INTERDISCIPLINARY WORKSHOP ON PERIODIC OSCILLATIONS

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List of abstracts

Dynamical models of exciton-polariton condensates

Natalia Berloff (University of Cambridge, UK)

Exiton-polariton condensates – recently achieved in semiconductor microcavities– can be modelled by complex Ginzburg -Landau equations. I will consider the effects of external trap, Josephson coupling and magnetic field on vortex formation in such condensates.

Periodic orbits of radially symmetric systems with a singularity

Alessandro Fonda (University of Trieste, Italy)

We study radially symmetric systems with a singularity which can be either attractive or repulsive. As a model, we have in mind the motion of a particle in a gravitational field (the attractive case), or in an electrical field with Coulomb-type force. In the presence of a radially symmetric periodic forcing, we show the existence of subharmonic solutions which rotate around the origin with different angular momenta. The proofs are carried out by the use of topological degree theory.

Detection and modelling of synchronization in cardiovascular system

Natalia Janson (Loughborough University, UK)

Synchronization is the result of interaction between coupled oscillators [1]. It occurs through two main mechanisms, phase locking and suppression of natural dynamics, which can be interpreted and compared in the languages of mathematics (saddle-node and Neimark-Sacker bifurcations, torus, periodic orbits, Poincare sections) and of physics (power spectra, amplitude, frequency, phase).

Within the cardiovascular system of a human three main rhythms interact: heart rate, respiration rate and the slow process with frequency 1Hz. Conventional methods for the detection of synchronization can be used when realizations are available for each process involved in interaction (e.g. electrocardiogramme for heart rate, respiratory effort for breathing, etc.). When one can only record a single signal that comes from a system in which several processes interact, conventional methods are not applicable. A method based on the analysis of threshold-crossing interspike intervals is presented and illustrated on cardovascular data. An approach to model interactions, based on evolution of phases of interacting processes, is introduced [2-5].

[1] A.G. Balanov, N.B. Janson, D.E. Postnov, and O. Sosnovtseva, ôSynchronization: from simple to complexö, Springer, 437 p (2009), ISBN: 978-3-540-72127-7

[2] N. B. Janson, A. G. Balanov, V. S. Anishchenko, and P. V. E. McClintock, "Phase relationships between two or more interacting processes from one-dimensional time series. I. Basic theory", Phys. Rev. E., Vol. 65, 036211 , 2002

[3] N. B. Janson, A. G. Balanov, V. S. Anishchenko, and P. V. E. McClintock, "Phase relationships between two or more interacting processes from one-dimensional time series. II. Application to heart-rate-variability data", Phys. Rev. E., Vol. 65, p. 036212, 2002.

[4] N.B. Janson, A. Balanov, N.B. Igosheva, O. Glushkovskaya-Semyachkina, T.G. Anishchenko, P.V.E. McClintock "Indices of cardiorespiratory synchronization from rat blod pressure data", Izv. VUZ Applied Nonlinear Dynamics, 11(3), pp. 120-132 (2003).

[5] N. B. Janson, A. G. Balanov, V. S. Anishchenko, and P. V. E. McClintock, "Modelling the dynamics of angles of human R R intervals", Physiological Measurement, Vol. 22, pp. 565-579, 2001

Oscillators coupled through an environment

Guy Katriel (Tel Aviv University, Israel)

We wish to understand the dynamics generated when many oscillators are indirectly coupled to each other by interacting with another system (the "environment"). Examples of models of biological phenomena which have this structure will be described. Mathematical results on the existence and (in)stability of synchronized and anti-synchronized oscillations will be presented, and some unsolved questions will be discussed.

On periodic oscillations of dark solitons/multi-solitons and vortices in Bose-Einstein condensates at zero and finite temperatures

Panayotis Kevrekidis (University of Massachusetts, USA)

In this talk, we will summarize some of the recent experimental activity on the dynamics of dark matter-wave solitons that has been enabled by the very controllable atomic physics setting of Bose-Einstein condensates, focusing especially on some of the most recent developments in quasi-1d settings. We will then illustrate how to connect these findings to numerical computations of one- and multi-soliton solutions and their linearization in appropriately tailored variants of the nonlinear Schrodinger equation which account for the transverse dimensions of the atomic clouds. We will motivate and compare numerical and experimental findings to simple particle-based theoretical models which capture the essential physics of the periodic oscillations observed in this system. Time permitting, we will generalize these considerations to finite-temperature condensates, as well as to multi-dimensional condensates and the precession of vortices within them.

The early history of self-sustained and relaxation oscillations : a case study in modeling

Jean Mawhin, (Catholic University of Louvain, Belgium)

We survey the early history of modeling the self-sustained and relaxation oscillations in mechanics and electronics, from Lord Rayleigh to Liénard, a beautiful example of the complicated and slow interaction between mathematical concepts and physical facts in the art of modeling.

