



## Control of neural chaos by synaptic noise

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### Abstract

We study neural automata – or neurobiologically inspired cellular automata – which exhibits chaotic itinerancy among the different stored patterns or memories. This is a consequence of activity-dependent synaptic fluctuations, which continuously destabilize the attractor and induce irregular hopping to other possible attractors. The nature of these irregularities depends on the dynamic details, namely, on the intensity of the synaptic noise and the number of sites of the network, which are synchronously updated at each time step. Varying these factors, different regimes occur, ranging from regular to chaotic dynamics. As a result, and in absence of external agents, the chaotic behavior may turn regular after tuning the noise intensity. It is argued that a similar mechanism might be on the basis of self-controlling chaos in natural systems.

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### 1. Motivation and model

We report on the complex dynamics and possible applications of a very simple neural automata, which is a neurobioinspired cellular automata (Wolfram, 1984). Although this is a crude representation of reality, neural automata exhibits dynamic associative memory, or switching behavior, which has been previously shown in neural networks with dynamic synapses (Cortes et al., 2006, 2004; Pantic et al., 2002). Our neural automata is formulated by incorporating a kind of fast stochastic fluctuations, which are activity-dependent processes, and operate on synaptic intensities (Cortes et al., 2006). This fast noise produces a kind of instabilities in the recalling

dynamics, which allow to solve efficiently some complex tasks in brain, as for instance the rapid response to highly changing stimuli which can play a functional role in attention, working memory or sequential processing of parallel sensory information. Instead of sequentially updating a small neighborhood at each temporal step, when the dynamics concern the whole, which means synchronously updated, fast noise produces, even in absence of external stimulation, chaotic switching among the different memories (Marro et al., 2006a). In this new variant, fast noise can control temporal oscillations of neural activity and other details of the dynamics, as for instance the complexity or chaoticity of these oscillations. Against another alternatives (Garfinkel et al., 1992; Schiff et al., 1994; Freeman et al., 1997; Molgedey et al., 1992), we suggest the possibility to design a mechanism in which noise intensity varies autonomously, which could be useful to self-control chaos in neural systems.

Our neural automata consists of  $N$  cooperative and, for simplicity, fully-connected neurons with stochastic

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dynamics.<sup>1</sup> A main feature is that, at each time step  $t$ , the individual states of  $N$  of its neurons are simultaneously updated. This is performed according to a modification of the Hopfield prescription (Amari, 1972; Hopfield, 1982; Amit, 1989), namely, we shall assume that each neuron  $s_i$ , endures a current – or is acted on by a local field – which has the form (Gardiner, 2004; Bibitchkov et al., 2002):

$$\bar{h}_i(\mathbf{S}) \equiv \int_{\mathbf{X}} h_i(\mathbf{S}, \mathbf{X}) \tilde{P}(\mathbf{X}|\mathbf{S}) d\mathbf{X}, \quad (1)$$

where  $\mathbf{S} = \{s_i; i = 1, \dots, N\}$  stands for a neuron configuration. This equation involves a set of random variables,  $\mathbf{X} = \{x_j\}$ , each affecting a postsynaptic neuron, of distribution  $\tilde{P}(\mathbf{X}|\mathbf{S})$ . This amounts to assume short-time, rapid synaptic fluctuations which, in fact, are known to influence and often determine the neuron activity in many natural processes—see, for instance (Marro and Dickman, 1999) for a technical justification of (1); see also (Abbott and Regehr, 2004) for a recent discussion on the role of synaptic “noise”.

This model has already been analyzed both analytically and numerically for certain choices of parameters. In particular, the case  $n = 1$  of “sequential updating” was shown to exhibit complex hopping between the attractors in some cases (Cortes et al., 2006), and we demonstrated recently (Marro et al., 2006a) that the hopping may become chaotic for Little dynamics, namely,  $n = N$ . We illustrate here a typical situation between these two limits by developing some computer simulations. The case with  $1 < n < N$  for which we present some results here happens to be relevant to understand the possibility of synaptic noise controlling chaotic neural activity.

