Geometric Analysis

- Hours: 6

- email: luis.guijarro@uam.es

- **Organizers:**

  - Pablo Angulo Ardoy (Universidad Autónoma de Madrid)
  - Jesús Gonzalo Pérez (Universidad Autónoma de Madrid)
  - Luis Guijarro Santamaría (Universidad Autónoma de Madrid)
  - Manuel Ritoré Cortés (Universidad de Granada)

- **Speakers:**

1) **Antonio Alarcón** (Universidad de Granada):*Calabi-Yau type problems.*

2) **María Calle García** (Universidad Autónoma de Madrid):*Existence of solutions of the Capillary problem in $M\times R$.*

3) **Isabel Fernández** (Universidad de Sevilla):*Minimal surfaces and harmonic maps in R^n.*

4) **Vicente Miquel Molina** (Universidad de Valencia):*Volume preserving mean curvature flow.*

5) **Pablo Mira** (Universidad Politécnica de Cartagena):*Constant mean curvature spheres in homogeneous 3-manifolds.*

6) **Frank Morgan** (Williams College):*Isoperimetry and the Log Convex Density Conjecture.*

7) **Francisco J. Palomo** (Universidad de Málaga):*Un teorema tipo Hopf para toros de Lorentz.*

8) **César Rosales** (Universidad de Granada):*The isoperimetric problem in the sub-Riemannian 3-space forms.*

9) **Graham Smith** (Centre de Recerca Matemàtica, Campus de Bellaterra):*Barrier techniques and the non-linear Plateau problem.*

10) **Gil Solanes** (Universitat Autònoma de Barcelona):*Total curvature of complete surfaces in hyperbolic spaces.*

Congreso de la Real Sociedad Matemática Española, Ávila, Febrero 1–5, 2011

Análisis Geométrico

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- email: luis.guijarro@uam.es

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Conferenciantes:

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2) **María Calle García** (Universidad Autónoma de Madrid):*Existence of solutions of the Capillary problem in $M\times R$.*
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7) **Francisco J. Palomo** (Universidad de Málaga):*Un teorema tipo Hopf para toros de Lorentz.*
8) **César Rosales** (Universidad de Granada):*The isoperimetric problem in the sub-Riemannian 3-space forms.*
9) **Graham Smith** (Centre de Recerca Matemàtica, Campus de Bellaterra):*Barrier techniques and the non-linear Plateau problem.*
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Calabi-Yau type problems

Antonio Alarcón

Calabi-Yau's problem is a well-known topic on minimal surfaces theory which dates back to the 60's and deals with the existence of complete bounded minimal surfaces in Euclidean 3-space. Since the construction of such a surface by N. Nadirashvili in 1996, large literature has appeared in this topic. We will show a complete solution to this problem.

Calabi-Yau's questions make sense for a wide range of surfaces and ambient manifolds. We will show in addition a complete solution to this problem for null curves in the complex 3-space C^3, holomorphic immersions in C^2, null curves in the complex Lie group $SL(2,C)$ and Bryant surfaces (constant mean curvature 1 surfaces) in the hyperbolic 3-space. In each case, we construct examples of arbitrary open orientable topological type and, in most of the cases, properly immersed in any convex domain of the ambient manifold.

Another question related to the Calabi-Yau problem is the following. Schoen and Yau proposed in 1985 the problem of determining the existence or not of hyperbolic minimal surfaces in R^3 properly projecting into a plane. We will show examples of such surfaces with arbitrary underlying conformal structure.

This is joint work with Francisco J. López.

Keywords: Minimal surfaces, Calabi-Yau type problems

Mathematics Subject Classification 2000: , ,

Departamento de Geometría y Topología, Universidad de Granada
Avda. Fuentenueva s/n, E-18071, Granada, Spain
alarcon@ugr.es

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Existence of solutions of the Capillary problem in $M \times \mathbb{R}$

María Calle García

The capillary problem considers an interface separating two fluids that lie adjacent to each other and do not mix. This imposes certain geometric conditions on the interface surface and its boundary. The existence, uniqueness and regularity of such surfaces has been widely studied in $\mathbb{R}^n$. Here we study the existence of a graph in $M \times \mathbb{R}$ with prescribed mean curvature and prescribed contact angle, where M is a Riemannian manifold. We follow the work of Korevaar to estimate the gradient of solutions, using the maximum principle. This is joint work with Leili Shariyari.

Departamento de Matemáticas
Universidad Autónoma de Madrid
maria.calle@uam.es

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Minimal surfaces and harmonic maps in $\mathbb{R}^n$.

Isabel Fernández

In this talk we will prove that for any open Riemann surface N, natural number $n \geq 3$, and non-constant harmonic map $h : N \to \mathbb{R}^{n-2}$, there exists a complete conformal minimal immersion from N into $\mathbb{R}^n$ whose last n-2 coordinate functions agree with h.

Also, we will obtain complete full non-decomposable minimal surfaces with arbitrary conformal structure and whose generalized Gauss map is non-degenerate and fails to intersect n hyperplanes of $\mathbb{CP}^{n-1}$ in general position are obtained. Finally, we will construct complete non-proper embedded minimal surfaces in $\mathbb{R}^n$, $\forall n > 3$.

Departamento de Matemática Aplicada
Universidad de Sevilla
isafer@us.es

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Volume preserving mean curvature flow.

Vicente F. Miquel Molina

We will review some of the solved and unsolved questions on mean curvature flows preserving volume, trying to stress the similarities and differences with the usual mean curvature flow.

