



## **Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education**

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### **Abstract**

The Joint ICMI/IASE Study was organised by the International Commission on Mathematical Instruction (ICMI; [www.mathunion.org/ICMI/](http://www.mathunion.org/ICMI/)) and the International Association for Statistical Education (IASE; [www.stat.auckland.ac.nz/~iase/](http://www.stat.auckland.ac.nz/~iase/)) to address the lack of attention to teaching statistics in schools. Results from this Study were reflected first in the Proceedings of the Joint ICMI/IASE Study Conference held in Monterrey in 2008 ([www.ugr.es/~icmi/iase\\_study/](http://www.ugr.es/~icmi/iase_study/)) and secondly in the Joint ICMI/IASE book that is to be published in the ICMI Study series by Springer. In this Session the main conclusions of this Study and the Study book will be presented.

*Keywords:* ICMI Studies; Teaching Statistics, Training Teachers.

Since the mid-1980s, the International Commission on Mathematical Instruction (ICMI, [www.mathunion.org/ICMI/](http://www.mathunion.org/ICMI/)) has involved itself directly in the identification and investigation of issues or topics of particular significance to the theory or practice of contemporary mathematics education, and invested many efforts in organising specific ICMI studies on these themes.

At the same time, in the past three decades a statistics education research community has developed, linking people from various backgrounds (statisticians involved in teaching statistics in service courses at the university, mathematics educators, and psychologists), leading to the creation of the International Association for Statistical Education (IASE, [www.stat.auckland.ac.nz/~iase/](http://www.stat.auckland.ac.nz/~iase/)) in 1991.

Conversations between ICMI and the IASE made clear there was a common interest in organising a Joint Study related to current problems in the teaching of statistics within school mathematics. This interest arose from the fact that, in spite of recommendations to increase the presence of statistics teaching at the school level, students in these levels do not acquire a statistical literacy adequate to function in an information-based society and to progress in the study of statistics at higher levels such as university or professional training.

The invitation from ICMI to collaborate on a Joint Study was accepted by the IASE. Subsequently, IASE suggested that this Joint Study merge with the next IASE Round Table Conference (June 30-July 4, 2008, Instituto Tecnológico y de Estudios Superiores, Monterrey, Mexico), just before the Eleventh International Congress on Mathematics Education (ICME-11; Monterrey, Mexico, July 6-13, 2008). Consequently, this specific Study brought the mathematics and statistics education communities together to work in collaboration and might serve to continue this collaboration in future work.

In this presentation, we describe the main conclusions from the papers presented and discussed in the conference (Batanero, Burrill, Reading & Rossman, 2008) that have later being developed in the Study book (Batanero, Burrill & Reading, 2011).

### Teaching statistics at school level

The usefulness of statistics for daily life, the important role of statistics in developing critical reasoning; and the instrumental role of statistics in other disciplines were critical reasons to introduce statistics in secondary schools since 1980. However, a recent tendency is that statistics is now taught at very early ages in many countries; in some, six year-old children start studying basic statistical concepts and continue to develop these concepts in all the curricular levels until secondary school, where students in countries like Spain may study elements of statistical inference.

The Principles and Standards for School Mathematics (NCTM, 2000) and the Guidelines for Assessment and Instruction in Statistics Education (GAISE) project (Franklin et al., 2005) were influential in further developing statistics education in the school curriculum in the United States of America and many other countries that followed the American example. In addition, quick innovation and globalization led to a new perceived complexity of education that affected the mathematics curriculum, with a shift from content knowledge to competences (Gattuso & Ottaviani, 2011) that has also been reflected in statistics.

Several papers included in the Joint Study Conference Topic 1 and in the Study book describe different perspectives and approaches to teaching statistics in the school curricula that vary in each country depending on national policies, availability of resources within a country, and the relevance given to different topics and grade level. A common tendency is that changes in what is expected in the teaching of statistics do not just concern the amount but also the teaching approaches. Until recently, statistics in many school curricula was reduced to a formula-based approach that resulted in students who were ill-prepared for tertiary level statistics and adults who were statistically illiterate. The current recommendations, even for primary school levels, suggest a data-orientated approach to the teaching of statistics where students are expected to design and carry on their own investigations and experiments with the aim of developing statistical thinking and reasoning. The importance of developing statistical thinking and not just statistical knowledge in the students is also being emphasized in many curricula that focus on developing statistical reasoning, which is essential to modern society and complement reasoning in other areas of mathematics (Scheaffer, 2006).

The following issues related to teaching statistics were also debated:

- *Mathematical and statistical thinking.* An ongoing discussion in the statistics education community is how to make teachers aware of statistical thinking as something different

from mathematical thinking, both of them being essential to modern society and complementing each other.

