PRE-SERVICE PRIMARY SCHOOL TEACHERS' PERCEPTION OF RANDOMNESS

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In this paper we present results of assessing perception of randomness in a sample of 200 prospective primary school teachers in Spain. We first compare three pairs of random variables deduced from a classical task in perception of randomness. Then, the written reports, where prospective teachers analyse these variables and explicitly conclude about their own intuitions are also analysed. Results show a good perception of the expected value and poor conception of both independence and variation.

INTRODUCTION

Probability is increasingly taking part in the school mathematics curriculum; yet most teachers have little experience with probability and share with their students a variety of probabilistic misconceptions (Stohl, 2005). Therefore it is important to assess teachers' probabilistic knowledge and find activities where teachers work with meaningful problems related to their professional development and are confronted to their own misconceptions in the topic (Batanero, Godino & Roa, 2004). This research was aimed at assessing pre-service primary school teachers' perception of randomness using two different tools: a) we first analyse some statistical variables deduced from a classical experiment related to perception of randomness that was carried out by the teachers; b) we secondly analyse the written reports produced by the teachers, which were part of an activity, directed to confront them with their own misconceptions of randomness.

Perception of randomness

Perception of randomness in adults has been extensively investigated (e.g., Wagenaar, 1972; Bar-Hillel and Wagenaar 1991) using a variety of stimulus tasks, which were classified by Falk and Konold (1997) into two main types. In generation tasks, subjects generate random sequences under standard instructions to simulate a series of outcomes from a typical random process (e.g. tossing a coin or locating points at random on a squared grill). In recognition tasks, people are asked to select the most random of several series of results that might have been produced by a random device. Systematic misconceptions and biases have consistently been found in this research, such as for example the gambler's fallacy, by which people believe that, after a long run of a same result in a random process, the probability of that event occurring in the following trial is lower. Related to this is the tendency of people to include too many alternations of different results (such as heads and tails in

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the flipping of a coin) in sequence generation tasks, in comparison to what would theoretically be expected in a random process. Various explanations have been provided for these biases such as local representativeness (Tversky & Kahneman. 1982), misperception of independence or convergence (Falk and Konold, 1997), difficulties in differentiating random and deterministic phenomena, or beliefs in the possibility of controlling random experiments (Fischbein & Gazit, 1984).

Green's (1989, 1991) research showed that the percentage of children recognizing random or semi-random distributions actually decreases with age. His study also demonstrated that children did not appear to understand the independence of the trials, and tended to produce series in which runs of the same result were too short compared to those that we would expect in a random process. Using Green's tasks, Batanero and Serrano's (1999) research with secondary school students showed a mixture of correct and incorrect properties associated by students to randomness. On one hand students perceived the local variability, lack of patterns in the lineal or spatial arrangement of outcomes, and unpredictability of the random processes. However some students did not perceived independence of trials or believed in the possibility of controlling a random process. Below we describe a formative activity, based on a statistical project that uses a generation task inspired by one of the tasks proposed by Green (1991) and is directed to assess the teachers' conceptions of randomness and help them overcome some of their misconceptions in the topic.

METHOD

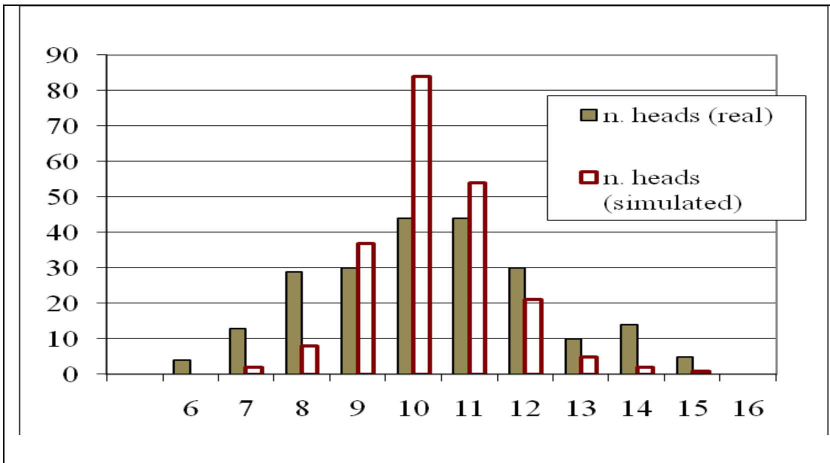
Participants in the sample were 200 prospective teachers in the Faculty of Education, University of Granada, Spain, from two different academic years; in total 6 different groups (35-40 pre-service teachers by group). All of them were following the same mathematics education course and had followed a mathematics course, which included descriptive statistics, the previous year. The data were collected as a part of a formative activity which is discussed in depth in Godino, Batanero, Roa and Wilhelmi (2008) as consisted of three sessions (90 minutes long each). The two main goals were: a) assessing pre-service teachers' conceptions of randomness; b) confronting pre-service teachers with their possible misconceptions in the topic.

