Synchronous Behavior of Two Electrically Coupled Chaotic Model Neurons

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May 16, 1998

Abstract

The role that subcellular processes play in the firing properties of individual neurons and circuit activity is not yet fully understood. We have built a two-compartment model of stomatogastric motor neuron to study synchronization, chaotic and regularization phenomena observed in real circuits. It is observed experimentally that two stomatogastric bursting neurons with natural electrical coupling can synchronize the slow voltage oscillations but not the more rapid spiking activity. We have studied the mechanisms underlying these synchronized chaotic oscillations using two LP model neurons coupled electrically. The effect of changing the coupling conductance and the role of calcium dynamics in the generation of the chaos and the regularization of the bursting behavior are discussed in this paper.

1 Introduction

It is observed experimentally [Elson et al., 1998] that two chaotic stomatogastric neurons with natural electrical coupling can synchronize the slow waves of their bursting activity, while the fast actions potentials on the top of these waves remain unsynchronized. The dynamic clamp technique allows altering the strength of this electrical coupling between the two neurons. Thus, it is possible to observe the effect that different coupling conductances make on the synchronization and regularization of the chaotic behavior of the two neurons.

The role that subcellular processes play in generating the different firing patterns of individual neurons and circuit activity is not yet well understood. We have built a two-compartment model of the lobster's stomatogastric LP neuron to study chaotic and synchronization phenomena observed in the stomatogastric central pattern generator (CPG). The model used here to describe the synchronization activity incorporates six active ionic currents and a detailed description of calcium dynamics that includes calcium diffusion through the endoplasmic reticulum (ER).

Simulations using two model neurons coupled electrically reproduce the experimental recordings showing that the degree of synchronization between the two cells depends on the strength of the coupling. We also show in this paper that calcium concentration oscillations inside the endoplasmic reticulum may have an important role in generating and regulating the chaotic bursting activity of these neurons.

2 The Model

The model incorporates six active ionic currents distributed in two compartments (soma-neuropil and axon) depending on their slow/fast evolution (see figure 1). The detailed calcium dynamics includes Ca^{2+} storage in the endoplasmic reticulum which can generate chaotic activity in the bursting and spiking behavior of neurons with the concentration of inositol 1,4,5-triphosphate (IP_3) receptor as a control parameter.

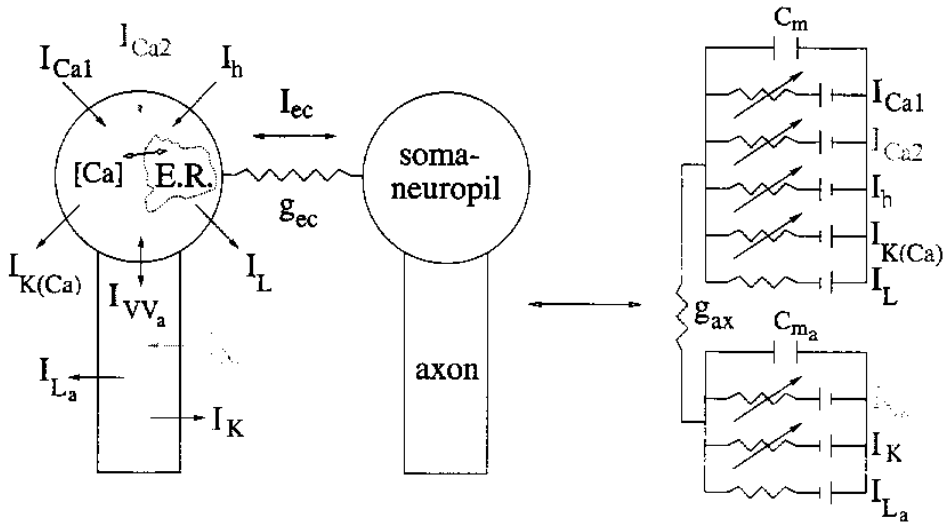

Figure 1: Model of two stomatogastric LP neurons coupled electrically. The model includes a detailed description of the $[Ca^{2+}]$ storage and diffusion in the endoplasmic reticulum of each neuron.

