CIRMIMNS

International meeting on numerical semigroups with applications

Levico Terme 2016

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July, 4-8
Levico Terme, Italy

CIRM
CENTRO INTERNAZIONALE PER LA RICERCA MATEMATICA

UNIVERSITÀ degli STUDI di CATANIA

List of abstracts

Abdallah Assi *Université d'Angers*

CANONICAL BASES FOR ONE DIMENSIONAL ALGEBRAS AND MODULES

Let $\mathbf{K}$ be a field and let $x_1(t), \ldots, x_n(t) \in \mathbf{K}[t]$. Let $\mathbf{A} = \mathbf{K}[x_1(t), \ldots, x_n(t)]$ be the $\mathbf{K}$-algebra generated by $\{x_1(t), \ldots, x_n(t)\}$. Given a nonzero polynomial $x(t) \in \mathbf{K}[t]$, we denote by $d(x(t))$ the degree in t of $x(t)$. If the length of $\mathbf{K}[t]/\mathbf{A}$ is finite, then the set $S = d(\mathbf{A}) = \{d(x(t)), x(t) \in \mathbf{A} - 0\}$ is a numerical semigroup and an algorithm for computing a system of elements $y_1(t), \ldots, y_r(t) \in \mathbf{A}$ such that $S = \langle d(y_1(t)), \ldots, d(y_r(t)) \rangle$ is given in [1]. Let $X_1(t), \ldots, X_s(t) \in \mathbf{K}[t]$ and consider the $\mathbf{A}$-module $\mathbf{M} = X_1(t)\mathbf{A} + \ldots + X_s(t)\mathbf{A}$. In this talk we show how to calculate a system of generators $Y_1(t), \ldots, Y_l(t)$ of $\mathbf{M}$ such that $d(Y_1(t)), \ldots, d(Y_l(t))$ is a system of generators of the ideal $I = \cup_{x(t) \in \mathbf{M}-0}(d(x(t)) + S)$ of S. We then focus on the case where $X_i(t) = x_i'(t)$ for all $i \in \{1, \ldots, n\}$ and we give some applications to the study of singularities of plane algebraic curves parametrized by polynomials.

References

[AGSM] Abdallah Assi, Pedro A. García-Sánchez, Vicenzo Micale, Bases of subalgebras of $K[\![x]\!]$ and $K[x]$, arXiv:1412.4089

Valentina Barucci *"Sapienza", Università di Roma*

ALMOST SYMMETRIC PROPERTY IN SEMIGROUPS AND RINGS

Almost symmetric numerical semigroups and almost Gorenstein one-dimensional analytically unramified rings were introduced many years ago by V. Barucci and R. Fröberg. After that, this concept has been studied by several authors and it has been generalized in different directions. It has been also used in various contexts and recently it was used by A. Oneto, F. Strazzanti and G. Tamone to give examples of decreasing Hilbert functions for the associated graded ring to a one-dimensional Gorenstein ring.
A survey of this concept will be given in the talk, looking at its stability with respect to certain constructions and mentioning some open problems about it.

Matheus Bernardini *Universidade de Campinas/Instituto Federal de São Paulo*

COUNTING NUMERICAL SEMIGROUPS OF A GIVEN GENUS BY USING γ-HYPERELLIPTIC SEMIGROUPS

Joint work with Fernando Torres.
Given a nonnegative integer g, we denote the number of numerical semigroups of genus g by $N(g)$. In 2008, Maria Bras-Amorós conjectured that

- $N(g-2) + N(g-1) \leq N(g)$,for all $g \geq 2$;
- $\lim_{g\to\infty} \frac{N(g)}{N(g-1)} = \frac{1+\sqrt{5}}{2}$;
- $\lim_{g\to\infty} \frac{N(g)}{N(g-1)+N(g-2)} = 1$.

In 2013, Alex Zhai proved last two statements, but the first one still remains as an open problem. Actually, even the inequality

$$N(g-1) \leq N(g),$$

which is true for all sufficiently large g, by the previous second statement, is not known if it holds for all g.
In this talk we introduce γ-hyperelliptic semigroups and use it to count numerical semigroups by genus.

Maria Bras-Amorós *Universitat Rovira i Virgili*

SEEDS OF NUMERICAL SEMIGROUPS

Joint work with Julio Fernández-González.
For a numerical semigroup, we introduce the concept of *seeds* as a generalization of those generators which are larger than the Frobenius number. This new concept allows to explore the semigroup tree in an alternative efficient way, since the seeds of each descendant can be easily obtained from the seeds of its parent. The talk is devoted to present the results which are related to this approach, leading to a new algorithm for computing the number of semigroups of a given genus.

Mircea Cimpoeaş *Simion Stoilow Institute of Mathematics of the Romanian Academy*

On intersection of complete intersection ideals

Joint work with Dumitru Stamate

Let K be a field and $\mathsf{S} = \mathsf{K}[x_1, \ldots, x_r]$ be the polynomial ring in the variables $x_1, ..., x_r$. An ideal $I \subset S$ is called a complete intersection (CI for short) if it is minimally generated by height(I) elements. This is a strong condition which is rarely preserved by taking intersections of such ideals. In this article we exhibit several infinite families of CI toric ideals in S such that any intersection of ideals in the same family is again a CI.