- *Fundamental ideas in the school statistics curriculum.* An important area of work was the identification of those statistical ideas that seem to be fundamental for understanding and being able to use statistics in the workplace, in personal lives and as citizens.
- *The role of probability in teaching and learning statistics.* Although the focus of the study is statistics, since statistics and probability are linked in school mathematics in many countries and within mathematics theory and practice, a reference to probability in the book was needed, as didactic problems still need to be solved in the teaching of probability.
- *Technology* has changed many aspects of modern life, and this change has been reflected in statistics education. In addition to exploring data, technology now is used to explore complex statistical ideas or processes via simulation. Computer software offers the opportunity for students to learn about modelling, enabling students to build their own models to describe data and to generate simulations that can be explored.
- *Teaching through project work.* Projects and investigations are ideal vehicles for student engagement, for learning to solve problems in context, and for synthesizing components of learning. Working with real data also helps students investigate issues that do not often appear in textbook problems: for example, recognising different types of data, managing missing or incomplete data, defining variables and categories of classification.
- *Assessment* of student learning is an important part in every educational process as it provides information about student achievement in relation to the intended learning outcomes.

### **Teachers' attitudes, conceptions and beliefs**

Although statistics as a content domain is widely accepted, typically statistics in the school curriculum is taught as part of mathematics. Consequently there is a need for a better preparation of primary and secondary school mathematics teachers, who are responsible for teaching statistics at these levels. The interest in the education and professional development of mathematics teachers has increased in the past 20 years, and there is now a body of research results on this issue; however, current literature seems to indicate that we have not come as far, in the specific case of statistics. This is evident in conferences (e.g., the ICMI Study 15), journals (e.g., *Journal of Mathematics Teacher Education*), surveys, and books that hardly take into account the particular case of statistics. The Joint ICMI/IASE Study was intended to address this omission by promoting research specifically focussed on the education and professional development of teachers to teach statistics.

Teacher education is usually focused on improving teachers' knowledge with relatively little attention paid to teachers' feelings, beliefs or attitudes. This is an important point, since, while teachers are willing to learn about and spend more time teaching statistics and acknowledge the practical importance of statistics, they feel their students experience greater difficulties in statistics than in other mathematical topics, and they consider themselves not well prepared to help their students face these difficulties.

Teachers' mathematics knowledge plays a significant role in the quality of their teaching since many activities of teachers, such as "figuring out what students know; choosing and

managing representations of mathematical ideas; selecting and modifying textbooks; deciding among alternative courses of action” involve mathematical reasoning and thinking (Ball *et al.*, 2001, p. 453). Teacher’s level of expertise in statistics depend on varied and different backgrounds and some may not have learned the content they now need to teach or how to teach in a way to meet the requirements of the curriculum. This is cause for concern as the research summarised in the Study book shows that many teachers unconsciously share a variety of difficulties and misconceptions with their students with respect to fundamental statistical ideas. For example, pre-service primary school teachers in Espinel’s (2007) research lacked the experience to interpret graphs, made errors involving symmetry, outliers and cumulative frequencies. They struggled with mean and median and thought mainly in terms of qualitative variables, thereby confusing histograms with bar graphs. They incorrectly identified the relevant variable and failed to interpret the data distribution as a whole, focusing instead on specific aspects, such as the average or an outlier.

There is a scarce research related to teachers’ statistical pedagogical content knowledge and moreover, this research suggests that this knowledge is often weak. Most teachers have little or no prior experience with using statistical investigation to conduct probability experiments or simulations. Thus, they may have difficulty implementing an experimental approach to teaching probability or teaching through statistical investigation. For example, in an experiment organised by Stohl (2005), although the participant teachers engaged students in statistical investigations through probability experiments, they often missed opportunities for deepening students’ reasoning. The teachers’ approaches to teach the frequentist approach to probability was unsuccessful, because they almost exclusively asked their students to work with only small samples sizes.

In González and Pinto’s (2008) qualitative research pre-service secondary school mathematics teachers had a scant knowledge of graphical representation, no training in matters related to the curriculum and the processes of learning and teaching; specifically they knew nothing about stem and leaf graphs. They did not perceive the different cognitive levels associated with graphs or the various components and processes linked to their interpretation. When asked to classify textbooks’ graphs they focused only on the procedural aspect of graphs and the teaching, according to their conception, should focus on the construction of graphs, the analysis of concepts and the application of algorithms and formulae.

Different models to describe the professional knowledge needed to teach statistics were discussed in the conference and in the book. Some of them derive from frameworks taken from mathematics education and others include specific components to take into account statistical thinking. For example, Burgess (2008) offered their own specific model of pedagogical content knowledge for statistics education that takes into account statistical reasoning (e.g., as described by Wild & Pfannkuch, 1999). Lee and Hollebrands (2008) developed a framework to describe the professional knowledge needed by teachers when they teach statistics and probability with technology tools.

Another issue raised was the need to prepare instruments to obtain a measure of teacher expertise in relation to pedagogical content knowledge for teaching statistics. Watson, Donne and Callingham (2008) prepared a questionnaire, where items asked teachers to predict a range of responses their students might produce if presented with a question, and then to explain how they might use the question in their classrooms, including how they might intervene to address inappropriate responses. Rasch analysis was used to obtain a measure of teacher ability in

relation to professional knowledge to teach statistics and three different levels of teacher ability were identified.

