
University of Salamanca
Spain

13. Formation of Higher-dimensional Topological Black Holes

We study higher dimensional gravitational collapse to topological black holes in two steps. Firstly, we construct some $(n+2)$–dimensional collapsing space–times, which include generalised Lemaitre–Tolman–Bondi–like solutions, and we prove that these can be matched to static Λ–vacuum exterior space–times. We then investigate the global properties of the matched solutions which, besides black holes, may include the existence of naked singularities and wormholes. Secondly, we consider as interiors classes of 5-dimensional collapsing solutions built on Riemannian Bianchi IX spatial metrics matched to radiating exteriors given by the Bizon-Chmaj-Schmidt metric. In some cases, the data at the boundary for the exterior can be chosen to be close to the data for the Schwarzschild solution.

JOSÉ NATÁRIO
University of Lisbon
Portugal

14. Some wave maps related to the Einstein equations

It is well known that the essential field equations satisfied by the Gowdy solutions of the vacuum Einstein equations are equivalent to the equations for a wave map from a fixed Lorentzian metric to the hyperbolic plane. This fact has proved extremely useful in analysing the global structure of this class of spacetimes. In this talk I will describe some generalizations of this fact. If the vacuum Einstein equations are replaced by the Einstein-Maxwell equations a wave map with values in the complex hyperbolic plane is obtained. In this case the correspondence is subject to a global condition. Other choices of matter fields lead to other target spaces. The same is true of solutions of the vacuum Einstein equations in higher dimensions. In all these cases the wave maps involved are invariant under certain discrete transformations. Replacing this condition by equivariance gives rise to generalizations of the Gowdy spacetimes whose topology is twisted. They represent nonlinear perturbations of homogeneous spacetimes including those of Bianchi type II.

ALAN RENDALL
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

15. Stability of cosmological solutions to the Einstein–Vlasov system with a positive cosmological constant

The subject of the talk is the Einstein–Vlasov–non–linear scalar field system (the Einstein–Vlasov system with a positive cosmological constant being contained as a special case). We begin by giving an introduction to this system and then address the question of stability in a cosmological setting. Moreover, we discuss the restrictions on the global topology of a spacetime imposed by the requirement that the causal past of any given observer is close to the corresponding set in a spatially homogeneous, isotropic and spatially flat model solution.

Hans Ringström
KTH Royal Institute of Technology
Sweden

16. Orbital stability of galactic dynamics in relativistic scalar gravity

A classical problem in theoretical astrophysics is to investigate the (non–linear) stability of galaxies in equilibrium. From a mathematical point of view, a lot of work has been developed in order to understand the stability of stationary solutions to the Vlasov–Poisson system during the last decades.However, the stability of the stationary solutions to the Einstein-Vlasov system is still an open problem. In this talk we will present a non-linear stability analysis to stationary solutions of the Nordström–Vlasov system in order to contribute to to a better understanding of this important problem. The latter provides a genuine relativistic generalization of the Vlasov–Poisson in the following sense: It is invariant under Lorentz transformations and its solutions converge to solutions of Vlasov–Poisson as the speed of light tends to infinity. In this analysis, we prove existence and non-linear (orbital) stability of a class of static solutions (isotropic polytropes) against general perturbations. The proof of orbital stability is based on a variational problem associated to the minimization of the energy functional under suitable constraints.

Oscar Sánchez
University of Granada
Spain

17. Null structure and regularity results for the Maxwell–Dirac system

In this talk I will discuss recent results on the regularity properties of the Maxwell-Dirac system in two and three space dimensions. This is partly joint work with Piero D'Ancona and Damiano Foschi.

Sigmund Selberg
Norwegian University of Science and Technology
Norway

18. The global geometry of T^2 symmetric spacetimes with weak regularity

I will present recent work in collaboration with P.G. LeFloch concerning the global behaviour of weakly regular solutions to the vacuum Einstein equations with T^2 symmetry. In the case of high regularity, the equations can be reduced to a system of wave map equations in $1+1$ dimension coupled to a certain first order system. In our work, the regularity conditions are such that the metric functions satisfying the wave map equations are in H^1. Under these regularity conditions, we prove a global existence theorem in areal coordinates.