A complete description of the currents (conductance variables described by Hodgkin-Huxley and Goldman-Hodgkin-Katz formalisms) and the calcium dynamics can be found in [Falcke et al., 1998]. For all the simulations described in this paper, IP_3 concentration was set in the regime where the neurons fire chaotic bursts ($[IP_3] \approx 0.356\mu M$).

3 Results of Model Simulations

We have reproduced four sets of experiments carried out using the dynamic clamp technique which allows us to change the conductance of the electrical coupling between two real neurons [1]. Depending on the strength of the coupling, different collective behaviors are

[1]The conductance associated with the electrical coupling of the two neurons is $g_{ec} \approx 100 - 200nS$ in the natural state

observed: from independent chaotic bursting to synchronized chaotic bursting and antiphase regular activity.

3.1 Independent chaotic behavior

When the two model neurons are coupled with null or small coupling conductance ($g_{ec} \approx 0.001\mu S$) independent chaotic behavior is observed. The widths of the membrane potential bursts range from half a second to two seconds without periodicity (see figure 2 -top-). The number of spikes on the top of the slow waves also changes from burst to burst.

In figure 2 it can be noted that local maxima of cytoplasmic calcium concentration ($[Ca^{2+}]$) mark the end of the burst plateaus. Calcium concentration inside the endoplasmic reticulum ($[Ca^{2+}]_{er}$) evolves slowly modulating (in antiphase) $[Ca^{2+}]$ faster oscillations and influencing on the length of the plateaus. We will discuss the evolution of these three variables for the different coupling strengths.

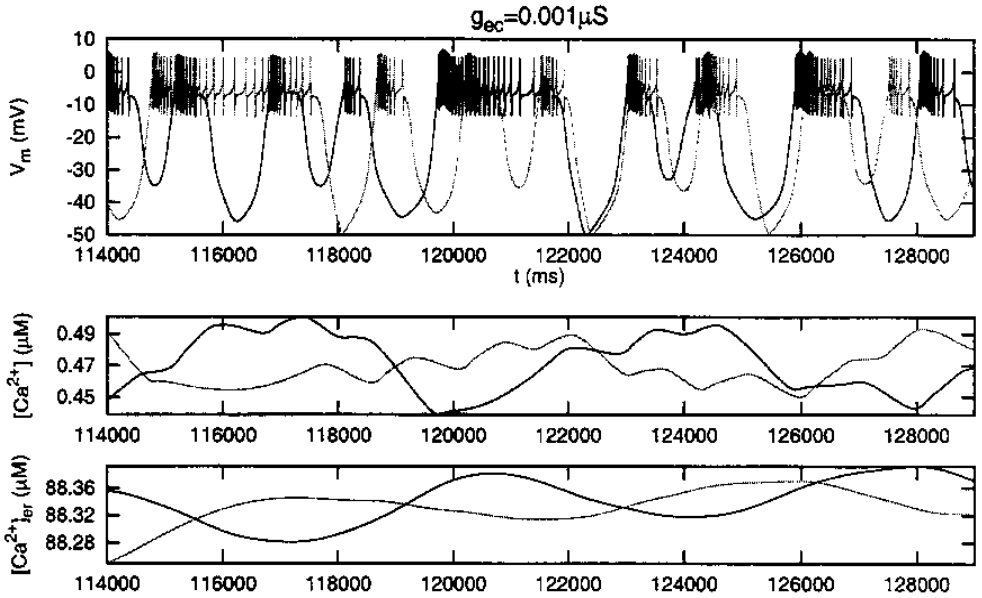

Figure 2: Independent chaotic bursting activity when the two LP model neurons are connected with a small coupling conductance ($g_{ec} = 0.001\mu S$). Activity for neuron one is plotted with a dark trace, neuron two is represented with a light trace. From top to bottom: membrane potential, cytoplasmic calcium concentration ($[Ca^{2+}]$), and calcium concentration inside the endoplasmic reticulum ($[Ca^{2+}]_{er}$).