In the first session the pre-service teachers were given the statistical project "Check your intuitions about chance". Prospective teachers were encouraged to carry out an experiment to decide whether the group had good intuitions on randomness or not. The experiment consisted of trying to write down apparent random results of flipping a coin 20 times (without really throwing the coin, just inventing the results) in such a way that other people would think the coin was flipped at random (simulated sequence). Participants recorded the simulated sequence on a recording sheet. Afterwards participants were asked to flip a fair coin 20 times and write the results on the same recording sheet (real sequence). At the end of the session, in order to confront these future teachers with their misconceptions, participants were given the data collected in their classroom. These data contained six statistical variables:

number of heads, number of runs and length of the longest run for each of real and simulated sequences from each student. Results are presented in Figures 1 to 3. Sample size for data analysed by the teachers in each group were smaller (30-40 experiments per group), although the shape of the distribution and summaries for each variable were very close to those presented in Figures 1 to 3. Teachers were asked to compare the variables collected from the real and simulated sequences, complete the analysis at home and write a report with a complete discussion of the project, including all the statistical graphs and procedures they used and their conclusions regarding people's intuitions about randomness. They were given freedom to select graphs or summaries in order to complete their reports. In a second session the reports were collected and the different solutions to the project given by the prospective teachers were collectively discussed in the classroom. In a third session a didactical analysis, was carried out in order to analyse the statistical knowledge needed to solve the project and the pedagogical content knowledge involved in teaching statistics in primary school through projects work.

RESULTS AND DISCUSSION

In order to assess pre-service teachers' conceptions of randomness we first analysed the number of heads, number of runs and longest run in each of the simulated and real sequences in the data collected by the teachers in their experiments. Here the data collected by the six groups of teachers (n=200) were analysed together, although similar results were found in each of the six subsamples.



	Sequences	
	Real	Simulated
Mean	10.45	10.29
Mode	10; 11	10
Median	10	10
Std. Deviation	2.05	1.22
Range	11	8

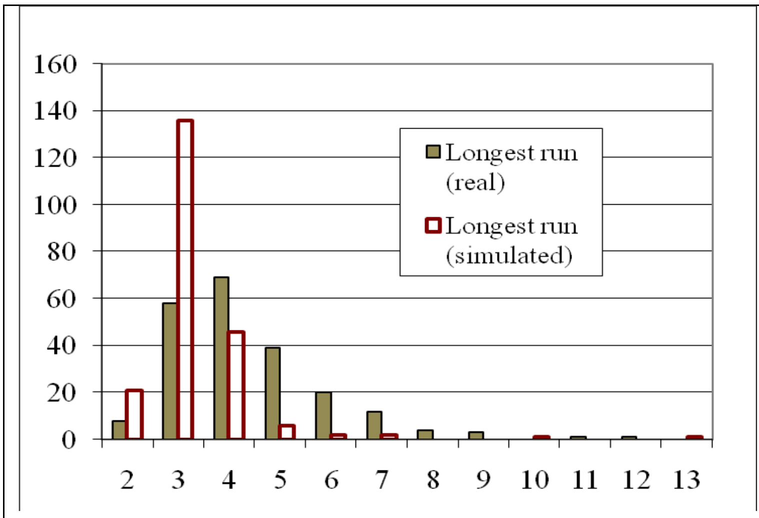
Figure 1: Distribution and summary statistics for Number of heads.

