MODELS FOR STATISTICAL PEDAGOGICAL KNOWLEDGE[1]

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Abstract: *The education of statistics teachers should be based on adequate models for pedagogical knowledge that guide the teachers' educators in implementing and assessing the training of teachers. In this chapter some models that are relevant for mathematics and statistics are analysed, and a new framework that complements the previously described models is proposed. The different facets and levels that should be taken into account when educating mathematics and statistics teachers are highlighted. Some implications for the training of teachers are presented and a formative cycle directed to increase the teachers' statistical and pedagogical knowledge simultaneously is briefly described.*

1. INTRODUCTION

One main conclusion in the Joint ICMI/IASE Study Conference was the need to elaborate models for statistical pedagogical knowledge that provide a foundation in training teachers to teach statistics. Research related to teacher education, development and thinking (Philipp, 2007; Sowder, 2007; Wood, 2008) includes diverse theoretical frameworks describing the knowledge that teachers need in order to enhance the students' learning and that are required in organizing the teachers' training designs and in assessing their efficacy. Although there is a general consensus that mathematics teachers should master the disciplinary content, there is no similar agreement about how such mastery should be achieved and how the discipline should be conceived. It is however recognized that mathematical or statistical knowledge alone does not assure professional competence and that other capabilities are required, including knowledge about how students learn, their conceptions, types of thinking, strategies, difficulties,

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and potential errors. Teachers should also be able to organize the teaching, design learning tasks, use adequate resources, and understand the factors that condition the teaching and learning processes (Ponte, 2008).

In this chapter the notion of pedagogical content knowledge proposed by Shulman (1987), which has been extensively applied in the teaching of mathematics, and other models created for mathematics education, are briefly described. Then, attention is focussed on the models for statistical pedagogical knowledge presented at the Joint ICMI Study Conference. In section 4 a model for teachers' pedagogical knowledge, which is based on a previous theoretical framework developed for mathematics cognition and instruction (Godino, Batanero, & Font, 2007), is presented. This model extends the components identified in the models described in sections 2 and 3 and can be adapted to the specific character of statistical knowledge (from both the epistemological and didactic points of view).

An implication of the analysis is the need to develop and assess teachers' competencies to carry out didactical analysis of their own practice, which takes into account the different components of pedagogical knowledge. A possible formative cycle that serves these purposes is briefly described in the final section.

2. MODELS FOR MATHEMATICAL PEDAGOGICAL KNOWLEDGE

A reason for the lack of impact of research into practice is that teachers, who are the main agents of change, are only viewed as simple components of the educational system, who automatically apply the information they receive. The complexity of teaching and the high level of initiative and autonomy required by the teacher are highlighted in the research on "teacher thinking" (beliefs, conceptions and attitudes) and on teacher professional knowledge and competencies. An increasing number of authors interested in this theme point to the insufficiency of mathematical knowledge alone to achieve truly effective teaching outcomes (Shulman, 1987; Hill & Ball, 2004). Consequently, this research is producing models of teacher knowledge, in order to design educational plans and elaborate tools for assessing the efficacy of such actions. In this section we present a synthesis of some models that were specifically developed for mathematics education.

Shulman (1987) identified seven categories of knowledge that underpin expert teaching: (a) content knowledge, or knowledge about the discipline; (b) general pedagogical knowledge; (c) curriculum knowledge; (d) pedagogical content knowledge, or pedagogical knowledge specific for the discipline (PCK); (e) knowledge of learners and their characteristics; (f) knowledge of education contexts; and (g) knowledge of educational ends, purposes and values. Ponte and Chapman (2006) emphasized PCK as an important component in the education of teachers. The categories of knowledge described by Shulman have played an important role in developing research programs and curricular materials and are still valid, although the initial interpretations and the names given to them have changed over time. For example, Ball and her colleagues (Ball, Lubienski, & Mewborn, 2001; Hill, Ball, & Schilling, 2008) developed the notion of mathematical knowledge for teaching (MKT) in which they distinguished six main categories.

