Geometric aspects on capillary problems and related topics

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TITLES AND ABSTRACTS

Yasunori AOKI
Uppsala University

Bounded and Unbounded Capillary Surfaces

A capillary surface, a liquid surface at equilibrium, can be modelled mathematically by a nonlinear elliptic partial differential equation with a nonlinear boundary condition called Laplace-Young equation. The PDE can simply be described in words as " Height of the capillary surface is proportional to the mean curvature" and the nonlinear boundary condition can be described as the "angle of contact of the capillary surface and the solid boundary only depends on the materials". In this talk we consider a cusp domain made by two vertical cylinders with different material thus with different angles of contact. It has been shown by Scholz in 2004 that the capillary surface at a cusp domain is unbounded if the angles of contact do not sum up to pi. In 2012, we have shown that the capillary surface is bounded if the angles of contact sum up to pi and if the curvatures of the boundaries are finite. At this point, the case where the angles of contact sum up to pi and infinite curvature boundaries remains to be an open problem. In 2014, we have investigated this case using Finite Element Approximation and made a conjecture where the capillary surface is bounded if and only if the angles of contact add up to pi.

In this talk I will first present the results in the following papers and then report on the recent progress on the analytic proof on the numerical conjecture we have proposed.

References:

Yasunori Aoki and David Siegel: Bounded and Unbounded Capillary Surfaces in a Cusp Domain, Pacific Journal of Mathematics, Volume 257, Issue 1, 2012, Pages 143-165.

Yasunori Aoki and Hans De Sterck: Numerical Study of Unbounded Capillary Surfaces, Pacific Journal of Mathematics, Volume 267, Issue 1, 2014, Pages 1-34.

Josef BEMELMANS
RWTH Aachen University

Capillary Surfaces and Floating Bodies

We consider a cylindrical container that is partially filled with fluid and with a rigid body that is floating on it. The surface that separates the fluid from the air above

is determined by capillary forces. Then the components of the boundary of this surface lie on the body and on the walls of the container. The capillary surface and the position and orientation of the body are determined by minimizing the total energy which takes into account gravity, cohesion of the fluid as well as adhesion between the fluid and both the walls of the container and the floating body. We prove existence of minimizers in the class of Cacciopoli sets and for surfaces that are graphs of real functions. The first variation leads to the capillary equation in a domain with a free boundary and to an equilibrium condition for the forces that act on the floating body. This is joint work with G.P. Galdi and M. Kyed.

References:

Bemelmans, J., Galdi, G.P., Kyed, M.: Fluid flows around floating bodies, I: The hydrostatic case. J. Math. Fluid Mech. 14 (2012) 751 -770

Bemelmans, J., Galdi, G.P., Kyed, M.: Capillary surfaces and floating bodies. Ann. Mat.193 (2014) 1185 -1200

Paolo CALDIROLI
Università do Torino

Surfaces with prescribed mean curvature in cones: existence and characterization as radial graphs

We deal with the Plateau problem for disc-type surfaces with prescribed mean curvature $H\colon \mathbb{R}^3 \to \mathbb{R}$, spanning a given Jordan curve Γ and contained in a convex cone. Under some growth condition on H we prove existence of a solution S characterized as minimizer of the energy associated to the problem. Moreover, under a suitable monotonicity assumption on H, we show that if the Jordan curve Γ has injective radial projection $\hat{\Gamma}$ on the unit sphere, then S is the radial graph of a mapping on the interior domain of the unit sphere bounded by $\hat{\Gamma}$. These results are contained in a recent, joint work with Alessandro Iacopetti (University of Turin).

Jaigyoung CHOE
Korea Institute for Advanced Study

Stable capillary hypersurfaces in a wedge

Let Σ be an immersed stable hypersurface of constant mean curvature in a wedge bounded by two hyperplanes in $\mathbb{R}^n$. Suppose that Σ meets those two hyperplanes in constant contact angles and is disjoint from the edge of the wedge. We will show that if $\partial\Sigma$ is embedded for $n = 3$, or if $\partial\Sigma$ is convex for $n = 4$, then Σ is part of the sphere. This is a joint work with M. Koiso.