For an affine semigroup $H \subseteq \mathbb{N}$, the semigroup ring $K[H]$ is the subalgebra in $K[t]$ generated by the monomials t^h, $h \in H$. If $a_1, \ldots, a_r$ generate H minimally, we define the toric ideal I_H as the kernel of the K-algebra map $\varphi : S \to K[H]$ letting $\varphi(x_i) = t^{a_i}$. Consider the list of integers $a = a_1 < a_2 < \cdots < a_r$. We denote $I(a)$ the kernel of the K-algebra map $\varphi : S \to K[\langle a \rangle]$ letting $\varphi(x_i) = t^{a_i}$, where we let $\langle a \rangle$ be the semigroup generated by $a_1, \ldots, a_r$. If they generate $\langle a \rangle$ minimally, we call $I(a)$ the toric ideal of $\langle a \rangle$. If k is any integer we let $a + k = (a_1 + k, \ldots, ar + k)$.

The study of properties of the family of ideals $\{I(a + k)\}_{k \geq 0}$ is a recent topic of interest. We use Gröbner bases techniques to derive new information about intersections of such CI ideals. Our main result, is where we prove that for a fixed a intersections of CI-ideals $I(a + k)$ with large enough shifts k produce another CI ideal.

Alexandru Ciolan and Pieter Moree *Bonn Univeristity/Max-Planck Institut für Mathematik*

Cyclotomic Numerical Semigroups

Joint work with P. A. García-Sánchez and P. Moree

Given a numerical semigroup S we can consider its associated *semigroup polynomial*

$$P_S(x) = (1 - x) \sum_{s \in S} x^s = 1 + (x - 1) \sum_{x \notin S} x^s.$$

A natural question is to what extent the properties of S are reflected in properties of $P_S(x)$. E.g., if all the roots of $P_S(x)$ are on the unit circle, what can we say about S (which we then call *a cyclotomic numerical semigroup*)? We discuss evidence for our conjecture with Pedro García-Sánchez that S is cyclotomic if and only if S is a complete intersection numerical semigroup.

Our talk recapitulates some material from a joint talk given by the speakers in Cortona. In addition some new results will be presented.

Manuel Delgado *Universidade do Porto*

On a question of Eliahou and a conjecture of Wilf

In a recent work Eliahou associated to a numerical semigroup S a constant $\mathcal{E}(S)$ and proved that numerical semigroups for which the associated constant is non negative satisfy Wilf's conjecture. The search for counterexamples for the conjecture of Wilf is therefore reduced to semigroups which have an associated negative Eliahou constant. Eliahou mentioned 5 numerical semigroups, which satisfy Wilf's conjecture and whose Eliahou constant is -1. The examples were computed by Fromentin who observed that these are the only ones with negative Eliahou constant among the over 10^{13} numerical semigroups of genus up to 60. We show that for any integer n there are infinitely many numerical semigroups S such that $\mathcal{E}(S) = n$, by explicitly giving families of such semigroups. We prove that all the semigroups in these families satisfy Wilf's conjecture, providing not previously known examples of semigroups for which Wilf's conjecture holds.

Shalom Eliahou *Université du Littoral Côte d'Opale*

Some new results on Wilf's conjecture

Let S be a numerical semigroup with conductor c and embedding dimension e. Let L be the left part of S, i.e. the subset of those elements of S which are smaller than c. In 1978, Wilf conjectured that the cardinality of L should be bounded below by c/e. In this talk, we exhibit new families of numerical semigroups that satisfy Wilf's conjecture.

Ignacio Farrán Martín *Unversidad de Valladolid*

Feng-Rao distances in Arf and inductive semigroups

Joint work with P. A. García-Sánchez
he parameters of one-point algebraic geometry codes are related to a certain Weierstrass semigroup, by means of the so-called Feng-Rao distances. For the classical (unidimensional) distance, strong results are known, but the problem becomes hard for the mutidimensional case. The asymptotical behaviour of such generalized Feng-Rao distances is related by the so-called Feng-Rao number. This work is focused on particular cases of numerical semigroups, like Arf semigroups and specially inductive semigroups, which are connected to towers of asymptotically good sequences of error-correcting codes.

Leonid Fel *Israel Institute of Technology*

Numerical semigroups generated by squares, cubes and quartics of three consecutive integers

We derive the polynomial representations for minimal relations of generating set of numerical semigroups $R_n^k = \langle (n-1)^k, n^k, (n+1)^k \rangle$, $k = 2, 3, 4$.
We find polynomial representations for degrees of syzygies in the Hilbert series $H(z, R_n^k)$ of these semigroups, their Frobenius numbers $F(R_n^k)$ and genera $G(R_n^k)$.