1. Common content knowledge (CCK): the mathematical knowledge teachers are responsible for developing in students.
2. Specialized content knowledge (SCK): the mathematical knowledge that is used in teaching, but not directly taught to students; for example, knowledge about why the algorithms for the arithmetic operations work.
3. Knowledge at the mathematical horizon: understanding the broader set of mathematical ideas to which a particular idea connects; for example, understanding some epistemological obstacles related to the historical development of probability.
4. Knowledge of content and students (KCS): the amalgamated knowledge that teachers possess about how students learn content.
5. Knowledge of content and teaching (KCT): the design of instruction, including how to choose examples and representations, and how to guide student discussions toward accurate mathematical ideas.
6. Knowledge of curriculum: ways to sequence and structure the development of a mathematical topic.

In fact components 4 to 6 are a decomposition of Shulman's PCK and comprise the competencies that are deeply embedded in the work of teaching knowing. For example, knowledge of what makes a topic difficult for students, ways in which learners tend to develop understanding of a particular idea, ways to sequence and structure the development of a mathematical topic, including representations likely to help students learn (Hill et al., 2008). As stated by Graeber and Tirosh (2008, p. 124), “the fact that many researchers do not offer a definition of PCK but rather attempt to characterise it with lists or examples is another indication that the concept is still somewhat ill defined”.

In addition to MKT, several researchers are proposing other tools to conceptualise the knowledge needed in teaching mathematics. Schoenfeld and Kilpatrick (2008, p. 322) offer a provisional framework for *proficiency in teaching mathematics* consisting of the following set of dimensions: (a) Knowing school mathematics in depth and breadth; (b) knowing students as thinkers; (c) knowing students as learners; (d) crafting and managing learning environments; (e) developing classroom norms and supporting classroom discourse as part of “teaching for understanding”; (f) building relationships that support learning; and (g) reflecting on their own practice.

A number of questions still need to be explored in research in teacher education, including the role of beliefs and values in the development of PCK, whether different teaching/learning paradigms require different components of PCK, what are adequate methods for assessing PCK; and what are more global theoretical models for describing the teachers’ knowledge, beliefs and affects, such as, teachers’ orientation, perspective and identity (Philipp, 2007).

3. MODELS FOR STATISTICAL PEDAGOGICAL KNOWLEDGE

Two key elements in the didactical analysis of teaching and learning processes are the epistemic (mathematical content) and cognitive (students’ learning) components. In anthropological and semiotic perspectives, mathematics is considered as a human activity arising from people’s practices when working with specific problem-solving situations. This point of view also takes into account the specificity of statistics (see Ottaviani & Gattuso, in this book), since the epistemic facet is specific for each particular content, and therefore for the case of statistics. Moreover, there are specific statistics problems, representations, and procedures that are different from those found in geometry, physics, or algebra. Basic statistical problems are related to inference and decision making under uncertainty (involving random variation) and involve specific statistical practices: randomization, collecting sample data, tabulation and transnumeration, data reduction, and using statistical models (Wild & Pfannkuch, 1999). These practices lead to the emergence of specific representations (e.g., specific graphs and terms), concepts (e.g., distribution, significance, correlation), procedures (e.g., analysis of variance), properties (e.g., bias, efficiency, independence) and arguments (e.g., the central limit theorem is given with a probabilistic statement, simulation is sometimes used to justify a result). Hence there are specific statistical practices and specific statistical objects and processes related to statistics problems. Consequently, since there is a specific epistemology of statistics, we should also recognize a specific didactics of statistics, given that the epistemic facet interacts with all the other facets of teachers' knowledge (cognitive, instructional, and curricular knowledge). This justifies the effort made by several statistics educators, in particular Burgess (2008), Garfield and Ben-Zvi (2008), and Watson, Calligham, and Donne (2008) to adapt and develop PCK or MKT models for statistical education.

Burgess (2008) defined teacher knowledge to teach statistics for the case when this teaching is based on statistical investigations. As research on teacher knowledge to teach statistics is scant and recent he based his approach on studies carried out in mathematics education. Burgess built a model for statistical pedagogical knowledge starting from Ball et al. (2001) and extending and adapting to statistics education, by including categories from the Wild and Pfannkuch's framework (1999) for statistical thinking. Hence, based on these two theoretical models, Burgess proposed a two-dimensional grid to analyse the statistical knowledge for teaching. In one dimension (mathematical knowledge for teaching) he considered four categories: common knowledge of content; specialised knowledge of content; knowledge of content and students; and knowledge of content and teaching. In the other dimension (statistical thinking in empirical inquiry) he included the following categories: four types of fundamental statistical thinking (need for data, transnumeration, variation, reasoning with models, integration of statistical and contextual); two components in the statistics research process (investigative cycle, interrogative cycle); and dispositions towards statistics.