Flavio F. CRUZ
Universidade Regional do Cariri

On the existence of radial graphs with constant scalar curvature

In this talk I will discuss about the problem of finding hypersurfaces of constant curvature and prescribed boundary in the Euclidean space, using the theory of fully nonlinear elliptic equations. The main result says that if the given data admits a suitable radial graph as a subsolution, then we can find a radial graph with constant curvature and that realizes the prescribed boundary. As an application we prove that if $\Omega \subset \mathbb{S}^n$ is a mean convex domain whose closure is contained in an open hemisphere of $\mathbb{S}^n$ then, for $0 < R < n(n-1)$, there exists a radial graph of constant scalar curvature R and boundary $\partial\Omega$.

Carlota M. CUESTA
Universidad del País Vasco

Interfaces determined by capillarity and gravity in a two-dimensional porous medium

We consider a two-dimensional model of a porous medium where circular grains are uniformly distributed in a squared container. We assume that such medium is partially filled with water and that the stationary interface separating the water phase from the air phase is described by the balance of capillarity and gravity. Taking the unity as the average distance between grains, we identify four asymptotic regimes that depend on the Bond number and the size of the container. We analyse, in probabilistic terms, the possible global interfaces that can form in each of these regimes. In summary, we show that in the regimes where gravity dominates the probability of configurations of grains allowing solutions close to the horizontal solution is close to one. Moreover, in such regimes where the size of the container is sufficiently large we can describe deviations from the horizontal in probabilistic terms. On the other hand, when capillarity dominates while the size of the container is sufficiently large,

we find that the probability of finding interfaces close to the graph of a given smooth curve without self-intersections is close to one.

Leonid G. FEL
Technion – Israel Institute of Technology

Stability of Axisymmetric Liquid Bridges

Based on the Weierstrass representation of second variation we develop a theory of stability for isoperimetric problem with minimized and constrained two-dimensional functionals of general type and free endpoints allowed to move along two given planar curves. We establish the stability criterion and apply this theory to the axisymmetric liquid bridge between two axisymmetric solid bodies without gravity and rotation to determine the stability of menisci with free contact lines. For catenoid and cylinder menisci and different solid shapes we determine the stability domain. Unduloid and nodoid menisci are considered between plates and spheres. We find the existence conditions of unduloid and nodoid menisci between two solid spheres and study their stability under axisymmetric perturbations (this is a joint work with B. Rubinstein).

Ailana FRASER
University of British Columbia

Existence and uniqueness theorems for free boundary minimal surfaces in the ball

Free boundary minimal surfaces in the ball are proper branched minimal immersions of a surface into the ball that meet the boundary of the ball orthogonally. Such surfaces have been extensively studied, and they arise as extremals of the area functional for relative cycles in the ball. They also arise as extremals of an eigenvalue problem on surfaces with boundary. In this talk I will describe existence and uniqueness theorems for such surfaces. This is joint work with R. Schoen.

Brian FREIDIN and Peter McGRATH
Brown University

Area Bounds for Free Boundary Minimal Surfaces in Conformally Euclidean Balls

We prove that the volume of a free boundary minimal surface $\Sigma^k \subset B^n$ where B^n is a geodesic ball in Hyperbolic space H^n is bounded from below by the volume of a geodesic k-ball with the same radius as B^n. More generally, we prove analogous results for the case where the ambient space is conformally Euclidean, spherically symmetric, and the conformal factor is nondecreasing in the radial variable. These results follow work of Brendle and Fraser-Schoen, who proved analogous results for surfaces in the unit ball in $\mathbb{R}^n$.

David HARTLEY
ICMAT

Stability of Unduloids under the Volume Preserving Mean Curvature Flow

The volume preserving mean curvature flow (VPMCF) moves hypersurfaces of Euclidean space by their mean curvature vector with an added constraint term to preserve the enclosed volume. Stationary solutions to this flow are given by the constant mean curvature (CMC) hypersurfaces, which in the axially symmetric case are spheres, cylinders, unduloids and nodoids. Here we consider the stability of the unduloids that are close to a cylinder of critical radius, above which the cylinder is stable and below which it is unstable. The analysis is performed using bifurcation theory to track how the critical eigenvalue changes along the continuous family of CMC hypersurfaces, which includes the cylinder. The unduloids of dimension greater than eleven are found to be stable, while those of a lower dimension are unstable.