Jacques Smulevici
University of Cambridge
United Kingdom

19. The stability of the Euler–Einstein system with a positive cosmological constant

The Euler–Einstein system models the evolution of a dynamic spacetime containing a perfect fluid. In this talk, I will discuss the nonlinear stability of the Friedmann–Lemaître–Robertson–Walker family of background cosmological solutions to the Euler–Einstein system in $1+3$ dimensions with a positive cosmological constant Λ. The background solutions describe an initially uniform quiet fluid of positive energy density evolving in a spacetime undergoing accelerated expansion. The main result is a proof that under the equation of state $p = c_s^2 \rho$, $0 < c_s^2 < 1/3$, the background solutions are globally future–stable under small perturbations. In particular, the perturbed spacetimes, which have the topological structure $[0, \infty) \times \mathbb{T}^3$, are future causally geodesically complete. The results I will present are extensions of previous joint work with Igor Rodnianski, which covered the case of an irrotational fluid, and of work by Ringström on the Einstein–non–linear–scalar–field system. Mathematically, the main result is a proof of small–data global existence for a modified version of the Euler–Einstein equations that are equivalent to the un–modified equations. The proof is based on the vectorfield method of Christodoulou and Klainerman.

It is of special interest to note that the behavior of the fluid in an exponentially expanding spacetime differs drastically from the case of flat spacetime. More specifically, Christodoulou has recently shown that on the Minkowski space background, data arbitrarily close to that of an initially uniform quiet fluid state can lead to solutions that form shocks. In view of this fact, we remark that the proof of our result can be used to show the following: exponentially expanding spacetime backgrounds can prevent the formation of shocks.

Jared Speck
University of Cambridge
U.S.A.

20. How much mass can you fit in a sphere?

In spherically symmetric spacetimes which look, at least topologically, like Minkowski space, restrictions on the eigenvalues of the Einstein tensor lead to bounds on $2m/r$, where m is the Hawking mass and r is the areal radius. The most famous such bound is that of Penrose, that $2m/r < 1$ when the Einstein tensor satifies the dominant energy condition, but better bounds can be proved under stronger assumptions on the Einstein tensor, including a number of physical interest. I will discuss recent work by myself and Paschalis Karageorgis which gives a uniform method of obtaining such bounds in the static case. I will also discuss related work and possible extensions.

John Stalker
Trinity College Dublin
Ireland

21. Static electrovacuum spacetimes with no event horizons

For the Einstein-Maxwell system with any given electromagnetic Lagrangian, we construct static, spherically symmetric, asymptotically flat solutions that have no event horizon and are either non-singular or only mildly singular on the axis of symmetry. We show that the solutions corresponding to a scaled version of the "ether" law can have any given ADM mass and charge. Finally we report on some work in progress on the radiation probing of static spherically symmetric spacetimes with a naked conical singularity on the axis.

Shadi Tahvildar Zadeh
Rutgers, the State University of New Jersey
U.S.A.

22. Price's law for linear waves on Schwarzschild/Kerr backgrounds

More than 30 years ago Price conjectured a t^{-3} local decay rate for linear waves on Schwarzschild backgrounds. Until not long ago, this had only been verified in special cases; a number of partial results were also obtained. This talk will describe recent work establishing this conjecture for a large class of stationary space–times which includes the Schwarzschild metric, as well as the Kerr metric with small angular momentum.

Daniel Tataru
University of California, Berkeley
U.S.A.

23. Some conformally–complete cosmological models

In his "Conformally–cyclic cosmology", Penrose is interested in positive Λ cosmologies with an initial singularity at which the Weyl tensor is finite or zero and a final space-like future null infinity. We show there exist both spatially–homogeneous solutions of the Einstein–Vlasov equations and spatially–homogeneous radiation–filled cosmologies with these properties. However, we shall see that these solutions aren't exactly what Penrose needs. We argue that an extra scalar field is needed and speculate how this might work.