Models of Fluid Adsorption in Structured Substrates

Carlos Rascón (Universidad Carlos III de Madrid, Spain)

When a gas is in contact with a substrate (a solid), the molecules of the gas usually form a liquid layer that covers the substrate. This liquid layer is generally nanoscopic in thickness and, therefore, is of primary importance in nanotechnology. After a brief introduction to the phenomenology of fluid adsorption, we will describe a number of physical models which are used to describe the thermodynamic behaviour of this liquid layer on substrates with different geometrical shapes. In most cases, these models give rise to non-linear differential equations, although in the most recent models integro-differential equations also appear.

Some qualitative aspects of electrical circuit dynamics

Ricardo Riaza (Universidad Politécnica de Madrid, Spain)

When addressing qualitative properties of dynamical systems arising in specific application fields, it is important to assess the conditions characterizing a given phenomenon in the language of that application. This is the case in electrical circuit theory, where qualitative aspects should be described in terms of the network topology and the electrical features of the circuit devices. From this point of view, we will survey in this talk some qualitative properties of equilibria in (possibly nonlinear) electrical circuits. These properties include asymptotic stability and hyperbolicity, the latter being related to oscillatory phenomena, as well as some local bifurcations. The mathematical discussion is directed to time-domain differential-algebraic circuit models, and makes systematic use of digraph theory.

Chaotic behavior in a one-dimensional cardiac model

David G. Schaeffer (Duke University, USA)

In electrocardiology, the term action potential refers to the behavior that, in response to a brief stimulus, the electrical potential across cardiac cell walls is elevated for an extended period. The duration of action potentials under periodic pacing is an important quantity clinically, physiologically, and mathematically. At slow to moderate pacing rates, every stimulus produces an action potential of the same duration, but at high pacing rates cardiac tissue often undergoes a bifurcation to what is called alternans: i.e., uniform APDÆs are replaced by an alternation between short and long action potentials. In a single cell or a small piece of cardiac tissue, this bifurcation is a familiar period-doubling bifurcation, but when propagation effects are important the nature of the bifurcation to alternans is far from clear. For example, the short/long alternation may suffer phase reversals at various locations in the tissue. This behavior, known as discordant alternans, is considered to be a precursor to ventricular fibrillation. In collaboration with my student, Shu Dai, I have studied these phenomena through an approximate equation (derived by Echebarria-Karma) for the modulation of nonuniform wave trains in one spatial dimension. In this lecture, after describing the context of the problem, I will report on this work. In particular, we have shown that: The modulation equation undergoes both Hopf and steady-state bifurcations; which bifurcation occurs first depends on a parameter derived from the speed of traveling waves; the competition between the two modes gives rise to interesting secondary bifurcation. Moreover, in certain parameter ranges, solutions of this equation exhibit chaotic behavior, as indicated in my title. While chaos has been observed in simulations of spiral waves in several dimensions, it is surprising to find it in a one-dimensional model.

The Nonlinear Dynamics of Seasonal Epidemics

Lewi Stone (Tel Aviv University, Israel)

Seasonality is a driving force that has major impact on the spatio-temporal dynamics of natural systems and their populations. This is especially true for the transmission of common infectious diseases (eg., influenza, measles, chickenpox, pertussis), and of great relevance for host-parasite relationships in general. Here we gain new insights into the nonlinear dynamics of recurrent diseases through the analysis of the classical seasonally forced SIR epidemic model. Despite the mathematical intractability of the forced model, we identify a new threshold effect and give clear analytical conditions for predicting the occurrence of either a future epidemic outbreak, or a "skip" – a year in which an epidemic fails to initiate. The threshold is determined by the population's susceptibility measured after the last outbreak, and the rate at which new susceptible individuals are recruited into the population. Moreover, the time of occurrence (i.e., phase) of an outbreak proves to be a useful new parameter that carries important epidemiological information. In forced systems, seasonal changes can prevent late-peaking diseases (i.e., having high phase) from spreading widely, thereby increasing population susceptibility, and controlling the triggering and intensity of future epidemics. These principles yield forecasting tools that should have relevance for the study of newly emerging and reemerging diseases.

Linked twist maps configurations in first order planar ODEs: some remarks.

Fabio Zanolin (University of Udine, Italy)

In the past three decades the study of linked twist maps (LTMs) defined on annular domains of the plane or the two-dimensional torus has been performed by different authors with the aim of proving the existence of complex dynamics. In more recent years the theory has found various applications also to the study of fluid mixing. In this talk we show some examples (and discuss a few consequences) for first order periodically perturbed planar ODEs when some geometric configurations related to LTMs naturally arise.