Constant Mean Curvature Invariant Surfaces in $\mathbb{L}^3$ and a Blaschke's Variational Problem

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ABSTRACT

In 1930, in [2], Blaschke studied the solutions of the variational problem for the energy $\mathbf{\Theta}(\gamma) = \int_\gamma \sqrt{\kappa}$ *acting on certain spaces of curves in the Euclidean 3-space* $\mathbb{R}^3$. *In particular, in* $\mathbb{R}^2$, *he obtained the catenaries.*

In this talk, for a fixed $\mu \in \mathbb{R}$, *we are going to extend this problem and we will consider curves in* $\mathbb{L}^3$ *which are extremals for the action*

$$\mathbf{\Theta}(\gamma) = \int_\gamma \sqrt{\kappa - \mu}\,. \qquad (1)$$

We are going to get all solutions of the Euler-Lagrange equations of (1) *in Minkowski 3-space* $\mathbb{L}^3$, *[1].*

Finally, making critical curves evolve under their associated Killing vector field ([3] and [4]), these solutions are going to be related with profile curves of constant mean curvature invariant surfaces of $\mathbb{L}^3$; *showing that a invariant surface of* $\mathbb{L}^3$ *has constant mean curvature, if and only if, it is geodesically foliated by critical curves of* (1), *[1]. This leads to another description of the well-known families of constant mean curvature surfaces in* $\mathbb{L}^3$, *([5] and [6]).*

Furthermore, our results can be extended to any Riemannian and Lorentzian 3-space form, [1].

References

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