The grid was used by the author to describe the knowledge put in practice by two statistics teachers, and the knowledge those teachers failed to apply even when they had the opportunity, in the context of teaching experiences based on statistical investigations. Results allowed Burgess to build a profile for each teacher's knowledge. His study served to describe the components of teacher knowledge that emerged during the teaching of statistics investigations and how lack of appropriate knowledge created missed opportunities in relation to the teaching and learning of statistics.

Garfield and Ben-Zvi (2008) described their experiences in training teachers in statistics, which were based on the application of six instructional design principles from Cobb and McClain (2004):

1. Focus on developing central statistical ideas rather than on presenting a set of tools and procedures.
2. Use real and motivating data sets to engage students in making and testing conjectures.
3. Use classroom activities to support the development of students' reasoning.
4. Integrate the use of appropriate technological tools that allow students to test their conjectures, explore and analyze data, and develop their statistical reasoning.
5. Promote classroom discourse that includes statistical arguments and sustained exchanges that focus on significant statistical ideas.
6. Use assessment to learn what students know and to monitor the development of their statistical learning as well as to evaluate instructional plans and progress.

Garfield and Ben-Zvi (2008) used these principles to design and teach courses. They teach also these principles to the students explicitly as they prepared to become teachers of statistics. Consequently, these prospective teachers had the opportunity to experience the learning of statistics following an instructional model that allowed them to know and understand the didactical knowledge incorporated in the principles mentioned.

The Cobb and McClain's (2004) principles of instructional design, adopted by Garfield and Ben-Zvi for teacher training courses, can be interpreted as an implicit model for teacher didactical knowledge. The first principle (focus on developing central statistical ideas) involves the epistemic component. Garfield and Ben-Zvi selected the following key statistical ideas: data, distribution, variation, central tendency, randomness, co-variation and sampling. The second principle is related to both the epistemic component (real data sets refer to statistical problems and related conjectures) and the affective component (students' motivation and commitment). The third principle calls on the instructional facet (classroom activities, exploration, discussion and argumentation, cooperative work) and the cognitive facet (development of students' reasoning). The fourth principle refers to tools and media. The fifth principle involves an interactional component: promoting classroom discourse that focuses on significant statistical ideas. Finally, the sixth principle highlights the role of assessment in teaching and learning.

Assessment and measurement are important tools in developing teachers' PCK, as highlighted by Watson et al. (2008). In their work presented at the Joint Study Conference, Watson et al. described and applied a questionnaire that was developed to assess the different components of Shulman's PCK (see also Calligham & Watson, this book). Their questionnaire, based on Watson (2001), also included some items measuring the teachers' beliefs about statistics and its teaching, and their confidence to teach particular statistical topics. Watson et al. viewed PCK as a general notion including the different categories initially proposed by Shulman, i.e., disciplinary content knowledge and pedagogical content knowledge related to students, curriculum, teaching: "This approach appears to treat PCK as the underlying and encompassing phrase to summarize Shulman's original intentions" (Watson et al., 2008, p. 1). Some items included by these authors in their questionnaire to assess teachers' PCK were based on the answers given by students to questions used in previous survey research carried out by Watson. "The major focus of PCK in items in this study is teachers' content knowledge, its reflection in knowledge of their students' content knowledge, and their PCK in using student responses to devise teaching intervention" (p. 1).

Although the models for PCK or MKT described in the previous paragraphs are useful for training teachers to teach statistics, their categories are still general and could be made more precise. It would be useful to develop models that provide detailed and further operative criteria that can be applied in designing procedures or materials directed to educating teachers. In the following section we describe a theoretical model that attempts to complement and expand those described in the previous sections. This model is applicable to both mathematics and statistics (consequently for preparing mathematics and statistics teachers).

