## Z_LINEAR_K (Version 5.2)

The program (options 1 and 2) provides the asymptotical inferences ($z$-statistics or confidence intervals CI) for the parameters $L = \Sigma\beta_i p_i$ (a lineal function of $K$ independent proportions with the $\beta_i$ knows) or $R = p_2/p_1$ (the relative risk), by means of the optimal procedures and the score procedure. The key are:

- $x_i \sim$ K independent binomials ($n_i$, $p_i$);
- Coefficients $\beta_i$: $B(+) = \sum_{\beta_i>0}\beta_i$, $B(-) = \sum_{\beta_i<0}\beta_i$, $B = B(+) + B(-)$
- Parameter of interest: $L$ (option 1) or $R$ (option 2).
- TEST: $H$: $L = \lambda$ *vs.* $K$: $L \neq \lambda$ ($\lambda$ is know) or $H$: $R = \rho$ *vs.* $K$: $R \neq \rho$ (error $\alpha$).
- CI: $L1 \leq L \leq L2$ or $R1 \leq R \leq R2$ (confidence $1-\alpha$).
- In some cases the results of the one-tailed test and CI are also provided, as the optimal methods may differ from those in the two-tailed case.

Additionally (option 3), the programme also determines the contrast $\{\beta_{i0}\}$ ($\Sigma\beta_{i0} = 0$) that makes the maximum of the score statistic $Z^2(\lambda = 0)$ (see bellow) which provides a homogeneity test for $2\times K$ tables alternative to the classical Pearson chi-squared. See paper (8).

### SCORE TEST

The score z-statistics for $H$: $L = \lambda$ without continuity correction is:

$$Z(\lambda) = \frac{\bar{L} - \lambda}{\sqrt{\Sigma \beta_i^2 \hat{p}_i \hat{q}_i / n_i}}$$

where $\bar{L} = \Sigma \beta_i \bar{p}_i$, $\bar{p}_i = x_i / n_i$, $\hat{q}_i = 1 - \hat{p}_i$ and $\hat{p}_i$ (= pi_LH in the output of the program) is the maximum likelihood estimator of $p_i$ under $H$. The score z-statistics with a continuity correction (cc) is:

$$Z_c(\lambda) = \begin{cases} 0 & \text{if } \left|\bar{L} - \lambda\right| \leq c \\ \left[\text{Sign}(\bar{L} - \lambda)\right] \dfrac{\left|\bar{L} - \lambda\right| - c}{\sqrt{\Sigma \beta_i^2 \hat{p}_i \hat{q}_i / n_i}} & \text{if } \left|\bar{L} - \lambda\right| > c \end{cases} \quad \text{where } c = \frac{\mathrm{B}(+) - \mathrm{B}(-)}{2\left\{\prod(n_i + 1) - 1\right\}}$$

When the test is for $H$: $R = \rho$, the expressions above are valid if they are changed $\beta_2$ for 1, $\beta_1$ for $-\rho$ and $\lambda$ for 0. Finally, the test conclude $K$ if $|Z(\lambda)|$ (or $|Z_c(\lambda)|$) $\geq z_{\alpha/2}$ (the $100\times(1-\alpha/2)$th percentile of the standard normal distribution). See paper (1).

### SCORE CI

The CI for $L$ (or $R$) is obtained to solve in $\lambda$ (or $\rho$): $|Z(\lambda)| = z_{\alpha/2}$ or $|Z_c(\lambda)| = z_{\alpha/2}$.

### *p*-VALUES

The program also provides the $p$-value of the test when the same makes sense (i.e. when the statistic is not obtained by increasing the data in a quantity that depends on the error $\alpha$).