Atsufumi HONDA
National College of Technology, Miyakonojo College

Type-changing CMC surfaces

In the Lorentz-Minkowski 3-space, if a maximal surface (i.e. spacelike zero mean curvature surface) has fold singular points, it can be extended to a time- like minimal surface across the singular point set analytically. Conversely, a type-changing zero mean curvature surface containing no lightlike lines can be constructed in this manner. On the other hand, it is known that, for a maxi- mal surface having conelike

singular points, its conjugate has fold singularities (so-called the 'duality of singularities'). In this talk, we show that such phenomena do not occur in the case of surfaces with non-zero constant mean curvature (CMC). More precisely, we prove that there exist no type-changing CMC surfaces. It is also shown that spacelike CMC surfaces do not admit any fold singular points. We also investigate the singularities which appear on the conjugates of spacelike CMC surfaces having conelike singular points.

This talk is based on joint works with Miyuki Koiso, Masatoshi Kokubu, Kentaro Saji, Masaaki Umehara and Kotaro Yamada.

Yoshihito KOHSAKA
Graduate School of Maritime Sciences, Kobe University

Stability of Delaunay surfaces as the stationary surfaces for the surface diffusion equation

The stability of the stationary surfaces for the surface diffusion equation will be studied. The surface diffusion equation was first derived by Mullins to model the motion of interfaces in the case that the motion of interfaces is governed purely by mass diffusion within the interfaces. Geometrically, the surface diffusion equation is obtained as the H^{-1}-gradient flow of the area functional for the evolving surfaces, so that it has a variational structure that the area of the surface decreases whereas the volume of the region enclosed by the surface is preserved. This provides the constant mean curvature surfaces as the stationary surfaces. In this talk, we will give a criterion of the stability of Delaunay surfaces as the stationary surfaces for the axisymmetric surface diffusion equation.

Miyuki KOISO
University of Kyushu

Local structure of the space of all triply periodic minimal surfaces in $\mathbb{R}^3$

We study the structure of the space of all triply periodic minimal surfaces (TPMS) in $\mathbb{R}^3$. The local dimension of this space around each "generic" TPMS is determined. We also discuss the existence of "singularities" of this space and its application to find infinitely many new examples of TPMS's. (Joint work with Paolo Piccione and Toshihiro Shoda.)

Jorge LIRA
Universidad Federal do Ceará

Maximum principle and minimal graphs in Riemannian manifolds

We survey some recent results about uniqueness and non-existence of entire minimal graphs in Riemannian manifolds endowed with a Killing vector field. These results are based on gradient estimates and rely on a variant of the maximum principle obtained in joint work with L. Alias and M. Rigoli.

Fernando MANFIO

Helicoidal flat surfaces in the 3-sphere
Universidad de Sao Paulo

Helicoidal surfaces are natural generalizations of rotation surfaces which, in the Euclidean space, have been known for a long time. In this talk we consider helicoidal at surfaces in the 3-dimensional sphere. A complete classi

cation of such surfaces is given in terms of their

rst and second fundamental forms and by linear solutions of the corresponding angle function. The classi

cation is obtained by using the Bianchi-Spivak representation for at surfaces and a representation for constant angle surfaces in s^3. Joint work with Joao Paulo dos Santos.

Bennett PALMER
Idaho State University

Free Boundary Problems with Line Tension

In a free boundary problem, line tension is an energy term assigned to the curve which is the intersection of the free surface with a supporting surface. In this talk we will first review the motivation for introducing the line tension. Then, we will discuss a result characterizing stable disc type equilibria contained in a ball.

Sung-Ho PARK
Hankuk University Foreign Studies

Minimal and cmc surfaces in $\mathbb{S}^3$ which are foliated by circles

First, we classify minimal and cmc surfaces in $\mathbb{S}^3$ which are foliated by circles. Secondly, we show that circle-foliated minimal surface in $\mathbb{R}^n$ is either a plane or a catenoid or the graph of $wz = c$ ($w, z \in \mathbb{C}$, c: real constant) in a 4-dimensional Affine subspace of $\mathbb{R}^n$. We use this result to construct minimal surfaces in $\mathbb{S}^4$ and $\mathbb{H}^4$ which are foliated by circles.

Julian SCHEUER
University of Freiburg

The inverse mean curvature flow perpendicular to the sphere

Usually the inverse mean curvature flow of hypersurfaces in $\mathbb{R}^{n+1}$ is driving the hypersurfaces to infinity. However, in this talk we consider the smooth inverse mean curvature ow of strictly convex hypersurfaces with boundary embedded in $\mathbb{R}^{n+1}$ which remain perpendicular to the unit sphere from the inside. We prove that the flow hypersurfaces converge to the embedding of a flat disk in the norm of $C^{1,\beta}$, $\beta < 1$. Several open questions about this ow can be discussed afterwards.