Ralf Fröberg *Stockholms Universitet*

Primary decomposition of powers of prime ideals for numerical semigroups

A numerical semigroup ring is isomorphic to a polynomial ring modulo a prime ideal. We consider the problem of primary decomposition of powers of such prime ideals.

Lenny Fukshansky *Claremont McKenna College, California*

Generalized Frobenius numbers: bounds and average behavior via geometric techniques

Let $n > 1$ be an integer, and let $1 < a_1 < \cdots < a_n$ be relatively prime integers. For a non-negative integer s, the s-Frobenius number of this n-tuple is defined to be the largest positive integer that has precisely s distinct representations as a linear combination of $a_1, \ldots, a_n$with non-negative integer coefficients. Hence the classical Frobenius number can be thought of as the 0-Frobenius number. I will demonstrate an approach, using techniques from discrete geometry, that allows to obtain bounds and average values for these generalized Frobenius numbers.

Evelia R. García Barroso *Universidad de La Laguna*

Milnor number and semigroup of branches in positive characteristic

Let $\mu(f)$ (respectively, $c(f)$) be the Milnor number (respectively, the conductor of the semigroup) of an irreducible power series $f \in \mathbf{K}[\![x, y]\!]$, where $\mathbf{K}$ is an algebraically closed field of characteristic $p \geq 0$. It is well-known that $\mu(f) \geq c(f)$. We give necessary and sufficient conditions for the equality $\mu(f) = c(f)$ in terms of the semigroup associated with f, provided that $p > \mathrm{ord} f$.
This talk is based on the results of [GP].

References

[GP] E.R. García Barroso and A. Płoski, The Milnor number of plane irreducible singularities in positive characteristic, Bull. London Math. Soc. **48** (2016), 94-98.

Juan Ignacio García-García *Universidad de Cádiz*

Asymptotic ω-primality of finitely generated cancelative monoids

Joint work with D. Marín-Aragón and A. Vigneron-Tenorio
For every element of an atomic monoid, its ω-primality is defined as follows:

Let S be an atomic monoid with set of units $S^\times$ and set of irreducibles $\mathcal{A}(S)$. For $s \in S \setminus S^\times$, we define $\omega(x) = n$ if n is the smallest positive integer with the property that whenever $x|(a_1 + \cdots + a_t)$, where each $a_i \in \mathcal{A}(S)$, there is a $T \subseteq \{1, 2, \ldots, t\}$ with $|T| \leq n$ such that $x| \sum_{k \in T} a_k$. If no such n exists, then $\omega(s) = \infty$. For $x \in S^\times$, we define $\omega(x) = 0$.

Every commutative, cancellative, reduced and finitely generated monoid is atomic, therefore the ω-primality is well-defined for their elements.

Associated with the ω-primality there is its asymptotic version, the asymptotic ω-primality or $\overline{\omega}$-primality. In this work we study this concept, we give a geometric interpretation, and we present an algorithm to compute it.

Felix Gotti *UC Berkeley*

PUISEUX MONOIDS AND THEIR ATOMIC STRUCTURE

In this talk, we will discuss different aspects of the atomic structure of the family of Puiseux monoids, i.e, additive submonoids of the rational numbers. Puiseux monoids are a natural generalization of numerical semigroups. Unlike the family of numerical semigroups, that one of Puiseux monoids contains non-finitely generated and non-atomic representatives. We plan to describe subfamilies of Puiseux monoids containing no atoms. Also, we will present several characterization criteria for Puiseux monoids to be atomic.

Lorenzo Guerrieri *Università di Catania*

LEFSCHETZ PROPERTIES OF GORENSTEIN GRADED ALGEBRAS ASSOCIATED TO THE APÉRY SET OF A NUMERICAL SEMIGROUP

Lefschetz Properties of Standard Graded Artinian algebras have been much studied in the last twenty years because of their many connections with other areas of mathematics. It is still unknown if both these Properties hold for all the Algebras of Codimension 3. In this work we show some methods to understand if Lefschetz Properties hold for a Gorenstein Graded Ring associated to the Apery Set of a Numerical Semigroup.

Benjamín Alarcón Heredia *Universidade Nova de Lisboa*

APÉRY SETS AND FENG-RAO NUMBERS OVER TELESCOPIC NUMERICAL SEMIGROUPS

Joint work with P. A. García-Sánchez and M. J. Leamer

Feng-Rao numbers are invariants of numerical semigroups, and they have an interpretation in AG codes. In this talk I show that the second Feng-Rao number of any telescopic numerical semigroup is the multiplicity of the semigroup.

I-Chiau Huang *Academia Sinica*

NUMERICAL SEMIGROUP ALGEBRAS

We study a numerical semigroup ring as an algebra over another numerical semigroup ring. As their tangent cones examined before, singularities are investigated from a relative point of view. Topics of interest include Cohen-Macaulyness, Gorensteiness and complete intersection.

