On the convexity of compact constant mean curvature spacelike surfaces in the Lorentz-Minkowski space

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A surface in the 3-dimensional Lorentz-Minkowski space $\mathbb{L}^3$ is said to be spacelike if its induced metric from $\mathbb{L}^3$ is Riemannian. In this talk we consider compact spacelike surfaces, with non-zero constant mean curvature, immersed in $\mathbb{L}^3$, and with (necessarily) non-empty, smooth and convex boundary. In this context, we ask about the influence of the geometry of the boundary on the surface. In particular, we prove that if the boundary is a planar curve, with the added property that it intersects any branch of any hyperbola at at most five points, then our surface is strictly convex. Let us observe that, under such assumptions, the boundary is always convex. Ellipses are a particular case of such curves.

The proof of our result follows the ideas of Chen and Huang [3], which were inspired by a previous argument by Alexandrov [2].

Finally, we also present an example that shows that, in general, the convexity of the boundary is not inherited by the surface. Therefore, we cannot omit the assumption on the minimum number of intersection points of our curve with a branch of a hyperbola.

The results are contained in a joint work with M. Caballero and R. López [1].

[1] A. L. Albujer, M. Caballero and R. López. Convexity of the solutions to the constant mean curvature spacelike surface equation in the Lorentz-Minkowski space. J. Differential Equations **258** (2015), 2364–2374.

[2] A. D. Alexandrov. Uniqueness theorems for surfaces in the large. Vestnik Leningrad. Univ. 11, no. 19 (1956); Amer. Math. Soc. Transl. **21** (1962), 341–354.

[3] J. T. Chen and W. H. Huang. Convexity of capillary surfaces in the outer space. Invent. Math. **67** (1982), 253–259.