3.2 Partial synchronization

A medium value ($g_{ec} \approx 0.05\mu S$) for the coupling conductance between the two model neurons causes burst synchronization but not spike synchronization (see figure 3 and also $V_1(t)$ vs $V_2(t)$ plot in figure 7).

This synchronization of the slow waves is the observed behavior for two real stomatogastric neurons interacting with their natural electrical coupling. Note that, in our simulations

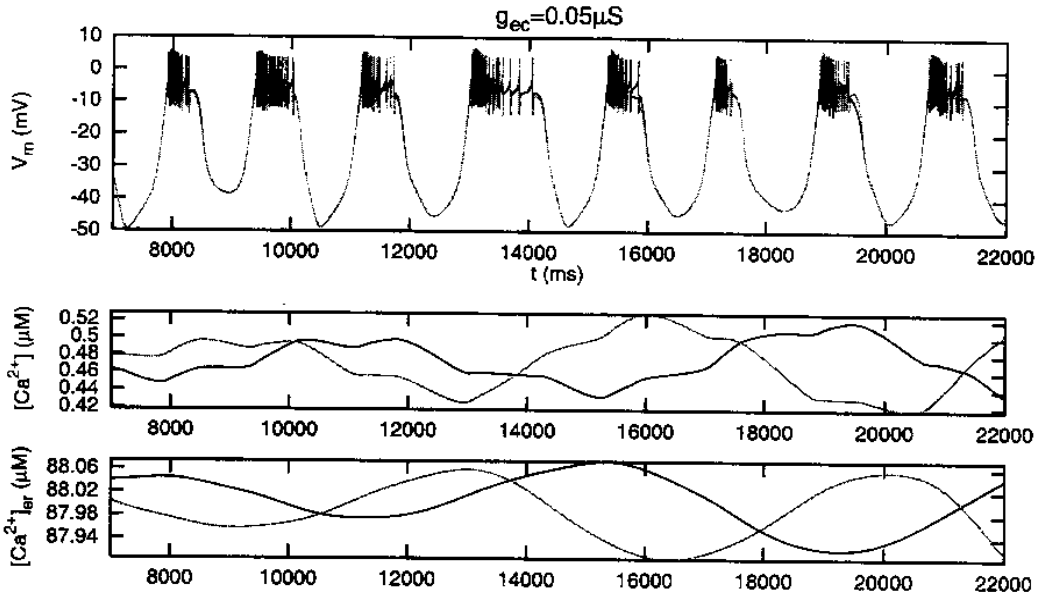

Figure 3: Burst synchronization when the two LP model neurons are connected with a medium value for the coupling conductance ($g_{ec} = 0.05\mu S$). Fast action potentials are not synchronized. See also figure 7.

for this conductivity range, $[Ca^{2+}]$ and $[Ca^{2+}]_{er}$ oscillate in a similar fashion for both neurons but they are not completely in phase, in spite of the existing burst synchronization for the membrane potential.

3.3 Total synchronization

Whenever the two model neurons are coupled with a high electrical conductance ($g_{ec} > 0.2\mu S$) complete synchronization (both for slow waves and fast action potentials) is observed (see figure 4 and also $V_1(t)$ vs $V_2(t)$ plot for the membrane potentials in figure 7). Note that $[Ca^{2+}]$ and $[Ca^{2+}]_{er}$ now oscillate more in phase between the two neurons than in the previous cases, but yet their trajectories do not overlap.

For all three cases discussed so far (small, medium and high positive coupling conductance) the bursting activity remains irregular no matter the degree of synchronization. Thus, synchronization occurs without regularization.

3.4 Antiphase synchronization and regularization

When the two neurons are coupled with a small negative conductance ($g_{ec} = -0.001\mu S$), thus inverting the sign of the current coming from the electrical coupling in both neurons, antiphase synchronization is observed (see figure 5) in the membrane potentials. Furthermore, the two neurons regulate their bursting behavior in the sense that the lengths of the burst are kept uniform. As can be noted in figure 5 (bottom plot), $[Ca^{2+}]_{er}$ remains nearly constant for the two neurons, while $[Ca^{2+}]$ oscillates regularly but in antiphase with respect to the other neuron. Note than in the previous cases $[Ca^{2+}]_{er}$ oscillated slowly with a wide amplitude (see bottom plots in figures 2, 3 and 4).