Bogdan Ichim *Simion Stoilow Institute of Mathematics of the Romanian Academy*

ON THE SCORE SHEETS OF A ROUND-ROBIN FOOTBALL TOURNAMENT

The set of (ordered) score sheets of a round-robin football tournament played between n teams together with the pointwise addition has the structure of an affine monoid.

Using Normaliz we study the most important invariants of this monoid, namely the Hilbert basis, the multiplicity, the Hilbert series and the Hilbert function.

Raheleh Jafari *Kharazmi University*

Huneke-Wiegand Conjecture for Numerical Semigroup Rings

Joint work in progress with Pedro A. García-Sánchez and Micah Leamer.

Let R be a one-dimensional Gorenstein domain and M be a finitely generated R-module which is not projective. It is a conjecture by Huneke and Wiegand (1994), that $M \otimes \mathrm{Hom}(M, R)$ has non-zero torsion elements. This conjecture is widely open even in the case where R is a numerical semigroup ring.

A survey of the numerical semigroup interpretation of the conjecture will be given in the talk, looking at equivalent translations, several ideas to deal with the problem and known classes of numerical semigroups that fulfill the conjecture.

Halil Ibrahim Karakaş *Baskent University*

Arf numerical semigroups with multiplicity less than or equal to six

The aim of this work is to exhibit all Arf numerical semigroups with multiplicity less than or equal to 6 and any conductor. The results obtained in this work will provide Arf numerical semigroups with multiplicity 3, 4, 5 or 6 of any conductor.

Philipp Korell *Technical University of Kaiserslautern*

Duality on value semigroups

The semigroup of values is a classical combinatorial invariant associated to a curve singularity. Due to Lejeune-Jalabert and Zariski, it determines the topological type of plane complex curves. As observed by Kunz in the irreducible case and generalized by Delgado to the reducible case, the Gorenstein property of a curve singularity is equivalent to a symmetry of its value semigroup. D'Anna reinterpreted this symmetry condition to characterize canonical ideals by their value semigroup ideals. Value semigroup ideals of fractional ideals satisfy certain axioms defining the class of so-called good semigroup ideals. Unifying the work of D'Anna and Pol we relate canonical ideals to canonical semigroup ideals, and we establish a purely combinatorial duality on good semigroup ideals which is compatible with the duality on fractional ideals.

Ernst Kunz *Universität Regensburg*

Numerical semigroups with nice properties

Let p, q be coprime integers with $3 \leq p < q$. In the lecture the semigroups H in the set

$$\mathbf{R}(p, q) := \{H \mid \langle p, q \rangle \subseteq H \subseteq \langle p, q, r \rangle\}$$

will be discussed where

$$r := \begin{cases} \frac{p}{2} & p \text{ even}, \\ \frac{q}{2} & q \text{ even}, \\ \frac{p+q}{2} & p \text{ and } q \text{ odd}. \end{cases}$$

These semigroups have nice properties.

a) There is a list for $\mathbf{R}(p, q)$ in which the semigroups are given by their minimal generators.

b) In terms of the minimal generators there are simple formulas for the genus, the Frobenius number, the deviation and the type.

c) There are strong relations between embedding dimension, deviation and type.

d) Many counting problems have an easy solution.

e) Wilf's question has a positive answer for the $H \in \mathbf{R}(p, q)$.

f) There is a "canonical" system of minimal relations in terms of the minimal generators.

g) There exists an algorithm to decide whether a numerical semigroup, given by a system of generators, belongs to $\bigcup_{p,q} \mathbf{R}(p, q)$.

David Llena *Universidad de Almería*

Delta sets for numerical semigroups with embedding dimension three

Joint work with P. A. García-Sánchez and A. Moscariello

Delta Sets measure distances between consecutive lengths for different factorizations of an element in a monoid or integral domain. These sets measure how far is such a moinoid (or domain) from being half-factorial (for every element, all its factorizations have the same number of irreducible elements). Our purpose, in this talk, is to show the relationship between Delta sets for numerical semigroups with embedding dimension three, and Euclid's algorithm used to compute the greatest common divisor for two integers.

María Ángeles Moreno-Frías *Universidad de Cádiz*

On divisor-closed submonoids and minimal distances in finitely generated monoids

Joint work with J. I. García-García and D. Marín-Aragón.

The aim of this talk is to provide a method for computing the set of minimal distances of a finitely generated cancellative monoid by studying the structure of its set of divisor-closed submonoids. Using the Archimedean components of the monoid, we also show how to obtain the set of divisor-closed submonoids. This set has a structure of complete finite lattice. For finitely generated submonoids of $\mathbb{N}^n$ (also known as affine semigroups), we give a geometrical characterization of such submonoids in terms of its cone.

For an affine semigroup H the lattice of its Archimedean components, the lattice of its divisor-closed submonoids and the lattice of the faces of the polyhedral cone generated by H are isomorphic. Finally, we use our results to give an algorithm for computing the set of minimal distance of finitely generated cancellative monoids.

