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Parabolic Weingarten surfaces in hyperbolic space

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Abstract. A surface in hyperbolic space $\mathbb{H}^3$ invariant by a group of parabolic isometries is called a parabolic surface. In this paper we investigate parabolic surfaces of $\mathbb{H}^3$ that satisfy a linear Weingarten relation of the form $a\kappa_1 + b\kappa_2 = c$ or $aH + bK = c$, where $a, b, c \in \mathbb{R}$ and, as usual, κ_i are the principal curvatures, H is the mean curvature and K is de Gaussian curvature. We classify all parabolic linear Weingarten surfaces in hyperbolic space.

1. Introduction

A surface S in 3-dimensional hyperbolic space $\mathbb{H}^3$ is called a *Weingarten surface* if there is some relation between its two principal curvatures κ_1 and κ_2, that is, if there is a smooth function W of two variables such that $W(\kappa_1, \kappa_2) = 0$. In particular, if K and H denote respectively the Gauss curvature and the mean curvature of S, the identity $W(\kappa_1, \kappa_2) = 0$ implies a relation $U(K, H) = 0$. In this paper we study Weingarten surfaces that satisfy the simplest case for W and U, that is, of linear type:

$$a\,\kappa_1 + b\,\kappa_2 = c \tag{1}$$

and

$$a\,H + b\,K = c, \tag{2}$$

where $a, b, c \in \mathbb{R}$. We say in both cases that S is a *linear Weingarten surface* and we abbreviate by LW-surface. In the set of LW-surfaces, it is worth mentioning

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