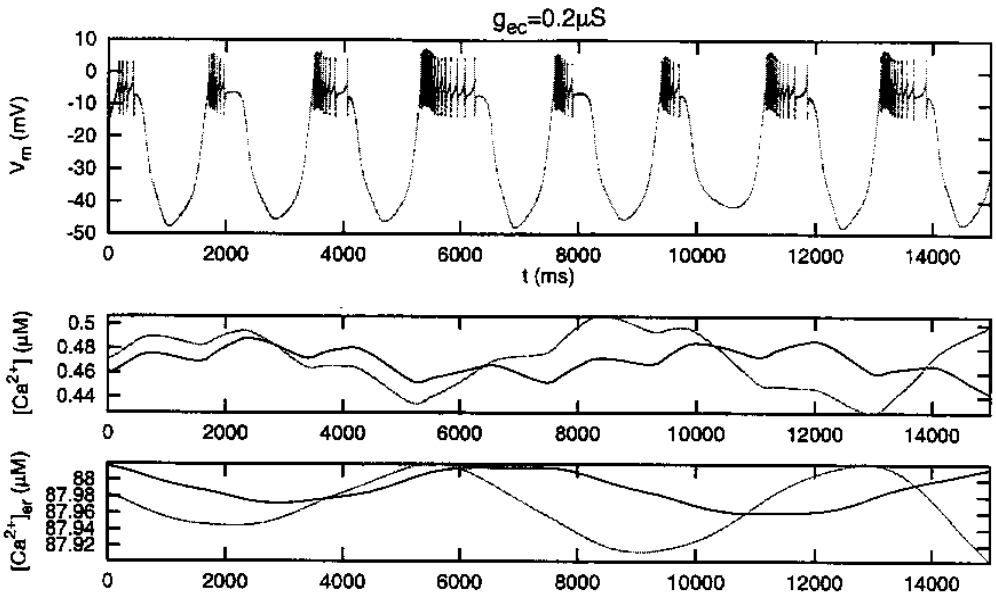

Figure 4: Burst and spike synchronization when the two LP model neurons are connected with a high value for the coupling conductance ($g_{ec} = 0.2\mu S$). Complete synchronization.

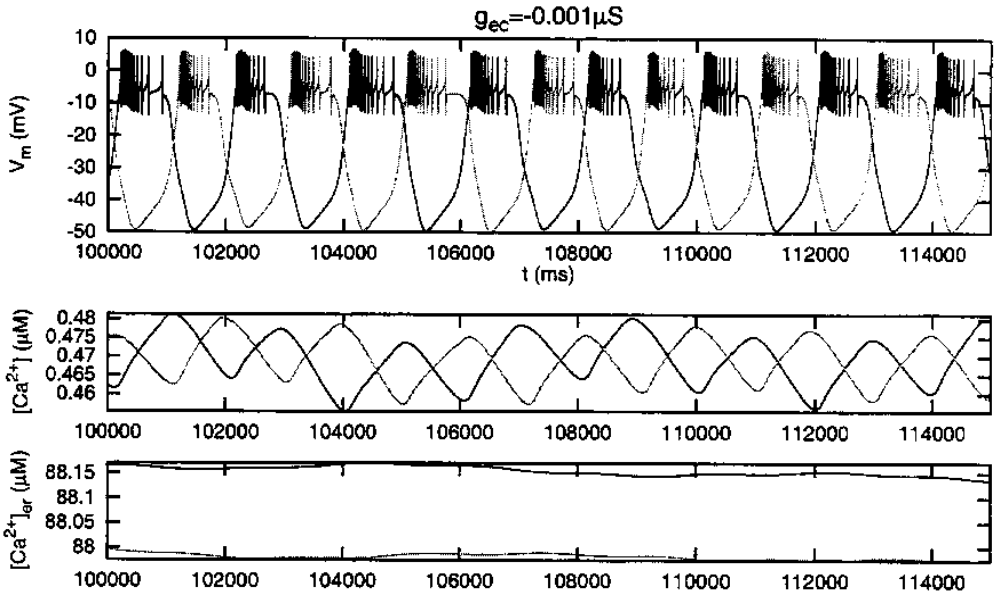

Figure 5: Antiphase behavior when the two neurons are coupled with small negative conductance inverting the sign of the current from the electrical coupling ($g_{ec} = -0.001\mu S$).