Julio J. Moyano-Fernandez *University Jaume I of Castellón*

Linear inequalities for the Hilbert depth of graded modules over polynomial rings

This is part of a joint work with Lukas Katthän and Jan Uliczka.

Let $R = K[X, Y]$ be a polynomial ring endowed with the $\mathbb{Z}$-grading given by $a := \deg(X)$, $b = \deg(Y)$, with $\gcd(a, b) = 1$.

In previous editions of this meeting we have reported on a characterization for positive Hilbert depth of a finitely generated graded module over $K[X, Y]$.

This characterization is based on some inequalities involving the numerical semigroup $\langle a, b\rangle$, but their meaning in merely commutative algebra terms was unknown to us.

In this talk we will present an algebraic interpretation of these inequalities, even in a more general setting.

Ignacio Ojeda *Universidad de Extremadura*

On critical binomial ideals

This is a joint work with D. Llena and P.A. García-Sánchez.

In this talk, a family of binomial ideals defining monomial curves in the $n-$dimensional affine space determined by n hypersurfaces of the form $x_i^{c_i} - \mathbf{x}^{\mathbf{u}} \in \mathbb{K}[x_1, \ldots, x_n]$ will be introduced. We prove that for each binomial ideal in that family, there exists finitely generated submonoid of $\mathbb{Z} \oplus T$, where T denotes finite commutative group. When $T = 0$, the submonoid is a numerical semigroup; in this case, we will give formulas for some classical invariants such as its genus, type and the Fröbenius number.

This research is partially supported by the projects MTM2015-65764-C3-1 and MTM2014-55367-P from the National Plan I+D+I and by Junta de Extremadura (FEDER funds).

Aureliano M. Robles-Pérez *Universidad de Granada*

Common behaviours in families of numerical semigroups: types of Frobenius varieties

Joint work with J.C. Rosales.

In order to collect common properties of several families of numerical semigroups, the Frobenius varieties were introduced by Rosales (Houston J. Math. 34 (2008), 339-348). Since some relevant families were out of this definition, two new concepts were given. Namely, the m-varieties (Bras-Amorós, García-Sánchez, and

Vico-Oton, Internat. J. Algebra Comput. 23 (2013), 1469-1483) and pseudo-varieties (Robles-Pérez and Rosales, Ann. Mat. Pura Appl. 194 (2015), 275-287). However, there are interesting families which are still out of these definitions. Therefore, now we present the concept of R-variety (or Frobenius restricted variety). In this talk we revise the relationships among these notions. In particular, we will show the tree structure that arises within them.
This work is supported by the project MTM2014-55367-P, which is funded by Ministerio de Economía y Competitividad and Fondo Europeo de Desarrollo Regional FEDER, and by the Junta de Andalucía Grant Number FQM-343.

Mesut Şahin *Hacettepe University*

On Pseudo Symmetric Monomial Curves

We talk about monomial curves, toric ideals and monomial algebras associated to 4-generated pseudo symmetric numerical semigroups. Namely, we determine indispensable binomials of these toric ideals, give a characterization for these monomial algebras to have strongly indispensable minimal graded free resolutions. We also characterize when the tangent cones of these monomial curves at the origin are Cohen-Macaulay. This will help solving Sally's conjecture about non-decreasing behaviour of the Hilbert function.

Alessio Sammartano *Purdue University*

0-th local cohomology of tangent cones of monomial space curves

Let $C \subseteq \mathbb{A}^n$ be an affine monomial curve and T its tangent cone at 0. The Cohen-Macaulayness of the coordinate ring of T is among the most important properties of the singularity of C. It is desirable to have measures of the failure of T to be Cohen-Macaulay. We will discuss two such measures, focusing mainly on the case of space curves: the length of the 0-th local cohomology module and the k-Buchsbaum property.

Indranath Sengupta *Indian Institute of Technology Gandhinagar*

Unboundedness of Betti numbers for certain toric ideals

H. Bresinsky [B1] produced examples of monomial curves (numerical semigroup rings) given by the parametrization $x_1 = t^{n_1}$, $x_2 = t^{n_2}$, $x_3 = t^{n_3}$, $x_4 = t^{n_4}$, such that $n_1 = a(a+1)$, $n_2 = (a+1)(a-1)$, $n_3 = a(a+1)+(a-1)$, $n_4 = a(a-1)$, where $a \geq 4$ is an even integer. This class of numerical semigroup rings require arbitrarily large numbers of polynomials to generate their defining toric ideals. Therefore, the first Betti number, which is nothing but the minimal number of generators for the defining toric ideal of the monomial curve is unbounded above. We would like to understand the unboundedness of the second Betti number as well. We consider another sequence of coprime integers $a, a+d, \ldots, a+kd$, b, $b+d$, such that a, b, d are positive integers with $\gcd(a, d) = \gcd(b, d) = 1$. We first try to see the Apery set of the numerical semigroup generated these numbers in order to investigate the boundedness of the minimal number of generators or the first Betti number of their defining ideal. We also study their Markov complexity, which may throw light on this study.