Perception of the Binomial distribution

In particular the theoretical number of heads in 20 trials can be modelled by the Binomial distribution *B (n, p)*, where n=20 and p= 0.5. The expectation and variance for this distribution is $\mu=np=10$; $Var=npq=5$. In Figure 1 the distribution and summary statistics for the real and simulated sequences are presented. Results show a good perception of the expected number, median and mode of binomial distribution (number of heads in 20 flipping of a coin) as it is shown in the mode, median and average number of heads in the simulated sequences, which are close to the theoretical value *np*=10 and in the non significant difference in the t- test of difference on averages between real and simulated sequences (*t* = -1.00; *p*=0.31). The perception of variability in the Binomial distribution was, however poor, as the standard deviation in the simulated sequences was almost half the theoretical value and the differences were statistically significant in the *F*- test (*F*=2.83, *p*=0.001).

Perception of independence

Perception of independence was poor, as prospective teachers produced in average shorter runs and higher number of runs than expected in a random process. This is visible in Figures 2 and 3. Results were significant in the t- tests of differences in averages (*t* = -7.76; *p* = 0.001 for the longest run; *t*=2.48; *p*= 0.01 for the number of runs). Some teachers recognized this difference in their reports: "In the simulated sequence we tend to produce many short runs" (AB). *Misconceptions of independence,* was also observed in some written reports by students who rejected the sequence as random because some runs were longer than expected: "Some students cheated and invented their sequences, since they have too many successive heads or tails to be random" (EA).

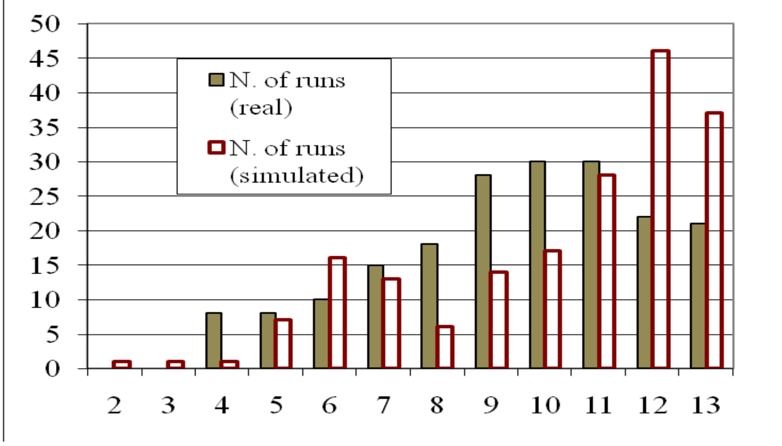


	Sequences	
	Real	Simulated
Mean	4.35	3.32
Mode	4	3
Median	4	3
Std. Deviation	1.6	1.12
Range	10	11

Figure 2: Distribution and summary statistics for Longest run.

Perception of variation

Variability is omnipresent throughout the statistical enquiry cycle and is fundamental to statistical thinking (Wild & Pfannkuch, 1999). *Perception of variation* was also poor in the longest run (see Figure 2) were the *F* test was statistically significant (*F*=2.06; *p*=0.0001). However, perception of variation was good as regards the number of runs (*F*=1.07; p=0.6; no significant). Some pre-service teachers made reference to variation in results as an important feature of random process or either justified randomness based on this variation (*randomness as variation*). "There is more variety in the random sequence. This is pretty logical, since these results were obtained by a random experiment that involved chance" (NC); " In the number of heads there is a difference, … since when we invent the data (in the simulated sequence) results are more even, but real sequence are more uneven, since they are due to chance" (IE). All the above results reproduced those obtained by Green (1991) and Batanero and Serrano (1999) with secondary school students, which is reasonable, because the statistics training that Spanish prospective primary teachers receive is reduced to their study of statistics along secondary education.



	Sequences	
	Real	Simulated
Mean	10.10	10.78
Mode	10; 11	12
Median	10	12
Std. Deviation	2.9	2.8
Range	14	13

Figure 3: Distribution and summary statistics for Number of runs

Teachers' conclusions

The majority of participants represented the data with graphs that varied in complexity and that have been analysed in a previous paper (Batanero, Arteaga, & Ruiz, 2009). Many participants also computed averages (mean, median or modes) and variation parameters (range or standard deviation). However, few of them got a correct conclusion about the group intuitions, which included a correct judgment of the collective perception of both average values and variation (Table 1). Only a small number of pre-service teachers explicitly were able to get a correct conclusion for the class' perception of expected values and variation in the different variables. Although the binomial distribution (number of heads) was more intuitive for the pre-service teachers, still the number of correct conclusions as regards the perception of the binomial distribution was very scarce. These results suggest that these pre-service teachers did not only hold some misconceptions of randomness, but they also were unconscious of their misconceptions and were unable to perceive them when confronted with the statistical data collected in the experiments.