References. Ben Lambert and Julian Scheuer, The inverse mean curvature ow perpendicular to the sphere, Math. Ann. (2015), 1–25.

Keomkyo SEO
Sookmyung Women's University

Characterizations of a Clifford hypersurface in a unit sphere

Let Σ be an $n(\geq 3)$-dimensional compact embedded hypersurface in a unit sphere with constant mean curvature $H \geq 0$ and with two distinct principal curvatures λ and μ of multiplicity $n-1$ and 1, respectively. It is known that if $\lambda > \mu$, there exist many compact embedded constant mean curvature hypersurfaces. In this talk, we prove that if $\mu > \lambda$, then Σ is congruent to a Clifford hypersurface. We also give a sharp curvature integral inequality for hypersurfaces in a unit sphere with

constant m-th order mean curvature and with two distinct principal curvatures, which generalizes Simons' integral inequality and gives a characterization of Clifford hypersurfaces in a unit sphere. This is a joint work with Sung-Hong Min.

Pieralberto SICBALDI
Université d'Aix-Marseille

Overdetermined problems, rigidity results and applications

Overdetermined elliptic systems appear in many problems in Physics and Applied Mathematics, and the classification of their solutions is a major topic in Analysis of PDEs. They appear also in Capillary theory, and I will present their role in that theory. In the last years, a surprising parallelism with constant mean curvature surfaces and De Giorgi's conjecture has been pointed out. In this talk I will present a new result about the rigidity of overdetermined elliptic problems in the plane, related to a conjecture by Berestycki, Caffarelli and Nirenberg. Joint work with A. Ros and D. Ruiz.

Rabah SOUAM
Université Pierre et Marie Curie-Université Paris Diderot

Stability of capillary hypersurfaces in slabs and halfspaces

We will discuss stability of immersed capillary hypersurfaces in slabs and halfspaces in R^n. We will show these hypersurfaces are necessarily of revolution under some conditions on their topology and the angle of contact.

Lucia TEALDI

Anisotropic mean curvature on facets and relations with capillarity

Given a norm ϕ on $\mathbb{R}^3$ (called anisotropy) and a solid set E, we shall recall the definition of ϕ-mean curvature of the boundary ∂E of E. The ϕ-mean curvature is defined as the divergence of any vector field solving a suitable minimum problem, related to the first variation of the perimeter functional; we call such a vector field an *optimal selection*. We will focus on non smooth and non strictly convex norms,

in particular crystalline and cylindrical ones, and consider the ϕ-mean curvature of a bidimensional facet F of ∂E. F is said ϕ-calibrable if its ϕ-mean curvature is constant, which is equivalent to the existence of a ϕ-subunitary vector field in F, with suitable normal trace, and constant divergence equal to the anisotropic mean curvature of F. When ϕ is the cylindrical anisotropy, this last problem has the same structure of the capillary problem in a vessel having F as cross section, in absence of gravity and with zero contact angle: this shows a remarkable connection between ϕ-calibrability and the study of a problem related to minimal surfaces in graph form over F. Still in the cylindrical case, we shall exhibit some examples of non ϕ-calibrable facets where it is possible to characterize the ϕ-mean curvature, and to exhibit an optimal selection.

References: S. Amato, G. Bellettini, L. Tealdi, *Anisotropic mean curvature on facets and relations with capillarity*, Geometric Flows. Volume 1, Issue 1, ISSN (Online) 2353-3382, DOI: 10.1515/geofl-2015-0005, September 2015.

Tatiana ZOLOTAREVA
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Free Boundary Minimal Surfaces in the Unit 3-Ball

In a recent paper A. Fraser and R. Schoen have proved the existence of free boundary minimal surfaces Σ_n in B^3 which have genus 0 and n boundary components, for all $n \geq 3$. For large n, we give an independent construction of Σ_n and prove the existence of free boundary minimal surfaces $\tilde{\Sigma}_n$ in B^3 which have genus 1 and n boundary components. As n tends to infinity, the sequence Σ_n converges to a double copy of the unit horizontal (open) disk, uniformly on compacts of B^3 while the sequence $\tilde{\Sigma}_n$ converges to a double copy of the unit horizontal (open) punctured disk, uniformly on compacts of $B^3 - \{0\}$.

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