References

[B1] H. Bresinsky, On prime ideals with generic zero $x_i = t^{n_i}$. *Proc. Amer. Math. Soc.* 47(2) (1975) 329–332.
[B2] H. Bresinsky, Monomial Gorenstein Ideals. *Manuscripta Math.* 29 (1979) 159-181.

Dario Spirito *Università di Roma Tre*

Star operations on numerical semigroups

Star operations are closure operations that are classically defined on the set of fractional ideals of an integral domain; their definition can be extended in a natural way to cancellative semigroups. In this talk, we study the set Star(S) of star operations on a numerical semigroup S, focusing in particular on its cardinality. More precisely, we are interested in determining, for a given positive integer n, the numerical semigroups with exactly n star operations: we show that, if $n > 1$, the number of such semigroups is finite, and, by estimating |Star(S)| through the invariants of S, we obtain a bound on the number of numerical semigroups S with $2 \leq$ |Star(S)| $\leq n$. We then show how these result can be generalized to the ring case and how to determine explicitly |Star(S)| when S has multiplicity 3.

Dumitru Stamate *University of Bucharest*

QUADRATIC NUMERICAL SEMIGROUPS AND THE KOSZUL PROPERTY

Joint work with Juergen Herzog.
Let H be a numerical semigroup. We give effective bounds for the multiplicity $e(H)$ when the tangent cone $gr(K[H])$ is defined by quadrics.
We conjecture that not all the values in the range are possible, and this correlates to a series of conjectures of Eisenbud, Green and Harris on a generalized Cayley-Bacharach statement. We classify Koszul complete intersection semigroups in terms of gluings.
We describe the Koszul property for several classes of numerical semigroups and we study the relationship with the Cohen-Macaulay property of the $gr(K[H])$.

Klara Stokes *University of Skövde*

WEIERSTRASS SEMIGROUPS OF COVERINGS OF GRAPHS

The Weierstrass semigroups at ramification points of coverings of curves have been studied with great interest for many years. Recently, an analogous theory of divisors for graphs has been developed. In this talk I will discuss some properties of numerical semigroups appearing in this way.

Francesco Strazzanti *Università di Pisa*

SYMMETRIC NUMERICAL SEMIGROUPS WITH DECREASING HILBERT FUNCTION

Joint work with A. Oneto and G. Tamone.
Let R be a one-dimensional Cohen-Macaulay local ring and let $\mathfrak{m}$ be the unique maximal ideal of R. By definition the Hilbert function H_R of R is that of its associated graded ring $\bigoplus_{i\geq 0} \mathfrak{m}^i/\mathfrak{m}^{i+1}$. The study of H_R is a classical topic in local algebra, for example when R is Gorenstein, M.E. Rossi asked if it has always non-decreasing Hilbert function. As a matter of fact, in the last ten years several authors found families of one-dimensional Gorenstein local rings that has non-decreasing Hilbert function, especially in the case of numerical semigroup rings. In particular in the latter case, if S is a numerical semigroup and $M = S \setminus \{0\}$, the Hilbert function of S is the map $H_S : \mathbb{N} \to \mathbb{N}$ defined as $H_S(n) = |nM \setminus (n+1)M|$ and the Rossi's problem simply means that a symmetric numerical semigroup has non-decreasing Hilbert function. In this talk we show that this is not true and, given $m, h \geq 1$ with $h \notin \{14 + 22k, 35 + 46k \mid k \in \mathbb{N}\}$, we explain how to construct explicit symmetric numerical semigroups such that $H_S(h) - H_S(h+1) > m$. Moreover we give other interesting examples and show how this construction can be generalized to find one-dimensional Gorenstein local rings with decreasing Hilbert function that are not numerical semigroup rings.

Grazia Tamone *Università di Genova*

HILBERT FUNCTION OF NUMERICAL SEMIGROUP RINGS

In this talk we deal with the behaviour of the Hilbert function of local one-dimensional semigroup rings: we show some recent results regarding classes of rings $R = k[\![S]\!]$, S semigroup, with decreasing Hilbert function H_R. In particular for semigroup rings of multiplicity e and embedding dimension ν verifying $\nu+3 \leq e \leq \nu+4$, the decrease of H_R gives an explicit description of the Apéry set of S. For $e = \nu+3$, the minimal multiplicity is $e = 13$: in this case the found conditions allow to know exactly the structure of the associated semigroups. Another consequence is that the Gorenstein local semigroup rings R with $e \leq \nu + 4$ have a non decreasing Hilbert function.

Apostolos Thoma *University of Ioannina*

ON THE MARKOV COMPLEXITY OF NUMERICAL SEMIGROUPS

Let $A = \{n_1, \cdots, n_s\}$ be a set of positive integers and S the numerical semigroup generated by A.
We denote by $A^{(r)}$ the r-th Lawrence lifting of A. We identify an element of $\ker_Z(A^{(r)})$ with an $r \times s$ matrix: each row of this matrix corresponds to an element of $\ker_Z(A)$ and the sum of its rows is zero.
The type of an element of $\ker_Z(A^{(r)})$ is the number of nonzero rows of this matrix.
The Markov complexity is the largest type of any vector in the universal Markov basis of $\ker_Z(A^{(r)})$ as r

varies.
We will discuss the Markov complexity of certain numerical semigroups.