In our model, whenever $[Ca^{2+}]_{er}$ remains nearly constant in both neurons, burst widths are kept fixed. This can also be seen if we look at the effect of $[Ca^{2+}]_{er}$ dynamics in an isolated neuron. Chaotic behavior is sustained in the single neuron model whenever the

$[Ca^{2+}]_{er}$ oscillations are present (see figure 6, top plot). If we keep constant $[Ca^{2+}]_{er}$, the model produces regular bursting activity (bottom plot in figure 6).

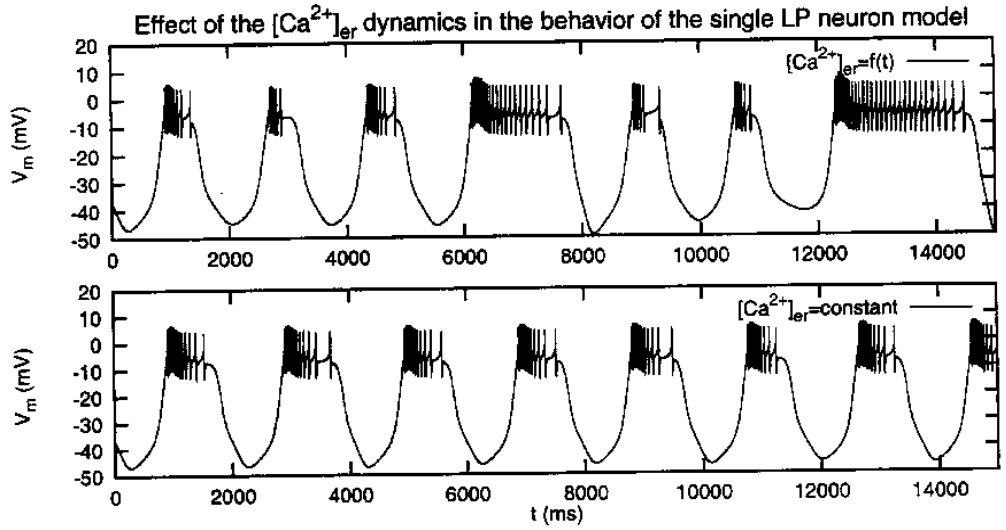

Figure 6: Top: irregular behavior in the isolated neuron when endoplasmic calcium diffusion is present in the model. Bottom: Regular bursting behavior when calcium concentration is kept constant inside the ER.

For a small negative electrical coupling, the calcium dynamics in the endoplasmic reticulum of both neurons is maintained constant since the fast oscillations of calcium in the cytoplasm are quick enough and regular enough to have no influence on the slower calcium diffusion through the endoplasmic reticulum membrane. And again, once the calcium concentration in the endoplasmic reticulum is kept constant, regularization of the chaotic behavior occurs.

All four cases discussed in this paper are summarized in figure 7. This figure displays $V_1(t)$, the membrane voltage in neuron 1, against $V_2(t)$ in neuron 2 showing the synchronization status for both slow and fast oscillations depending on the value of the electrical coupling conductance g_{ec} between the two neurons. g_{ec} is the only model parameter changed in these four simulations.

4 Conclusions and Discussion

Simulations using a model of two electrically coupled chaotic neurons show that: 1) the degree of membrane potential synchronization depends on the magnitude of the total coupling conductance, 2) values of the coupling conductance in the physiological range produce firing patterns similar to the experimental data (synchronized burst, but unsynchronized action potentials), 3) small negative coupling conductances induce anti-phase bursting behavior between the two neurons and regularization of burst width. The range of negative coupling conductances that sustain this behavior is smaller than the range of positive conductances that cause partial or total synchronization.

