Simulation as a tool to train Pre-service School Teachers

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1. Introduction

Probability is increasingly taking part in the school mathematics curriculum; yet most teachers have little experience with probability and share with their students a variety of probabilistic misconceptions (Stohl, in press). Since an education which only focuses on technical skills is unlikely to help these teachers overcome their erroneous beliefs, we should find new ways to teach probability to them, while at the same time helping to bridge conceptualisation and pedagogy (Ball, 2000). Moreover, we should present trainee teachers with some activities based on a constructivist and social approach to teaching (Jaworski, 2001). In this paper first we present results from an initial assessment that show that Spanish trainee teachers frequently hold three probabilistic misconceptions, then we analyse two experiments carried out at the Faculty of Education, University of Granada, where simulation served to confront trainee primary school teachers with their probability misconceptions. We conclude that a better prior training for teachers as well as permanent support for these teachers from University departments and research groups is an urgent necessity. Other complementary details are described in Batanero, Godino & Roa (2003) and Godino, Cañizares & Díaz (2003).

2. Initial assessment

Spanish trainee teachers only study probability with a formal, mathematical approach on the first year of secondary school level (14 years-old) for four weeks, and it is reasonable to expect that they should hold some probabilistic misconceptions that have been widely reported amongst adults. To assess the extent of these misconceptions amongst trainee teachers, we gave a questionnaire to a sample of 132 students at the Faculty of Education, University of Granada (Garfield, 2003) developed to measure statistics and probabilistic reasoning. In Table 1 we reproduce 5 items, which refer to the following probabilistic misconceptions:

- *Representativeness.* Item 2 is adapted from Kahneman et al (1982) to assess whether trainee teachers appear to reason according to the "law of small numbers" when judging probabilities. This is a special case of the representativeness heuristic, because people tend to judge small samples as being equally representative of a population as large samples.
- *Equiprobability:* Items 3 and 4 (Lecoutre, 1992) assess whether trainee teachers tended to assume all the events to be equiprobable.
- *Outcome approach:* Students choosing answer c in item 5 or answer b in item 1 might be reasoning according to the outcome approach (Konold et al, 1993) in which people confuse epistemic (subjective probability that refers to

just a result) with frequentist probability (that refers to the probability of an event in a series of trials).

Table 1. Percentages of responses to probabilistic items

	%
Item 1. Five faces of a fair die are painted black, and one face is painted white. The die is rolled six times. Which of the following results is more likely?	
a. Black side up on five of the rolls; white side up on the other roll (correct)	31.1
b. Black side up on all six rolls	64.4
c. a and b are equally likely	5.5
Item 2. Half of all newborn children are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?	
a. Hospital A (with 50 births a day) (correct)	24.4
b. Hospital B (with 10 births a day)	13.7
c. The two hospitals are equally likely to record such an event.	61.9
Item 3. When two dice are simultaneously thrown it is possible that one of the following two results occurs: Result 1: A 5 and a 6 are obtained. Result 2: A 5 is obtained twice. Select the response that you agree with the most:	
a. The chances of obtaining each of these results is equal	66.2
b. There is more chance of obtaining result 1 (correct)	6.2
c. There is more chance of obtaining result 2.	2.3
d. It is impossible to give an answer.	25.3
Item 4. When three dice are simultaneously thrown, which of the following results is MOST LIKELY to be obtained?	
a. Result 1: "A 5, a 3 and a 6" (correct)	14.6
b. Result 2: "A 5 three times"	0.8
c. Result 3: A 5 twice and a 3"	3.1
d. All three results are equally likely	81.5
Item 5. When three dice are simultaneously thrown, which of these three results is LEAST LIKELY to be obtained?	
a. Result 1: "A 5, a 3 and a 6"	3.1
b. Result 2: "A 5 three times" (correct)	23.1
c. Result 3: A 5 twice and a 3"	73.1
d. All three results are equally unlikely	0.8

Results in Table 1 suggest that representativeness (item 2, option c), equiprobability (item 3, option a; item 3, option d) and outcome approach (item 1, option b; item 4 option d) are widely extended among trainee teachers in this sample. It is then likely that these teachers will not feel confident when they teach probability to their students and that they might even help to spread their misconceptions among them.

2. Simulation and the Teaching of Probability

Since the time available for teaching probability to teachers is scant, the best we can do is to use the teaching time to make them aware about their probabilistic misconceptions, help them to overcome some of them and increase their interest in probability and its teaching. Fortunately, we count on simulation, where we can operate and observe the results in a simulated experiment to obtain information about a real situation. For example, we can find an estimate of the probability of having more than 60% females amongst 100 newborn babies by repeating the experiment of throwing 100 coins at the same time a large number of times. Even in this simple example, simulation condenses time and space in the experiment. It is also a specific and algorithmic model of reality, since it allows intuitive work on the model without resorting to mathematical formalization.