Simone Ugolini *Università di Trento*

ON NUMERICAL SEMIGROUPS CLOSED WITH RESPECT TO THE ACTION OF AFFINE MAPS

In this talk I will present a recent work of mine [U] about numerical semigroups containing a given positive integer and closed with respect to the action of an affine map.
If we take a positive integer a and a non-negative integer b, then we can define the map

$$\begin{aligned} \theta_{a,b} : \mathbb{N} &\rightarrow \mathbb{N} \\ x &\mapsto ax+b. \end{aligned}$$

We say that a subsemigroup G of $(\mathbb{N},+)$ containing 0 is a $\theta_{a,b}$-semigroup if $\theta_{a,b}(y) \in G$ for any $y \in G\backslash\{0\}$. Now consider a positive integer c greater than 1 and coprime to b. We denote by $G_{a,b}(c)$ the smallest $\theta_{a,b}$-semigroup containing c. Indeed, $G_{a,b}(c)$ is a numerical semigroup. For such a semigroup we find a minimal set of generators, the embedding dimension, the genus and the Frobenius number.
References
[U] Simone Ugolini, On numerical semigroups closed with respect to the action of affine maps, accepted for publication in Publicationes Mathematicae Debrecen, Preprint arXiv: `http://arxiv.org/abs/1505.06580`

Alexey Ustinov *Russian Academy of Sciences, Khabarovsk*

ON THE DISTRIBUTION OF FROBENIUS NUMBER

For given relatively prime positive integers $a_1,\ldots, a_n$, Frobenius number $g(a_1,\ldots,a_n)$ is the largest natural number that is not representable as a non-negative integer combination of $a_1,\ldots,a_n$.
It is more natural to consider the function $f(a_1,\ldots,a_n) = g(a_1,\ldots,a_n) + a_1 + \ldots + a_n$, which gives the largest integer number that is not representable as a positive integer combination of $a_1,\ldots, a_n$. For example $f(a,b) = ab$ as $g(a,b) = ab - a - b$.
Davison (1994) conjectured that the value of $f(a,b,c)$ for a "random" triple is likely to be of the order $\sqrt{abc}$. More formally if

$$X_N = \{(a,b,c) : 1 \le a,b,c \le N; (a,b,c) = 1\}$$

then

$$\lim_{N\to\infty} \frac{1}{|X_N|} \sum_{(a,b,c)\in X_N} \frac{f(a,b,c)}{\sqrt{abc}}$$

exists and is finite. Arnold (1999) conjectured that for all $n \ge 2$ function $f(a_1,\ldots,a_n)$ has weak asymptotic of the form $c_n \sqrt[n-1]{a_1 \ldots a_n}$.
The talk will be devoted to the case $n = 3$ which can be studied with analytical methods based on a bountds of Kloosterman sums.

Theorem 1 (weak asymptotic). Let a be positive integer, $x_1, x_2, \varepsilon > 0$ and

$$M_a(x_1,x_2) = \{(b,c) : 1 \le b \le x_1 a, 1 \le c \le x_2 a, (a,b,c) = 1\}.$$

Then

$$\frac{1}{|M_a(x_1,x_2)|} \sum_{(a,b,c)\in M_a(x_1,x_2)} \left(f(a,b,c) - \frac{8}{\pi}\sqrt{abc} \right) = O_{x_1,x_2,\varepsilon}(a^{4/3+\varepsilon}).$$

Corollary (a stronger form of Davison's cojecture). For all $a \ge 1$, $\varepsilon > 0$

$$\frac{1}{|M_a(1,1)|} \sum_{(b,c)\in M_a(1,1)} \frac{f(a,b,c)}{\sqrt{abc}} = \frac{8}{\pi} + O_\varepsilon(a^{-1/6+\varepsilon}).$$

Theorem 2 (density function for normalized Frobenius numbers). For all $a \ge 1$, $x_1, x_2, \varepsilon > 0$

$$\frac{1}{|M_a(x_1,x_2)|} \sum_{\substack{(a,b,c)\in M_a(x_1,x_2) \\ f(a,b,c)\le \tau\sqrt{abc}}} 1 = \int_0^\tau p(t)\,dt + O_{\varepsilon,x_1,x_2,\tau}(a^{-1/6+\varepsilon}),$$

with

$$p(t) = \begin{cases} 0, & \text{for } t \in [0, \sqrt{3}]; \\ \frac{12}{\pi}\left(\frac{t}{\sqrt{3}} - \sqrt{4-t^2}\right), & \text{for } t \in [\sqrt{3}, 2]; \\ \frac{12}{\pi^2}\left(t\sqrt{3}\arccos\frac{t+3\sqrt{t^2-4}}{4\sqrt{t^2-3}} + \frac{3}{2}\sqrt{t^2-4}\log\frac{t^2-4}{t^2-3}\right), & \text{for } t \in [2, +\infty). \end{cases}$$

Alberto Vigneron-Tenorio *Universidad de Cádiz*

On proportionally modular affine semigroups

Joint work with J. I. García-García and M.A. Moreno-Frías.

