

BEHAVIOR OF THE GAUSSIAN CURVATURE OF TIMELIKE MINIMAL SURFACES WITH SINGULARITIES

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ABSTRACT

A timelike minimal surface in the 3-dimensional Lorentz-Minkowski space is a surface with a Lorentzian metric whose mean curvature vanishes identically. One of the most important differences between spacelike surfaces (or surfaces in the Euclidean space) and timelike surfaces is the diagonalizability of the shape operator of a surface. For the minimal case, the diagonalizability of the shape operator corresponds to the sign of the Gaussian curvature of a timelike minimal surface away from flat points. More precisely the shape operator is diagonalizable over the real number field on points with negative Gaussian curvature and diagonalizable over the complex number field on points with positive Gaussian curvature. Flat points consist of umbilic points and quasi-umbilic points. The aim of this talk is to investigate the behavior of the Gaussian curvature near regular points and singular points on timelike minimal surfaces.

First we prove that the sign of the Gaussian curvature of any timelike minimal surface is determined only by the orientations of two null curves that generate the surface, and flat points are characterized by the degeneracies of these null curves.

Next we also discuss the behavior of the Gaussian curvature near singular points on timelike minimal surfaces. In particular, about relations between the Gaussian curvature and singular points, we prove that near a non-degenerate singular point of a timelike minimal surface which is not a cuspidal edge, there is no flat points. Moreover in this case, the sign of the Gaussian curvature is negative (resp. positive) near a singular point if and only if the surface has (resp. has not) the structure of a wave front at the singular point. On the other hand, near cuspidal edges, we can not determine the sign of the Gaussian curvature in general, but we can determine the behavior of the Gaussian curvature if cuspidal edges accumulate to another non-degenerate singular point. We also determine when flat points accumulate to a non-degenerate singular point of a timelike minimal surface. In this talk we try to give detailed proofs of results and related ideas. This talk is based on the preprint arXiv:1701.00238.