4. EXPANDING THE ANALYSIS OF MATHEMATICAL AND STATISTICAL PEDAGOGICAL KNOWLEDGE

In this section we describe a specific model, which is based on a theoretical integrative framework developed for research in mathematics education. The onto-semiotic approach (synthesized in Godino, Batanero, & Font, 2007) combines three dimensions in mathematical knowledge and teaching: (a) the epistemological component, which is conceived from an anthropological and socio-cultural perspective; (b) the cognitive component, which is given a semiotic foundation; and (c) the instructional component, which is based on social constructivism. Mathematics is conceived as a human activity linked to solving certain types of problem-situation, whereas mathematical objects are viewed as emerging from the systems of practices carried out to solve these problems. The above assumptions are also applicable to statistics, and hence the categories of teachers' knowledge derived from the onto-semiotic approach also serve to characterize the statistical pedagogical knowledge. The different types of mathematics and statistics objects considered in this perspective are first clarified, then the different facets and levels considered in the mathematical or statistical pedagogical knowledge are described and finally the idea of didactic suitability and its components are expanded.

4.1. Types of mathematical and statistical objects

Different types of knowledge are put in practice when carrying out mathematical or statistical practices and when interpreting their results. For example, when comparing two distributions (statistical problem) some symbolic or graphical representations, concepts, propositions and procedures are used to elaborate the argument needed to make a decision as regards those distributions (such as justifying whether the differences in averages or spread for these distributions are statistically significant). In the example, the following types of mathematical objects, introduced in the onto-semiotic approach to describe the mathematical practices, are identified:

1. *Language:* terms, expressions, symbols, graphs used to represent the distributions, their parameters or the operations carried out with them.
2. *Situations:* extra or intra-mathematical problems or applications, for example, comparing the two distributions or carrying out a statistical test for the differences in

averages or spread.

3. *Concepts:* given by their definitions or descriptions (variable, distribution, parameter, average, standard deviation).
4. *Propositions:* properties or attributes of concepts (e.g., the sum of frequencies is equal to the number of cases; two distributions with very different means are different).
5. *Procedures:* operations, algorithms, techniques (computing the mean and standard deviations; computing the significance of differences).
6. *Arguments:* used to validate and explain the propositions and procedures (deductive or inductive reasoning).

By considering these six types of mathematical or statistical objects, the traditional distinction between conceptual and procedural knowledge, which is insufficient to describe all the objects that intervene and emerge in mathematical or statistical activity, is expanded. Problem-situations are the origin and reason of mathematical or statistical activity; language is needed to represent the other types of objects and is an instrument for action; arguments justify the procedures and propositions that relate different concepts. These and other theoretical tools, as well as a classification of mathematical processes, are described in detail in Godino et al. (2007).

4.2. Facets and levels of mathematical and statistical knowledge for teaching

A statistics teacher needs a deep knowledge of statistics, which includes competence in understanding and applying the different types of objects described in section 4.1 for the particular statistical content he or she is teaching. Moreover, the teacher needs a deep mathematical or statistical knowledge for teaching. Teaching and learning processes involve a group of students, the teacher, and some didactic resources, all of them interacting within an institutional context. Consequently the mathematical or statistical knowledge for teaching should also include the different facets or components that are necessary to study teaching and learning processes and that are synthesized in Figure 1. Didactic research is producing a substantial amount of knowledge for each of these facets that teachers should acquire and apply to achieve efficient teaching.

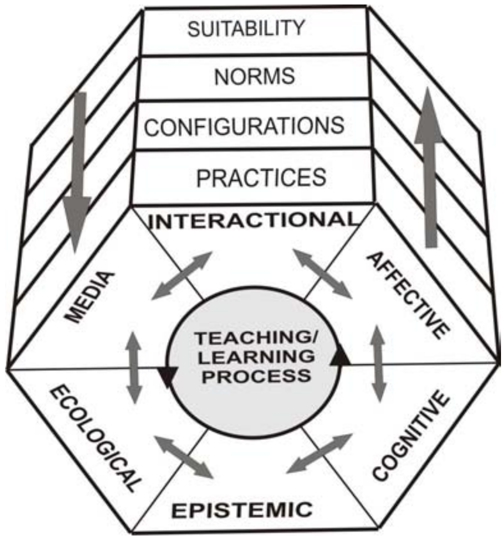

Figure 1. Facets and levels of teachers' knowledge

A short description of the facets of the model is given below (see Godino, Batanero, & Font, 2007) for a more complete description):