In order to deal with model simulations that remain versatile enough, we need to introduce some simplifications in the following; notice, however, that some of them may turn irrelevant to the resulting emergent behavior. Most convenient is to restrict ourselves to binary neurons, i.e.,  $s_i = \pm 1$ , which are known to capture the essentials of cooperative phenomena (Abbott and Kepler, 1990; Pantic et al., 2002). Concerning the stochastic variable, we need to determine both its nature and its distribution. A simple choice is to assume that synaptic intensities are of the form  $w_{ij} = w_{ij}^L x_j$ , where  $w_{ij}^L$  are average weights which, also for the sake of simplicity, we shall consider to be of the Hebbian type. That is,  $w_{ij}^L = N^{-1} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$ , where  $\xi_i^{\mu}$  (with  $\mu = 1, \dots, M$ ) stands for  $M$  (binary) patterns that are assumed hereafter to be stored in the system. It then naturally follows

stochasticity of the presynaptic currents in (1) which are given by  $h_i(\mathbf{S}, \mathbf{X}) = \sum_{j \neq i} w_{ij}^L x_j s_j$ . This is consistent with actual features of natural systems such as, for example, variations of the glutamate concentration in the synaptic cleft, and differences in the potency released from different locations on the active zone of the synapses (Franks et al., 2003). These and similar “noises” which cause synaptic fluctuations are typically very fast compared to the time relaxation of the whole neuron system. Therefore, it seems sensible to assume that, in the time scale for the neuron activity, neurons behave as in the presence of a steady distribution for the synaptic fluctuations. This is taken into account by means of the distribution  $\tilde{P}(\mathbf{X}|\mathbf{S})$  in (1), a situation which is discussed with further detail in (Marro and Dickman, 1999).

## 2. Synaptic noise

Recent neurobiological findings (Abbott and Regehr, 2004), concerning activity-dependent processes may help in determining  $\tilde{P}(\mathbf{X}|\mathbf{S})$ . In particular, it has been reported short-time synaptic depression (Tsodyks et al., 1998), i.e., that synaptic weights tend to decrease under repeated presynaptic activation. A simple way of implementing this in (1) is by taking:

$$\tilde{P}(\mathbf{X}|\mathbf{S}) = \prod_i \{p(\vec{\mathbf{m}})\delta(x_j + \Phi) + [1 - p(\vec{\mathbf{m}})]\delta(x_j - 1)\}, \quad (2)$$

where the factorization is for simplicity and  $\vec{\mathbf{m}} = \vec{\mathbf{m}}(\mathbf{S})$  is the  $M$ -dimensional overlap vector of components  $m^{\mu}(\mathbf{S}) = N^{-1} \sum_i \xi_i^{\mu} s_i$ . In accordance with the mentioned observation, (2) implies that increasing the mean firing rate, which will increase the probability function  $p(\vec{\mathbf{m}})$ , will make more likely that synaptic intensities decrease by a factor of  $\Phi$ . The Hopfield model, for which such depressing noise is absent, corresponds here to the limit  $\Phi \rightarrow -1$ . Finally, in order to fully determine the model, one may use the choice (Cortes et al., 2006)  $p(\vec{\mathbf{m}}) = (1 + \alpha)^{-1} \sum_{\nu} [m^{\nu}(\mathbf{S})]^2$ , where  $\alpha = M/N$  is the network load parameter (Hertz et al., 1991). After some straightforward algebra, one obtains the effective currents as:

$$\bar{h}_i(\mathbf{S}) = \left(1 - \frac{1 + \Phi}{1 + \alpha} \sum_{\mu} [m^{\mu}(\mathbf{S})]^2\right) \sum_{\mu} \xi_i^{\mu} m^{\mu}(\mathbf{S}). \quad (3)$$

In addition to the discussed synaptic stochasticity, that we represent here by means of the variable  $x$ , there are independent causes for assuming an stochastic dynamics of the neuron system. That is, a neuron may sometimes

<sup>1</sup> Some consequences of other network topologies have been studied in (Torres et al., 2004), for instance.

remain silent even if it endures a large current. This is naturally modelled by introducing a “temperature” parameter  $T$ . In practice, one usually assigns a probability which depends on  $(h_i - \theta_i)/T$ , where  $\theta_i$  is a threshold, to the change according to  $\text{sig}(h_i) = s_i$  at time  $t$ . This mechanism is equivalent to assume the existence of a hypothetical “thermal bath” which induces stochasticity of the neuron activity by means of a master equation. In general, this equation implies a tendency towards equilibrium. However, in the present case, the canonical tendency competes with the stochastic changes of  $h_i$ , which impedes equilibrium, and the system goes asymptotically to a non-equilibrium steady state (Marro and Dickman, 1999). This complex, non-equilibrium situation is at the origin of the intriguing behavior we describe next.