Moreover, between 42.0% and 62.0% of these teachers justified their wrong conclusions as regards some of the variables in the project by making explicit their own misconceptions of randomness that reproduced some of the misconceptions described by Batanero and Serrano (1999) in secondary school students. Some participants perceived *randomness as unpredictability;* in their responses they assumed they could not reach a conclusion about the differences in distribution for the number of runs, number of heads or longest run because anything might happen in a random process: “I want note that it is impossible to make a prediction of results since in this type of experiment any result is unpredictable” (AA) “Results of random experiments cannot be predicted until they happens” (SG). In these responses the “outcome approach” (Konold, 1991) that is the interpretation of probability questions in a non probabilistic way may operate.

Conclusion about the intuitions (n=200)	Correct	Incorrect	No conclusion
Perception of the expected value in the binomial distribution (number of heads)	32 (16.0)	114 (57.0)	54 (27.0)
Perception of independence (expected longest run)	6 (3.0)	87 (43.5)	107(53.5)
Perception of independence (expected number of runs)	9 (4.5)	100 (50.0)	91 (45.5)
Perception of variation (number of heads)	25 (12.5)	121 (60.5)	54 (27.0)
Perception of variation (longest runs)	8 (4.0)	85 (42.5)	107 (53.5)
Perception of variation (number of runs)	11 (5.5)	98 (49.0)	91 (45.5)

Table 1: Frequency (percent) of prospective teachers’ conclusions.

A few subjects perceived *randomness as equiprobability* (in the classical approach to this concept) and stated that "Any result is possible, since this is a random experiment; there is equal probability for each result". “The probability for head and tails is the same, so in 20 throwing there is the same probability to obtain 20 heads, 20 tails or any possible combination of heads and tails” (EB). These teachers only saw an event as random if there was the same probability for this event and for any other possible event in the experiment.

Some participants associated randomness to *lack of model or pattern* a view close to that by Von Mises (1952/1928) who indicated that a sequence was random whenever it was not possible to get an algorithm that produced the sequence “You cannot find a patter, as it is random” (BS). However, in fact in the analysis of the project data a variety of models appear: Binomial distribution, runs, geometrical distribution, etc. and therefore, randomness can also been interpreted as multiplicity of models. Other teachers described randomness as something that cannot be controlled (*randomness as lack of control*), a vision common until the Middle Ages according Batanero and Serrano (1999): “Despite our inability to control randomness, we got equal number of heads and tails” (AG). It was also observed the *illusion of control*, by which some participants believed they could predict the result of random experiments. For example, one participant classified the students in the group according their capacity for predicting the results: “Only 21.7% students guessed the number of heads in the experiment; 13% were very close because they had an error of (± 1); the remaining students failed in their prediction” (LG). Finally, other views included randomness as *lack of order*: “it is not random, it is too ordered” (SG).

DISCUSSION AND IMPLICATIONS FOR TRAINING TEACHERS

The above analysis suggests these teachers present different misconceptions of randomness they could transmit to their future students. It also show the usefulness of working with activities similar to the one described in this report to help these teachers make these conceptions explicit. In order to overcome these misconceptions, after working with the project, it is important to continue the formative cycle. In our experience, in the second session the correct and incorrect solutions to the project were debated and the different conceptions of randomness explicit in the teachers’ responses were discussed.

Our results also indicate that these prospective teachers failed to complete the last part of the modelling process. According to Chaput, Girard & Henry (2008), the first step of a modelling process consists of describing the concrete situation (in this case, checking the intuitions on randomness) in usual language and the creation of hypotheses which are intended to interpret the situation (for example, accepting the equiprobability of heads and tails in the coin). Next, the second step of the modelling process is translating the *working hypotheses* into *model hypotheses* and working with the model. The teachers translated the problem to statistical terms (comparing three pairs of distributions); they built and worked with some statistical models that represented the variables deduced from this problem (summaries and graphs). The third, and final, step consists first of interpreting the mathematical results, then giving them a meaning to create answers to the original problem. In our research few teachers were unable to translate the results from working with statistical models to the real situation (they could not understand what the statistical results indicate about the intuitions in the group). It is important that teachers’ educators develop teachers’ ability to explore and learn from data if we want succeed in implementing statistics education at school level.

Acknowledgements

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