1. *Epistemic facet*: The intended and implemented institutional meaning for a given mathematical or statistical content, that is, the set of problems, procedures, concepts, properties, language, and arguments included in the teaching and its distribution over the teaching time.
2. *Cognitive facet:* Students' levels of development and understanding of the topic, and students' strategies, difficulties, and errors as regards the intended content (personal meaning).
3. *Affective facet:* Students' attitudes, emotions, and motivations regarding the content and the study process.
4. *Media facet:* Didactic and technological resources available for teaching and the possible ways to use and distribute these resources over time.
5. *Interactional facet:* Possible organisations of the classroom discourse and the interactions between the teacher and the students that help solve the students' difficulties and conflicts.
6. *Ecological facet:* Relationships of the topic with the official curriculum, other mathematical or statistical themes and with the social, political and economical settings that support and condition the teaching and learning.

Teaching and learning processes can also be analysed from four different levels or points of view that provide additional categories for teachers' knowledge.

a. *Mathematical-statistical or didactic practices:* Mathematical or statistical actions that students carry out to solve the problems posed, as well as the actions carried out by the teacher in order to promote learning and contextualise the content.
b. *Configurations of mathematical or statistical objects and processes:* Mathematics objects (e.g., problems, procedures, concepts, properties, language or arguments) and processes (e.g., generalization, representation) that intervene and emerge in the aforementioned practices.
c. *Norms:* Rules, habits and conventions that condition and make possible the study process and affect each facet and their interactions.
d. *Didactic suitability*: Objective criteria that serve to improve the teaching and learning and guide the evaluation of the teaching/learning process.

Teachers' progressive knowledge in each of these facets and levels for a specific content develops their understanding of the teaching complexity and their competence in finding possible causes for learning conflicts. Although in Figure 1 the components and levels of teachers' knowledge are separated, in order to highlight their difference, in fact all of them interact. As an example, below, the interactions of didactic suitability with the facets 1 to 6 in the teachers' knowledge are analysed.

4.3. Didactic suitability

Didactic suitability for a particular teaching and learning process should be evaluated for each of the six facets described in section 4.2 because the teaching process may be suitable from the statistical point of view and not suitable, for example, from the affective point of view. Consequently six different types of suitability can be considered (Godino, Wilhelmi, & Bencomo, 2005):

1. *Epistemic suitability* measures the extent to which the implemented meaning (statistical content implemented in a classroom or course) represents adequately the intended meaning (the curricular guidelines for this course or classroom).
2. *Cognitive suitability* is the degree to which the implemented meaning is appropriate to the students' cognitive development. That is, the degree to which the

implemented meaning is included in the students' *zone of proximal development*, and whether the students' learning (personal meaning achieved) is close to the intended meaning.

3. *Emotional suitability* describes the students' involvement (interest, motivation, attitudes) in the study process.
4. *Media suitability* reflects the availability and adequacy of material and temporal resources in the teaching process.
5. *Interactive suitability* is the extent to which the organisation of the teaching and the classroom discourse serve to identify and solve possible conflicts and difficulties that appear during the instructional process.
6. *Ecological suitability* is the extent to which the teaching process is in agreement with the school and society educational goals, and takes into account other possible social and cultural factors.

The different categories for teacher knowledge in the models described in sections 2 and 3 include to a greater or lesser extent the facets assumed in the onto-semiotic model. The levels of analysis crossing each facet in this last model involve a deepening in the analysis of the knowledge needed to design teacher education and to assess teacher knowledge. Moreover the idea of suitability and the different suitability criteria provide a guide to design, implement and assess teacher professional development plans, and to support the teachers' reflection on their own practice.

5. IMPLICATIONS FOR TEACHERS' EDUCATION

Statistics teachers should develop competence to recognize the statistical objects and processes that intervene in the students' statistical practices, be aware of the norms that support and condition learning, affect, resources and interactions in the classroom. Consequently, the education and assessment of teachers' professional knowledge should take into account the different facets and levels described in section 4. The multi-dimensional and systemic nature of this knowledge also requires multiple strategies for developing and assessing this knowledge, such as those described in other chapters in this book.

A main challenge for teacher educators is finding suitable ways to articulate the teachers' learning of statistics and transmitting an epistemological vision of statistics in agreement with social constructivism, as well as developing teachers' statistics pedagogical knowledge. A possible tool is the a formative cycle designed by Godino Batanero, Roa, and Wilhelmi (2008), which was tried in an experience with prospective primary school teachers.