### 3. Computer simulations

The above programme was implemented in the computer by iterating the following Monte Carlo algorithm:

- (1) Store  $M$  different patterns  $\xi_i^{\mu}$  as the average weights  $w_{ij}^{\mu}$  according to the chosen, e.g., Hebb’s learning rule.
- (2) Set any state  $\mathbf{S} = \{s_i\}$  at random.
- (3) Compute the  $N$  local fields  $\bar{h}_i(\mathbf{S})$  as defined in (3).
- (4) Choose a site (neuron) at random, repeat the choice  $N$  times and keep only the  $n < N$  sites which differ from each other (this procedure lets you with  $n \approx (2/3)N$  sites—for the values of  $N$  of interest here).<sup>2</sup>
- (5) Perform the changes  $s_i \rightarrow -s_i$  at the chosen  $N$  sites using the standard rate  $\omega(s'_i \rightarrow s_i) = (1/2)\{1 - s'_i \tanh[\beta \bar{h}_i(\mathbf{S}')]\}$ , being  $\beta = 1/T$ .
- (6) Increase time in one unit, and go to step (3).

Fig. 1 illustrates the resulting behavior for a single pattern, i.e., it corresponds to the limit  $\alpha \rightarrow 0$ . This shows a complex hopping process between the pattern,  $\xi^1$ , and the anti-pattern,  $-\xi^1$ . The figure compares the evolution at finite temperature with that in absence of thermal fluctuations to demonstrate that hopping is not a consequence of the latter. Consequently, in order to avoid the short-length oscillations shown in the bottom graph of Fig. 1, which are induced by the thermal noise, we are concerned in the following with simulations at  $T = 0$ .

<sup>2</sup> Both Monte Carlo simulations and analytical results (Marro et al., 2006b) are in full-agreement and, in the thermodynamic limit, they satisfy that  $n/N = 1 - (1/e)$ .

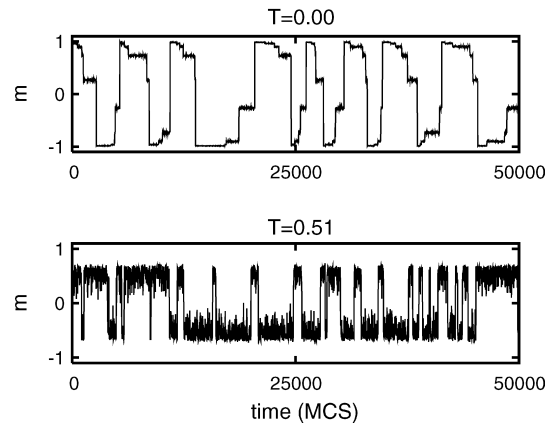


Fig. 1. Time variation of the overlap  $m \equiv m^1(\mathbf{S})$  between the current neural activity,  $\mathbf{S}$ , and a single pattern stored in the synaptic weights, i.e.,  $M = 1$ , as obtained in a Monte Carlo simulation with  $N = 3600$  neurons and a depressing factor  $\Phi = 0.043$ . The top graph is for  $T = 0$ , i.e., in absence of thermal fluctuations, while the bottom graph is for  $T = 0.51$ .

Fig. 2 illustrates a main result, namely, that the frequency and other details of the hopping strongly depend on the value of the parameter  $\Phi$  which modulates the fast synaptic noise. An appropriate measure

