Riemannian extensions and quasi-Einstein metrics

M. Brozos-Vázquez, E. García-Río, X. Valle-Regueiro

MBV: Departamento de Matemáticas, Escola Politécnica Superior, Universidade da Coruña, Spain

EGR-XVR: Departamento de Matemáticas, Facultade de Matemáticas, Universidade de Santiago de Compostela, Spain

miguel.brozos.vazquez@udc.gal eduardo.garcia.rio@usc.es javier.valle@usc.es

ABSTRACT

A Walker manifold (M, g) is a pseudo-Riemannian manifold that admits a null parallel distribution. Riemannian extensions are Walker metrics realized on the cotangent bundle of an affine manifold (Σ, D) and defined in terms of the torsion-free connection D. Different modifications of these metrics give rise to Walker manifolds with special curvature properties (see [3] and references therein).

A pseudo-Riemannian manifold (M, g) is called generalized quasi-Einstein if there is a solution $f \in \mathcal{C}^\infty(M)$ of the equation

$$\mathrm{Hes}_f + \rho - \mu df \otimes df = \lambda g \tag{1}$$

for some $\mu \in \mathbb{R}$ and $\lambda \in \mathcal{C}^\infty(M)$ [1]. This definition includes as particular cases important geometric structures such as Einstein metrics, gradient Ricci solitons, gradient Ricci almost solitons and quasi-Einstein metrics [2, 4].

The aim of the talk is to present a classification of self dual generalized quasi-Einstein manifolds in dimension 4, showing the importance of Riemannian extensions when the level set hypersurfaces of the solution f of Equation (1) are degenerate [1].

References

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