### REST OF THE TEST AND CI

It comes in a similar way, but to other definitions of the statistic Z (see references). Equivalences between the outputs of the program and methods of the paper in references are indicated in the following table:

| *Case* | *Output program* | *Method in the paper* |
|---|---|---|
| $K \geq 3$ Parameter $L$ | Score (without cc) | S0 in (2) |
| | Score (with cc) | S0c in (2) |
| | Adjusted Wald | W3 in (2) |
| | Peskun | P0 in (2) |
| $K = 2$ and parameter $L$ with $\beta_1 \neq -\beta_2$ | Score (without cc) | S0 in (3) |
| | Score (with cc) | S0c in (3) |
| | Adjusted Wald (a) | W2 in (3) |
| | Adjusted Wald (b) | W4 in (3) |
| | Adjusted likelihood ratio | LR1 in (7) |
| $K = 2$ and parameter $L$ with $\beta_1 = -\beta_2$ (**difference of proportion**) | **Test for lambda = 0** | |
| | Score (without cc) | Two-tailed: ZE0 in (6)<br>One-tailed: $\tilde{Z}$E0 in (11) |
| | Score (with cc) | Two-tailed: ZE0c in (6)<br>One-tailed: $\tilde{Z}$W2 in (11) |
| | Adjusted Score (without cc) | Two-tailed: ZE3 in (6) |
| | Adjusted Score (with cc) | Two-tailed: ZE3c in (6) |
| | Adjusted Wald (without cc) | Two-tailed: ZW4 in (11) |
| | Adjusted Wald (with cc) | One-tailed: $\tilde{Z}$W2 in (11) |
| | Adjusted Arc Sine (a) (with cc) | Two-tailed: $\tilde{A}$E1 in (11)<br>One-tailed: $\tilde{A}$E5 in (11) |
| | **Test for lambda ≠ 0 or CI for *L*** | |
| | Score (without cc) | Two-tailed: ZE0 in (6)<br>One-tailed: ZE0 in (11) |
| | Score (with cc) | Two-tailed: ZE0c in (6)<br>One-tailed: $\tilde{Z}$E0 in (11) |
| | Adjusted Arc Sine (a) | Two-tailed: AE1 in (6) |
| | Adjusted Arc Sine (a) (with cc) | One-tailed: $\tilde{A}$E5 in (11) |
| | Adjusted Arc Sine (b) | Two-tailed: AL1 in (7) |
| | Adjusted Wald | Two-tailed: ZW4 in (6) |
| | Peskun (b) | One-tailed: ZP'0 in (11) |
| | Peskun (a) | One-tailed: ZP0 in (11) |
| $K = 2$ and parameter $R$ (**relative risk**) | **Test for rho = 1** | |
| | Score (without cc) | ZE0 in (5) and (12) |
| | Score (with cc) | ZE0c in (5) |
| | Adjusted Score (without cc) | ZE3 in (5) |
| | Adjusted Score (with cc) | ZE3c in (5) |
| | Adjusted score approx. | ZA1 in (5) and (12) |
| | Adjusted log transfor. | LW2 in (5) and (12) |
| | Peskun | ZP0 in (5) and (12) |
| | Adjusted score Wald | ZW2 in (5) and (12) |
| | **Test for rho ≠ 1 and CI for *R*** | |
| | Score (without cc) | ZE0 in (5) and (12) |
| | Adjusted score approx. | ZA1 in (5) and (12) |
| | Adjusted Wald | ZW4 in (5) |
| | Adjusted log transfor.<br>Log transfor. | Two-tailed: LW1 in (5)<br>One-tailed: LW0 in (12) |
| | Adjusted_1 arc sin | AE1 in (5) |
| | Adjusted_2 arc sin | AE2 in (5) |

| *Case* | *Output program* | *Method in the paper* | |
|---|---|---|---|
| | Peskun | ZP0 in (5) and (12) | |
| | Adjusted_5 arc sin | AE5 in (5) and (12) | |
| $K = 1$<br>Parameter $L$<br>(**one proportion**) | **Test or CI for *L*** | | |
| | Score (without cc) | Two-tailed:<br>One-tailed: | S0 in (4)<br>S0 in (10) |
| | Score (with cc) | Two-tailed:<br>One-tailed: | S0c in (4)<br>S0c in (10) |
| | Adjusted Arc Sine (a) | Two-tailed: | A1 in (4) |
| | Adjusted Wald | Two-tailed:<br>One-tailed: | W2 in (4)<br>W5 in (10) |
| | Modified Score | Two-tailed: | MS in (9) |