Dantal (1997) suggested the following steps in the teaching of probability using simulation: 1) Observation of reality, 2) simplified description of reality, 3) construction of a model, 4) mathematical work with the model, and 5) interpretation of results in the real setting. He also suggested that teachers are too interested in steps 3 and 4, the "real mathematics", because they are the easiest to teach, although all the different steps are equally relevant in the students' learning. Between the domain of reality, where real random situations are to be found, and the theoretical domain where we build a probability model, Coutinho (2001) locates the pseudo-concrete domain where we work with simulation. Whereas in the real world we carry out specific actions and have experiences and in the theoretical domain we use formal or symbolic representations, in the pseudo-concrete domain we carry out mental and physical operations. There, the student is outside of reality and works with an abstract ideal situation. The didactical role of the pseudo-concrete model is to implicitly induce the theoretical model for the student, when mathematical formalisation is not possible (Henry, 1997).

1. Simulation Experiments based on Random number Tables

Our first experiment was carried out within a compulsory course of Mathematics and its Didactics (90 hours long). All the students (26 trainee teachers) had been introduced to elementary probability ideas before the experiment. The aim was to introduce the students to simulation with concrete manipulatives and random number tables to solve and reflect on counter-intuitive probability problems. Below we describe the different steps and main difficulties observed in practice.

Step 1. Solving the two dice problem. Students were asked to solve a problem very close to item 3. The results matched the assessment described in section 1, since most students considered the three results were equally likely. Even when the lecturer argued that it is always possible to distinguish the two dice (either by their colour or by the order of throwing) many of them were not convinced of this relevance of order.

Step 2. Simulation with small samples. Each student was asked to simulate 10 throws of two dice and record his/her results in a table. Then they were asked to compare results with other classmates and with their answers to the problem and explain why different students obtained different results in the simulations. Due to the small sample size, many students in fact got equiprobability in their experiments, so that they were confirmed in their incorrect intuitions. We also observed some difficulties in using the random number tables, since the digits in the table ranged from 0 to 9, while the dice results range from 1 to 6. This leads to the need for increasing the number of simulations, since some numbers should be disregarded, which was not obvious for some students.

Step 3. Simulation with large samples. Each student was asked to simulate 100 throws of two dice, compare the results with other classmates and explain the differences in the simulations carried out in steps 3 and 4. In this second simulation the results were much more convincing and approached more to the correct responses. A debate was organised in the classroom where students had to explain why experimental results contradicted their previous expectations, with the help of a tree diagram and enumeration of the different possibilities in the mathematical model. Students were asked to summarise their conclusions in a written report and apparently the misconception seemed to have been overcome.

Step 4. Solving the hospital problem. Students were asked to solve a problem very close to item 2 and to write their responses in their report. Most of them reasoned according to representativeness heuristics and considered the event to be equally likely in both hospitals.

Steps 5 and 6. Simulations with small and large samples. The students were asked to carry out 10 simulations for each of the hospitals A and B and record for each of them the number of days for which more than 60% of babies were female. Again the results with small samples were too variable and did not allow the students to overcome their incorrect intuitions. The results for all the students were collapsed to produce 260 simulations for each of the two hospitals (26 students x 10 simulations) and a debate was held regarding the results. Again simulation with large samples clearly showed the correct response and served to organise a debate about the counter-intuitive results in probability and about the usefulness of simulation and probability to solve real problems. Finally students were asked to write their conclusions and the differences between the two hospitals and the simulation results with small and large samples. They were also asked to describe the implications that using small samples might have on the conclusions from different experiments.

A summary of the results is given in Table 2, where we present the responses to the two problems in a sample of 26 trainee teachers before and after carrying out the simulations. Before simulation, most students presented equiprobability bias in problem 1 and representativeness in problem 2. The activity seemed to influence a change of conceptions for a sizeable part of students with a previous misconception (30% improvement in problem 1 and 15.4% improvement in problem 2). These students suggested in their written reports that results are more reliable in large samples. However, a large proportion of the students were still unable to give a response after simulation.

Table 2: Percentage of responses before and after simulation

Problem 1 (throwing two dice)	Before simulation	After simulation
Odd-odd more likely	3.8	0.0
Odd-even more likely (correct)	34.6	65.3
Equiprobability	61.5	3.8
Don't know	0.0	26.9
Problem 2 (Hospital)	Before simulation	After simulation
Big hospital	7.7	7.7
Small hospital (correct)	53.8	69.2
Equiprobability	30.7	3.8
Don't know	7.7	15.3