### **Current practices in the training of teachers**

Few current teacher training programmes do adequately educate teachers for their task to prepare statistically literate citizens. Even when many prospective secondary teachers have a major in mathematics, few of them have received specific preparation in applied statistics and they also need education in the pedagogical content knowledge related to teaching statistics.

The situation is even more challenging for primary teachers, since teaching statistics to young children needs different approaches, than teaching statistics in secondary or high school students, and few primary school teachers have had suitable training in either theoretical or applied statistics. Research in statistics education shows that textbooks and curriculum documents prepared for primary and secondary teachers might not offer enough support. Sometimes they present too narrow a view of concepts (for example, only the classical approach to probability or inference is shown); applications are at other times restricted to games of chance or are not based on analysis of real data; finally in some of them the definitions of concepts are incorrect or incomplete (Cardeñoso, Azcárate & Serradó, 2005). There were also presented some examples of successful courses specifically directed to train teachers to teach statistics in different countries some of them based on theoretical models of how this training should be (e.g., Garfield & Everson, 2009).

Presentations from Botswana, Central- America, China, Iran, the Philippines, South Africa and Uganda, among other countries, showed that the problems concerning the way in which teachers are specifically educated to teach statistics in developing countries were similar to those described for developed countries.

### **Empowering teachers to teach statistics**

In many countries, statistical offices and agencies are providing resources that can be used to support the introduction of statistical literacy in schools. However, without wide-reaching education and professional development of teachers, such resources are unlikely to have an impact on students. Moreover, in order for teachers to develop a deep and meaningful understanding of statistics that later they can use to help students develop the ability to think and reason statistically, it is important to promote teachers' statistical literacy (Ridgway, Nicholson, & McCusker, 2011) and statistical reasoning (Pfannkunch & Ben-Zvi, 2011). Different suggestions and experiences in the education of teachers were presented at the Joint Study Conference.

A conclusion of the Conference discussion was that teachers should experience the full cycle of research with statistical projects, if the goal is to change how statistics is experienced in the classroom. Moreover, when time available for working with teachers is scarce, some papers (e.g., Godino et al., 2008) suggested that a formative cycle where teachers are first given a statistical project and then carry out a didactical analysis of the project can help to simultaneously increase the teachers' statistical and pedagogical knowledge.

Although technologies are becoming more common in statistics classrooms, teachers'

abilities to use these tools effectively depend on many factors, including their statistical and pedagogical knowledge. Technology can be used to engage teachers in tasks that simultaneously develop their understanding of statistical ideas and allow them to experience how technology tools can be useful in teaching statistics (Lee & Hollebrands, 2008).

Connecting teacher education to their own practice and promoting collaborative work among teachers is essential to improving professional practice. It is through the exchange of ideas and materials among teachers who have common problems and needs that new ideas emerge for the introduction of new activities, new practices or new competencies (Arnold, 2008). For example, analysing collective case studies and discussing teaching experiences and students' responses to given tasks can reveal the teachers' lack of specific knowledge of some statistical concepts and promote their statistical and pedagogical content knowledge. Through reading and discussing cases, teachers can acquire knowledge of general principles of statistics while also developing reasoning skills necessary for teaching (Groth & Shihong, in press). The affordances offered by modern Internet technologies provide new distance-learning opportunities for the pre-service and in-service training of teachers, making it possible to overcome the restrictions of shrinking resources and geographical locations and to offer high quality learning experiences to geographically dispersed teachers (Meletiou & Serrado, in press).

### **Collaboration in teacher education.**

Because of the inter-disciplinary nature of statistics, cooperation is both natural and beneficial for those involved in statistics education. The preparation of mathematics teachers has historically been the responsibility of mathematicians and mathematics educators, although recently statisticians have started to play a major role in teacher preparation in a few countries. For example, the GAISE framework ([www.amstat.org/education/gaise/](http://www.amstat.org/education/gaise/)) was written, in collaboration between mathematics educators and statisticians, to provide guidance to those involved with teacher preparation. In addition, in many countries statistical offices and associations are increasingly involved in producing materials and organising initiatives to help increase statistical literacy of both students and teachers. Examples analysed in the Study book deal with experiences of collaborations by statistical offices in Canada, New Zealand, South Africa, and Portugal, as well as international experiences such as the CensusatSchool or the International Statistical Literacy Project.

### **Final thoughts**

The success of the Joint ICMI/IASE Study indicated that the time was ripe for collaboration between mathematicians and statisticians to address challenges related to the advancement of both teaching and research in statistics education. However, continuous changes and the rapid development of statistics education imply that this collaboration should continue in the coming years. Thus, the hope is that the analyses, research and case studies presented in the conference and analysed in the Study book will provide a rich starting point for new research and, consequently for improving the preparation of teachers and statistics education at school level.

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