Paul Tod
University of Oxford
United Kingdom

24. A geometric invariant measuring the deviation from Kerr data

I will discuss the construction of a geometrical invariant for regular asymptotically Euclidean data for the vacuum Einstein field equations. This invariant vanishes if and only if the data corresponds to a slice of the Kerr black hole spacetime —thus, it provides a measure of the "non–Kerrness" of generic data. In order to motivate the construction of the geometric invariant, I will discuss a characterisation of the Kerr spacetime using valence–2 Killing spinors —this will lead to the notion of approximate Killing spinors. An approximate Killing spinor is a valence–2 symmetric spinor satisfying a system of second order linear elliptic equations. The existence of solutions to this elliptic system with the appropriate behaviour at infinity will be discussed. This is work in collaboration with T. Bäckdahl.

Juan Antonio Valiente Kroon
Queen Mary, University of London
United Kingdom

25. Minimal data at a point for solutions to certain geometric systems

In this poster recent results in the characterization of solutions to certain geometrical system of equations in a three dimensional Riemannian manifold will be presented. The system of equations has been constructed as to include several physically interesting systems of equations, such as the stationary Einstein vacuum field equations or harmonic maps coupled to gravity in three dimensions. A characterization of its solutions in a neighbourhood of a given point through sequences of symmetric trace free tensors is given and necessary and sufficient conditions on the data for the existence of the solution are presented, thus providing a complete characterization of all the solutions around the given point.

Andrés Aceña
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

26. Steady states of the relativistic Vlasov–Maxwell–Fokker–Planck system

We prove the existence of steady states of the relativistic Vlasov–Maxwell–Fokker–Planck system with an external confining potential. Moreover we show that the steady states are minimizers of a properly defined free energy functional, which is decreasing along solutions of the time dependent problem.

José Antonio Alcántara Félix
Universidad de Granada
Spain

27. The Einstein–Klein–Gordon–Friedrich system. On the non–linear stability of scalar–field cosmologies

We construct a first-order quasi-linear symmetric hyperbolic system for a homogeneous scalar-field minimally coupled to gravity, using the Friedrich formulation. We then show that for some classes of potentials, small nonlinear perturbations of an expanding Klein-Gordon-Robertson-Walker background, exponential decay or converge to a constant value.

Artur Alho
Universidade do Minho
Portugal

28. The Einstein field equations for cylindrically symmetric elastic configurations

In the context of relativistic elasticity it is interesting to study axially symmetric space-times due to their significance in modeling neutron stars. To approach this problem, here, a particular class of these space-times is considered. A cylindrically symmetric elastic space-time configuration is studied, where the material metric is taken to be flat. The components of the energy-momentum tensor for elastic matter are written in terms of the invariants of the strain tensor, here chosen to be the eigenvalues of the pulled-back material metric. The Einstein field equations are presented and a condition confirming the existence of a constitutive function, which satisfies the Einstein field equations is obtained. This condition leads to particular cases for which we hope to find examples of solutions.

Irene Brito
Universidade do Minho
Portugal

29. Initial data characterisations in General Relativity

The Einstein equations have been studied mainly from two points of view: the "initial value problem" point of view and the "exact solution" point of view. Both approaches have received wide attention in recent years but they have followed generically different procedures for different aims. In this work we seek to find conditions on an initial data set of the Einstein field equations which ensure that the data development is a subset of some known exact solution. This problem is a combination of the two points of view just mentioned and we describe how the conditions are found for the Kerr solution (Kerr initial data) and the Schwarzschild solution (Schwarzschild initial data). Some applications of these results are discussed.