The formative cycle started with a statistical project that was completed by the prospective teachers in teams, following a socio-constructivist instructional design. Collecting data to complete the project led the future teachers to compare frequency distributions, and thus justify the introduction of statistical tables, graphs and summaries. Another feature of this project was the multivariate approach to data analysis, which is also specific to statistics, as decision making in random situations often requires taking into account, not just one variable, but a multiple approach. The project also provided the prospective teachers with a teaching model where the traditional *knowledge division* in textbooks (concepts versus procedures) was overcome and where statistical concepts and techniques were justified by a real problem, so that these concepts acquired a *situational meaning* for the teachers.

In a second stage, the project served to provoke didactical reflection on pedagogical content knowledge. After discussing the solution to the problem posed and the statistical conclusions for the research project, the prospective teachers were asked to analyse the different facets and suitability criteria described in section 4 in the teaching/learning process they had lived in their own classroom. Many prospective teachers in the Godino et al. (2008) experience had difficulties in analysing the different components for pedagogical knowledge and in assessing the didactical suitability of the teaching process. This outcome was reasonable, given the scarce time devoted to preparing the teachers who took part in the experience and the complexity of pedagogical knowledge. However, the activity proved to be useful to introduce systematic reflection on the different facets affecting the teaching and learning of statistics. Moreover, responses by even the most advanced future teachers showed some underlying conceptions about teaching and learning mathematics that should be made explicit and confronted. It also provided a multivariate approach to didactical analysis by including the different dimensions that interact with the teaching and learning processes of statistics that were described in the previous sections.

To conclude we suggest the need to improve the models for the didactic knowledge required to teach statistics that take into account the specificity of statistics. Improving the statistics education of school teachers will also require significant changes in the initial teachers' preparation syllabus and assigning more time to teachers' statistics education.

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REFERENCES

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

Batanero, C., Burrill, G., Reading, C., & Rossman, A. (2008) (Eds.). *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference.* Monterrey, Mexico: International Commission on Mathematical Instriction and International Association for Statistical Education. Online: www.stat.auckland.ac.nz/~iase/publicatons.

Burgess, T. A. (2008). Teacher knowledge for teaching statistics through investigations. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008).

Cobb, P., & McClain, K. (2004). Principles of instructional design for supporting the development of students' statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 375–396). Dordrecht, Netherlands: Kluwer.

Garfield, J., & Ben-Zvi, D. (2008). Preparing school teachers to develop students' statistical reasoning. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008).

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics*

Education, *39*(1-2), 127-135.

Godino, J. D., Batanero, C., Roa, R., & Wilhelmi, M. R. (2008). Assessing and developing pedagogical content and statistical knowledge of primary school teachers through project work. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008).

Godino, J. D., Wilhelmi, M. R., & Bencomo, D. (2005). Suitability criteria of a mathematical instruction process. A teaching experience of the function notion. *Mediterranean Journal for Research in Mathematics Education, 4*(2), 1–26.

Graeber, A., & Tirosh, D. (2008). Pedagogical content knowledge. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 117-132). Rotterdam, Netherlands: Sense Publishers.

Hill, H., & Ball. D. L. (2004). Learning mathematics for teaching: Results from California´s mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39,* 372-400.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age Publishing and National Council of Teachers of Mathematics. National Council of Teachers of Mathematics.

Ponte, J. P. (2008). Preparing teachers to meet the challenges of statistics education. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008).

Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practice. In A. Gutierrez & P. Boero (Eds.), *Handbook of research of the psychology of mathematics education: Past, present and future* (pp. 461-494). Rotterdam, Netherlands: Sense Publishing.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Towards a theory of profiency in teaching mathematics. In D. Tirosh & T. Wood (Eds.), *Tools and processes in mathematics teacher education* (pp. 321-354). Rotterdam, Netherlands: Sense Publishers.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-22.

Swoder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp.157-223). Charlotte, NC: Information Age Publishing and National Council of Teachers of Mathematics.

Watson, J. M. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education, 4*, 305-337.

Watson, J. M., Callingham, R. A., & Donne, J. M. (2008). Establishing PCK for teaching statistics. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008).

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67* (3), 223-265.

Wood, T. (Ed.) (2008). *The international handbook of mathematics teacher education.* Rotterdam, Netherlands: Sense Publishers.