## References


(1) Martín Andrés, A., Álvarez Hernández, M. and Herranz Tejedor, I. (2011). Inferences about a linear combination of proportions. *Statistical Methods in Medical Research 20*, 369-387. DOI: 10.1177/0962280209347953. Erratum in *Statistical Methods in Medical Research 21(4)*, 427–428, 2012. DOI: 10.1177/0962280211423597.

(2) Martín Andrés, A., Herranz Tejedor, I. and Álvarez Hernández, M. (2012). The optimal method to make inferences about a linear combination of proportions. *Journal of Statistical Computation and Simulation 82 (1)*, 123-135. DOI: 10.1080/00949655.2010.530601.

(3) Martín Andrés, A. and Álvarez Hernández, M. (2013). Optimal method for realizing two-sided inferences about a linear combination of two proportions. *Communications in Statistics - Simulation and Computation 42*, 327-343. DOI: 10.1080/03610918.2011. 650263.

(4) Martín Andrés, A. and Álvarez Hernández, M. (2014). Two-tailed asymptotic inferences for a proportion. *Journal of Applied Statistics 41 (7)*, 1516–1529. DOI: 10.1080/02664763.2014. 881783.

(5) Martín Andrés, A. and Álvarez Hernández, M. (2014). Two-tailed approximate confidence intervals for the ratio of proportions. *Statistics and Computing 24,* 65-75 (published online: 28 September 2012). DOI: 10.1007/s11222-012-9353-5. (2015) "Erratum to: Two-tailed approximate confidence intervals for the ratio of proportions", *Statistics and Computing* 26(3), 743-744, 2016. DOI: 10.1007/s11222-015-9619-9.

(6) Martín Andrés, A., Álvarez Hernández, M. and Herranz Tejedor, I. (2012). Asymptotic two-tailed confidence intervals for the difference of proportions. *Journal of Applied Statistics 39 (7)*, 1423-1435. DOI: 10.1080/02664763.2011.650686.

(7) Álvarez Hernández, M. and Martín Andrés, A. (2017). New asymptotic inferences about the difference, ratio and linear combination of two independent proportions. *Communications in Statistics - Simulation and Computation 46 (2)*, 1557-1568.

(8) Martín Andrés, A. and Álvarez Hernández, M. (2015). Simultaneous inferences: new method of maximum combination. *Statistical Papers 56*, 1099–1113. DOI: 10.1007/s00362-014-0627-1.

(9) Martín Andrés, A. and Álvarez Hernández, M. (2016). Comment on “An improved score interval with a modified midpoint for a binomial proportion”. *J. Stat. Comput. Simul. 86 (2)*, 388-393. DOI: 10.1080/00949655.2015.1015128.

(10) Álvarez Hernández, M., Martín Andrés, A. and Herranz Tejedor, I. (2016). One-sided asymptotic inferences for a proportion. *Journal of Applied Statistics 43(9)*, 1738-1752. DOI: 10.1080/02664763.2015.1117595.

(11) Álvarez Hernández, M., Martín Andrés, A. and Herranz Tejedor, I. (2018). One-tailed asymptotic inferences for the difference of proportions: analysis of 97 methods of inference. *Journal of Biopharmaceutical Statistics 28 (6)*, 1090–1104, DOI: 10.1080/10543406.2018.1452028.

(12) Martín Andrés, A., Álvarez Hernández, M. and Herranz Tejedor, I. (2022). One-sided asymptotic inferences for the relative risk: comparison of 63 inference methods. *Communications in Statistics – Theory and Methods*. 51 (5), 1330-1348. DOI: 10.1080/03610926.2020.1760299.