Alfonso García-Parrado
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

30. Curvature conditions of pseudo-symmetric type on Riemannian manifolds

Pseudo–symmetric Riemannian spaces ($R \cdot R$ and $Q(g, R)$ are linearly dependent at every point of the manifold) constitute a generalization of spaces of constant sectional curvature, along the line of locally symmetric ($\nabla R = 0$) and semi–symmetric spaces ($R \cdot R = 0$). This notion arose during the study of totally umbilical submanifolds of semi–symmetric spaces, as well as during the consideration of geodesic mappings.

The concept has applications in di?erent topics. Many space–time metrics have been shown to satisfy a curvature condition of pseudo–symmetric type, which in combination with the Einstein equations reect some symmetry of the physical space.

In our approach we consider a di?erent perspective, using sub- geodesic mappings which are a natural generalization of geodesic map- pings on Riemannian manifolds. We study ξ–subgeodesically related spaces, extending some known results concerning pseudo-symmetric spaces admitting geodesic mappings. Using properties of concircular transformations of metrics, conformal related spaces are characterized. Concircular semi-symmetric Riemannian spaces ($R \cdot Z = 0$) which are geodesically related are also studied.

Iulia Elena HIRICĂ
University of Bucharest
Romania

31. Isotropization of non-diagonal Bianchi I–symmetric spacetimes with collisionless matter at late times assuming small data

Assuming that the space-time is close to isotropic in the sense that the shear parameter is small and that one has certain control over the maximal velocity of the particles, we have been able to show that for non-diagonal Bianchi I-symmetric spacetimes with collisionless matter the asymptotic behaviour at late times is close to the special case of dust. We have been able to show that the Kasner exponents converge to $\frac{1}{3}$ and also an asymptotic expression for the induced metric has been obtained. The key has been a bootstrap argument..

Ernesto NUNGESSER
Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Germany

32. Bianchi type I string cosmological models in Brans Dicke theory

Homogeneous and anisotropic exact bianchi type -I string cosmological model is investigated in Brans- Dicke theory with an special case when the sum of total of rest energy density and tension density for cloud of strings vanishes. To get determinate solutions it is assumed that expansion is proportional to the eigen value of the shear tensor.Some physical and Geometrical properties of the models are also discussed.

Ghanshyam Singh RATHORE
Mohanlal Sukhadia University Udaipur
India

33. Perturbation method for particle–like solutions of the Einstein–Dirac–Maxwell equations

We prove by a perturbation method the existence of solutions of the coupled Einstein–Dirac equations and of the coupled Einstein–Dirac–Maxwell equations for a static, spherically symmetric system of two fermions in a singlet spinor state and, in the case of the Einstein–Dirac–Maxwell equations, with the electromagnetic coupling constant $(\frac{e}{m})^2 < 1$. We show that the nondegenerate solution of Choquards equation generates a branch of solutions of the Einstein–Dirac equations and of the Einstein–Dirac–Maxwell equations.

Simona Rota Nodari
CEREMADE
France

34. Constant Mean Curvature Slicing of Reissner–Nordström Black Holes

In order to specify a foliation of spacetime into a spacelike hypersurfaces we need to prescribe a way to calculate the lapse function ? which measures the proper time interval between neighboring surfaces hypersurfaces along the normal direction. There are different possible lapse choices that can be classied in the following way 1) prescribed slicing conditions where the lapse is specied as a a priori known function of space and time, 2) algebraic slicing conditions where the lapse is specied as some algebraic function of the geometric variables of the hypersurface 3) evolution type slicing conditions where the time derivative of the lapse is evolved as just another dynamical quantity 4) elliptic slicing conditions where the lapse is obtained by solving an elliptic differential equation at every time time step that typically enforces some geometric condition on the spatial hypersurfaces. Here the lapse will be a prescribed slicing know as the constant mean curvature (CMC) slicing. This slicing condition will be applied to the Reissner–Nordström metric and the resultant slices will be investigated and then compared to those of the extended Schwarzschild solution.

Patrick Tuite
University College Cork
Ireland