2. Simulation Experiments based on Computers

A second experiment was carried out within an optional introductory statistics course (90 hours long) with 53 students, most of whom were trainee primary school teachers; others came from pedagogy, psychology or economics courses. The course was organised in such a way that half the sessions were carried out in a traditional classroom, where the lecturer presents the students with the written material that had been provided beforehand, and proposed some problems to lead students to discover certain concepts and properties for themselves. The remaining sessions were held in the computer lab where students performed data analysis activities with the help of Statgraphics software in groups of 2-3 students. Before the experiment the students had studied the basis of descriptive statistics and were familiar with the use of the software. The aim of the experiment was to introduce the students to the use of the simulation options in the Statgraphics software; to make them carry out random simulations with this option and to use the simulation to solve and reflect on counter-intuitive probability problems. This program produces graphs and numerical tables and simulates a variety of different theoretical models of probability distributions, including discrete uniform, binomial and normal distributions. We also used some Internet applets to carry out one of the simulations. Below we describe the different steps and main difficulties observed in practice.

Step 1: Introducing the simulation option in the software. Students were taught the different options and were asked to represent different discrete uniform distributions. They were also asked to comment on the differences in the probability distribution and cumulative distribution plots when changing the parameters. The main difficulty here was to identify which menu should be used to change the parameters, since this is a secondary option of the programme.

Step 2: Solving the two dice problem. Similar to experiment 1. Again most students considered the three results were equally likely and there was confusion about the relevance of order.

Step 3. Simulation with small samples. Similar to experiment 1. Simulation was carried out with Statgrahics, instead of using random number tables. One advantage was that students were able to obtain random results ranging only from 1 to 6, so no observation needed to be disregarded. They all were able to use Statgraphics to obtain tables and graphs of simulations results. Again results in small samples confirmed some students in their incorrect intuitions. Some students showed difficulties in distinguishing experimental (results from simulation) from theoretical (given by the model) probability.

Step 4. Simulation with large samples. Each student was asked to simulate 100, 1,000 and 5,000 throws of two dice with the help of the "What are your chances" resource, produced by the USA National Centre for Education Statistics (http://nces.ed.gov/nceskids/probability/index.asp). Then they were asked again to compare results with other classmates and with their answers to the problem and to describe how regularity was achieved in the progressively increasing sample size. Since these students had previously studied tree diagrams, they were asked to use them to explain why results contradicted their previous expectations. Students seemed to overcome the equiprobability bias for this problem and they were also able to explain the higher probability for central values in the distribution of sums.

Step 4. Solving the hospital problem. Similar to experiment 1. Most of them reasoned according to the representativeness heuristics and considered even to be equally likely in both hospitals.

Steps 5 and 6. Simulations with small and large samples. Again the students were asked to carry out 10 simulations for each of the hospitals A and B and record for each of them the number of days, for which more than 80% of the babies were female. Again the results with small samples were too variable and did not allow the students to overcome their incorrect intuitions. The results for all the students were collapsed to produce 200 simulations for each of the two hospitals (20 groups of students x 10 simulations) and a debate was held about the results. Again simulation with large samples clearly showed the correct response and served to organise a debate about the counter-intuitive results in probability and about the usefulness of simulation and probability to solve real problems. Finally students were asked to write their conclusions about problem 2, and the differences between the two hospitals and the simulation results with small and large samples.

Some precautions

As suggested by Biehler (1997) the temporal and spatial features of many random phenomena make them difficult to observe. Simulation serves to build a simplified model for the phenomena, where irrelevant features are disregarded, and the phenomena are condensed in time and available for the students' work. In particular computers offer students a variety of simulation tools to explore and discover concepts and principles that otherwise would be much more abstract. This may reinforce probabilistic intuition, whose relevance was described by Fischbein (1975).

Today many complex problems are solved by simulation so showing the students some simple examples of this technique may serve to show them their applicability to real problems. In teacher training, simulation may also help students to recognise the differences between theoretical and experimental probability and to show them a model for didactic situations they can use with their own students. We should, however not be extremely optimistic as regards the difficulties of the simulation activity. In our experiments we observed the following difficulties that were also described by Countinho (2001):

- Difficulty in operating the software when this is not familiar to the students. Hence we deduce the need for user-friendly software that does not add any additional complexity to simulation activities.
- Distinguishing the estimation of probability given through simulation from the real theoretical value of probability, which is also accessible by formal calculus, if possible.
- Since simulation and the frequentist approach to probability only provide an estimate for the problem solution and not the reason why this solution is valid it has no explanatory power. Consequently, it lacks the validation value that only the classical approach and formal probability calculus can provide.

We cannot be satisfied only with the student's ability to go from the real world domain to the pseudo-concrete domain, even when this step has a main didactic role in preparing the student to gain understanding in the formal domain where he/she can finally carry out the mathematical activity of formalisation. However, this formal study will be facilitated if students have previously built correct probability intuitions that might be supported by simulation, which provides pseudo-concrete models or reality. We finally suggest this is one way to be explored in the training of primary school teachers.

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