Departamento de Geometría y Topología
Universidad de Valencia
vicente.f.miquel@uv.es

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Constant mean curvature spheres in homogeneous 3-manifolds

Pablo Mira

A beautiful conjecture in the theory of constant mean curvature (CMC) surfaces in homogeneous manifolds is the following one. Let X be a simply connected homogeneous 3-manifold. Then for each value of H there is at most one (up to ambient isometries) CMC H sphere in X, which inherits all symmetries of the ambient manifold. Moreover, if X is not homeomorphic to R^3, those spheres exist for all values of H, while if X is homeomorphic to R^3, there is a constant $h_0 > 0$ associated to the space such that CMC H spheres exist exactly when $|H| > h_0$, and all of them are embedded.

In this talk we will expose some recent advances on the solution to this conjecture, obtained in collaboration with W.H. Meeks, J. Pérez and A. Ros.

Keywords: Constant mean curvature spheres, Homogeneoues 3-manifolds

Mathematics Subject Classification 2000: , ,

Departamento de Matemática Aplicada y Estadística
Universidad Politécnica de Cartagena
Paseo Alfonso XIII, 52
30203 Cartagena, Murcia (Spain)
Pablo.Mira@upct.es

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Isoperimetry and the Log Convex Density Conjecture

Frank Morgan

Isoperimetric problems, puzzling and inspiring mathematicians for thousands of years, now flourish under the recent surge of interest in densities on manifolds following Perelman's proof of the Poincaré Conjecture. We'll discuss some recent results and open problems, including the Log Convex Density Conjecture on isoperimetry for radial densities on $\mathbb{R}^n$.

Departament of Mathematics and Statistics
Williams College

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Un teorema tipo Hopf para toros de Lorentz

Francisco J. Palomo

We will give certain integral inequalities involving the Ricci tensor and conjugate points along lightlike geodesics in a compact Lorentz manifold. These inequalities allow us to show that a conformaly flat Lorentz torus without conjugate points is flat. The result is sharp since we cannot weaken our hypothesis to the absence of conjugate points only along geodesics of a single causal type. We will also present several open questions.

Keywords: Conformally stationary Lorentz tori, conjugate points, manifold of lightlike directions.

Mathematics Subject Classification 2000: 53C50, 53C22, 53C25.

Departamento de Matemática Aplicada.
Universidad de Málaga.
Edificio de Ingenierías, Campus de Teatinos.
fjpalomo@ctima.uma.es

Referencias

[1] L.W. Green. A theorem of E. Hopf. *Mich. Math. J.* **volumen**(5), 31–34, 1958.

[2] M. Gutiérrez, F.J. Palomo and A. Romero. Lorentzian manifolds with no null conjugate points. *Math. Proc. Camb. Phil. Soc.* **volumen**(137), 363–375, 2004.

[3] E. Hopf. Closed surfaces without conjugate points. *Proc. Nat. Acad. Sci. U.S.A.* **volumen**(34), 47–51, 1948.

[4] F.J. Palomo and A. Romero. Conformally stationary Lorentzian tori with no conjugate points are flat. *Proc. Amer. Math. Soc.* **volumen**(137), 2403–2406, 2009.

[5] M. Sánchez. Structure of Lorentzian tori with a Killing vector field. *Trans. Amer. Math. Soc.* **volumen**(349), 1063–1080, 1997.

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The isoperimetric problem in the sub-Riemannian 3-space forms

César Rosales

The most simple and symmetric spaces in the sub-Riemannian geometry of contact 3-manifolds are the space forms $\mathbb{M}(\kappa)$. They can be obtained in different ways. For example, they appear as limits in the Hausdorff-Gromov topology of some relevant Thurston spaces: the Heisenberg group, the standard 3-sphere, and the universal cover of $SL(2,\mathbb{R})$.

In this talk we gather recent progress on the isoperimetric problem in $\mathbb{M}(\kappa)$, with emphasis on variational questions. Precisely, we will show classification results obtained jointly with A. Hurtado and M. Ritoré for first and second order minima of the area under a volume constraint. As a consequence, we shall deduce sub-Riemannian counterparts to some classical statements in Riemannian 3-space forms, such as the Alexandrov theorem and the description of complete stable surfaces. Finally, we will discuss further development and open questions.

Keywords: Isoperimetric problem, sub-Riemannian 3-space forms

Mathematics Subject Classification 2000: 53C17, 53C42

Departamento de Geometría y Topología, Universidad de Granada
Avda. Fuentenueva s/n, E-18071, Granada, Spain
crosales@ugr.es

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Barrier techniques and the non-linear Plateau problem.

Graham Smith

We show how barrier techniques are used to solve the non-linear Plateau problem for a large class of curvature functions - arXiv:1008.3545.

Centre de Recerca Matemàtica,
Campus de Bellaterra, Edifici C,
08193 Bellaterra, Barcelona
gsmith@crm.cat

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Total curvature of complete surfaces in hyperbolic spaces.

Gil Solanes Farrés

We present a Gauss-Bonnet formula for the integral of the Lipschitz-Killing curvature of complete surfaces in hyperbolic space of any dimension. This formula involves a contribution from the ideal boundary curve of the surface, which is a functional for closed space curves, invariant under Möbius transformations. This invariant can be interpreted as a regualrization of the volume of the set of spheres linked with the curve. This is related to Banchoff-Pohl's notion of area enclosed by a space curve. Some connections with the theory of knot energies will also be described.

Departamento de Matemáticas
Universidad Autónoma de Barcelona.

solanes@mat.uab.cat