DOUBLE ROTATIONAL SURFACES IN LORENTZ-MINKOWSKI 4-SPACE

Wendy Goemans – Abstract for the Young Researcher Workshop on Differential Geometry in Minkowski Space, Granada, Spain, April 17-20, 2017

One of the most basic examples of surfaces in 3-dimensional Euclidean space is a rotational surface or a surface of revolution, that is, a surface which is the trace of a planar curve that is rotated about an axis in its supporting plane. Its simple construction makes that it is appealing to geometers, but also is open to alteration. One possible generalization is to subject a planar curve to two simultaneous rotations. The resulting surface is called a twisted surface and studied in [3, 4] (see also the references therein). Another possibility is to extend the concept of a rotational surface to higher dimensional ambient spaces, see for instance [5, 6].

Combining these two points of view leads to the construction of a double rotational surface in 4-space: perform on a planar curve in 4-space two simultaneous rotations, possibly at different rotation speeds. In [1], double rotational surfaces in 4-dimensional Euclidean space are defined and curvature properties on it are examined. The relation of flat double rotational surfaces with newly defined Clelia curves in Euclidean 4-space is also highlighted there. These results turn out to be analogous to the 3-dimensional case (for an overview of properties of Clelia curves in 3-space and their relation with flat twisted surfaces, see [2] and the references therein).

In this contribution, double rotational surfaces in 4-dimensional Lorentz-Minkowski space are defined. Two problems arise when transferring the construction of subjecting a planar curve, the profile curve, to two simultaneous rotations in Lorentz-Minkowski 4-space. Firstly, the supporting plane of the profile curve has one of three causal characters, namely spacelike, timelike or lightlike (null). Secondly, because of the existence of these different causal characters, there exist different rotations, depending on the causal character of the planes that are left invariant. Therefore, different possibilities for the construction must be considered. Since constructions involving only rotations which leave invariant spacelike or timelike planes are very similar to the construction in Euclidean 4-space (see [1]), most attention will be paid to the cases in which so-called 'null-rotations' are involved. As to be expected, incorporating rotations which leave invariant lightlike planes, leads to results deviating from the ones in Euclidean 4-space. Focus will be on obtaining explicit parameterizations of these surfaces. To conclude, some interesting curvature properties (flatness, minimality) for these double rotational surfaces are studied and illustrated with examples.

References

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