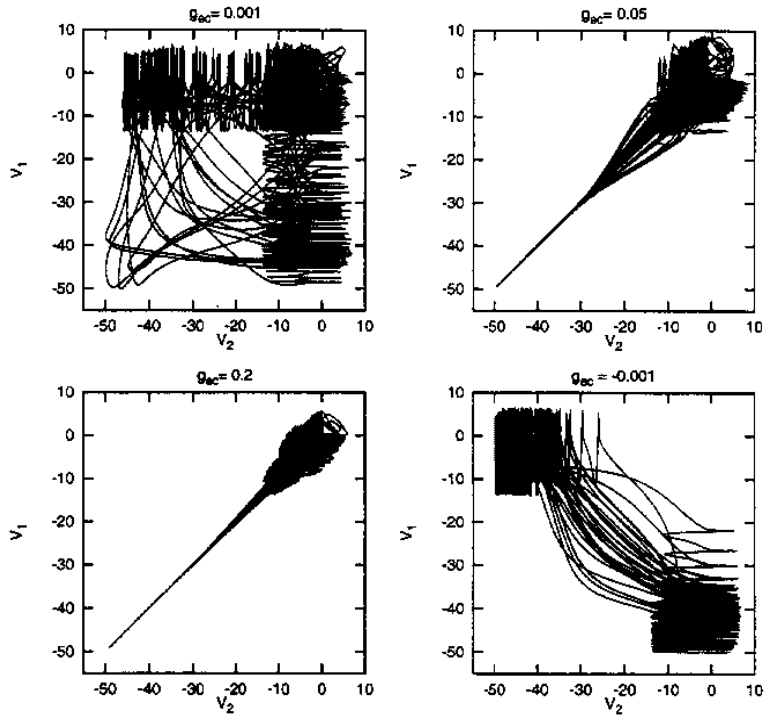

Figure 7: Portraits of membrane potential V_1 vs V_2 showing the synchronization phenomena for all four cases discussed previously. From left to right and top to bottom: independent bursting activity, slow wave synchronization, total synchronization and antiphase synchronization.

The synchronization of chaotic oscillations in biological systems has been traditionally studied using models with few degrees of freedom (see for example [Abarbanel et al., 1996]). These models reproduce qualitatively some of the synchronization phenomena observed in biological circuits. However, they can not be used to answer questions about the role of subcellular mechanisms involved in the generation and regularization of chaotic behavior.

The model of two electrically coupled LP neurons presented in this paper incorporates enough realistic variables to allow a direct comparison with experimental results. It can be used to test hypotheses related to the role played by subcellular processes in the origin and control of irregular behavior observed in real neurons and circuits. The essential question raised by the recognition of chaos in the individual neurons of CPGs is how an assembly of such neurons can cooperatively act to produce regular ordered signals to motor systems. In this paper we have explained how calcium oscillations inside the endoplasmic reticulum may regulate chaotic bursting activity between two electrically coupled neurons. In the model, as long as the luminal calcium concentration remains constant the bursts will be regular. This prediction that could be tested experimentally is supported by recent studies showing that the presence of calcium oscillations inside the endoplasmic reticulum has important effects in the cytoplasmic membrane potential oscillations [Li et al.,1995a, Li et al., 1995b, Li et al., 1997].

Our model makes some assumptions that still need to be tested. As a parameter in the model, IP_3 receptor concentration regulates the degree of chaos in the bursting activity of

the single neurons. Whether this receptor is present in the LP neuron is not yet known. However, there are several reports on the presence of IP_3 in the ER of many excitable cells and neurons [Otsu, 1990, Satoh, 1990, Walton, 1991].

Beyond reproducing quantitatively the features of the chaotic oscillations of LP cells, our model of two electrically coupled neurons provides an attractive starting point for the modeling of the entire pyloric CPG.

Acknowledgments

This work has been supported by grants from ORD (H. D. I. A.), DOE (H. D. I. A., M. I. R) and Universidad de Granada (J. J. T.).

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