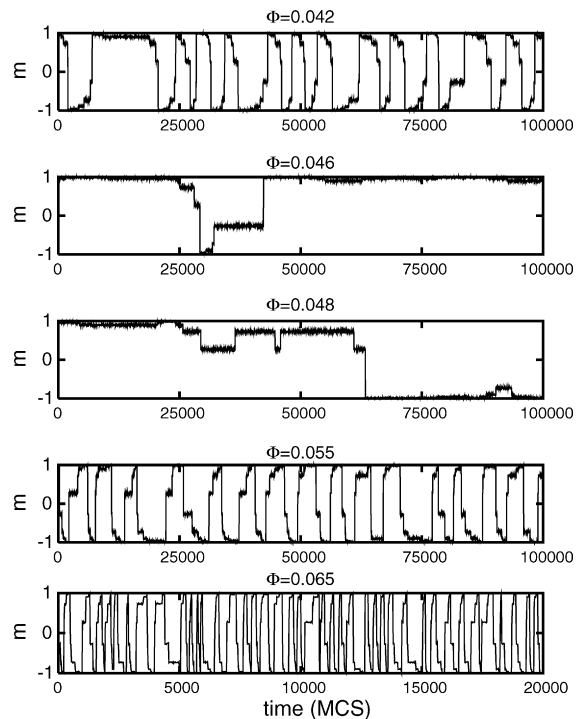


Fig. 2. Monte Carlo simulations show the effect of varying the synaptic noise parameter  $\Phi$ . The graph is depicted in absence of thermal fluctuations,  $T = 0$ , for a single stored pattern,  $M = 1$ , and  $N = 3600$  neurons. The resulting hopping shows dramatic variations of temporal scale and degree of complexity as one varies  $\Phi$ .

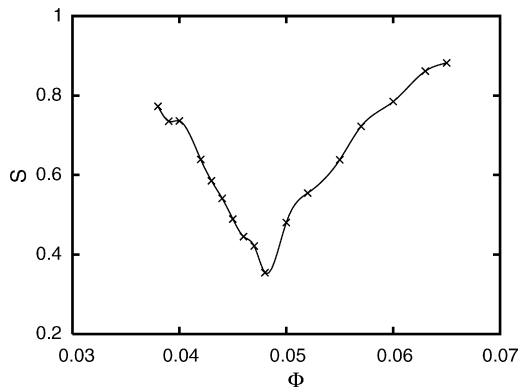


Fig. 3. The entropy function, as defined in the main text, for different time series obtained during Monte Carlo simulations of neural automata for different values of the synaptic noise parameter  $\Phi$ . Decreasing values of the entropy indicate a tendency towards regularization for complexity at time series. The graph reveals different regimes of chaoticity.

of the associated entropy will provide a quantitative description of the complexity of such hopping. Using standard fast Fourier transform algorithms, we computed the power spectra  $P(\eta)$ . The normalized probability  $p_\eta = P(\eta) / \sum_\eta P(\eta)$  then allows one to define a regular entropy as  $S \equiv -\sum_\eta p_\eta \log_2 p_\eta$ . This quantity has been used before to detect regularity out of chaotic activity in actual neurons (Varona et al., 2001). As a matter of fact,  $S > 0$  is to be associated with chaotic behavior while  $S = 0$  would correspond to periodic dynamics.

Fig. 3 depicts the entropy which results in our case as a function of  $\Phi$ . This shows a minimum, which corresponds to the smallest degeneration in the time series of Fig. 2 (second graph from the top). Decreasing  $S$  indicates a tendency to regularization or smaller chaoticity, while higher chaos and irregularity in the time series corresponds to larger values of  $S$ .

#### 4. Conclusions

We have introduced a class of hybrid neural automata with two main features. On one hand, these models provide a convenient arena to analyze the influence of fast synaptic noise on the retrieval process. On the other hand, they may describe a continuous transition from sequential, single-neuron updating to the case of Little dynamics or parallel updating as one varies the model parameter  $n$ . The synaptic noise is modelled in the automata trying to mimic recent observations, namely, the noise occurs in a short-time scale and conveniently couples to the neuron activity to induce synaptic depression. Depending on the intensity of this depression, the model exhibits a varied emergent behavior, including chaotic hopping between

the attractors. This results in a rather complex pattern of neural activity. Monitoring the entropy suggests how a fast noise might provide a mechanism to control chaos in living systems. The design of a mechanism in which noise intensity varies autonomously could be useful to the self-control of chaos. Notice in this respect that manipulating  $N$  in the model might be convenient for that purpose (Marro et al., 2006b). That is, two main cases follow together from the present analysis and some previous work (Cortes et al., 2006; Marro et al., 2006a): (1)  $n = 1$ , for which the system is sensible to an external stimulus, which may destabilize the attractor, but it does not exhibit autonomous hopping between attractors and (2)  $n > 1$ , for which hopping occurs autonomously, without the need for any external stimulus. In the latter case, as far as  $n < N$ , the parameter  $\Phi$  allows for a control of the hopping, while this always occurs at high frequency for  $n = N$ . For  $n \approx (2/3)N$ , the case for which we report some results here, the time the neuron activity stays at or nearby each attractor may be varied by tuning  $\Phi$ , as illustrated in Fig. 2.

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