Proportionally modular numerical semigroups were introduced in [J. C. Rosales, P. A. García-Sánchez, J. I. García-García, J. M. Urbano-Blanco, Proportionally modular Diophantine inequalities, J. Number Theory 103 (2003) 281-294] from the non-negative integer solutions of the inequality $ax \mod b \le cx$ with a, b and c positive integers.

The aim of this talk is to present their natural generalization by defining *proportionally modular affine semigroup*, that is, the set

$$\{x \in \mathbb{N}^p \mid f(x) \mod b \le g(x)\},$$

where f and g are linear functions $f, g : \mathbb{Q}^p \to \mathbb{Q}$, and $b \in \mathbb{N}$.

In particular, we prove these semigroups are finitely generated and we give an algorithm to compute their minimal generating sets. Besides, for proportionally modular affine semigroups in $\mathbb{N}^2$, it is provided a faster algorithm to compute their minimal system of generators and some properties are studied.

Caterina Viola *University of Dresden*

Catenary and tame degree in embedding dimension 4 numerical semigroups

We study the relationship between catenary degree and the tame degree in numerical semigroups with embedding dimension 4 that are symmetric but not complete intersections.

Rolf Waldi *Universität Regensburg*

On the deviation and the type of certain local Cohen-Macaulay algebras and numerical semigroups

Joint work with E. Kunz.

I want to present certain local Cohen-Macaulay algebras, for which close relations between deviation $d(R)$, type $t(R)$ and embedding dimension $\mathrm{edim}(R)$ exist.

For $a, b \in \mathbb{N}$ let $R = K[\![X_1, \dots, X_a, Y, Z_1, \dots, Z_b]\!]/I = K[\![x_1, \dots, x_a, y, z_1, \dots, z_b]\!]$ where the x_i, y, z_j are the images of the X_i, Y, Z_j in R. Assume that the system of generators $\{x_1, \dots, x_a, y, z_1, \dots, z_b\}$ of $\mathfrak{m}_R$ is minimal, hence $a + b + 1 = \mathrm{edim}(R)$, and let $\{z_1, \dots, z_b\}$ be a maximal regular sequence, hence $b = \dim(R)$ and R is a Cohen-Macaulay algebra.

Theorem a) If $x_i x_j \in (y, z_1, \dots, z_b)$ $(i, j = 1, \dots, a)$, then

$$d(R) \ge \binom{\mathrm{edim}(R) - \dim(R)}{2} - (\mathrm{edim}(R) - \dim(R))$$

and

$$t(R) \le \mathrm{edim}(R) - \dim(R).$$

b) If even $x_i x_j \in (z_1, \dots, z_b)$ $(i, j = 1, \dots, a)$, then there exists $\delta_1 \in \{0, \dots, a\}$ such that

$$d(R) = \binom{\mathrm{edim}(R) - \dim(R)}{2} - \delta_1$$

and $\delta_2 \in \{0, 1\}$ such that

$$t(R) = \mathrm{edim}(R) - \dim(R) - \delta_2.$$

Moreover $\delta_2 = 0$ if and only if $\delta_1 = 0$.

c) If $x_i x_j \in (z_1, \dots, z_b)$ $(i, j = 1, \dots, a)$ and if there exists an element $s \in \mathfrak{m}_R \setminus (x_1, \dots, x_a, z_1, \dots, z_b)$ with $s \cdot (y, x_1, \dots, x_a) \subset (z_1, \dots, z_b)$, then $\delta_1 = \delta_2 = 0$.

d) If $x_i x_j \in (z_1, \dots, z_b)$ $(i, j = 1, \dots, a)$ and $x_i y \in (z_1, \dots, z_b)$ $(i = 1, \dots, a)$, then $\delta_1 = \delta_2 = 0$ as well.

The observations can be applied to numerical semigroups (or equivalently monomial curves) and lead also to some further cases, generalizing E. Kunz, On the type of certain numerical semigroups and a question of Wilf. Semigroup Forum (to appear), with ring-theoretic proofs, in which a question of H. Wilf, A circle-of-lights algorithm for the money-changing problem. Amer. Math. Monthly 85 (1978) 562-565, has a positive answer.

Santiago Zarzuela *Universitat de Barcelona*

HOMOGENEOUS NUMERICAL SEMIGROUPS, THEIR SHIFTINGS, AND MONOMIAL CURVES OF HOMOGENEOUS TYPE

Joint work with R. Jafari

Given a numerical semigroup $S = \langle n_1, \ldots, n_d \rangle$ we introduce the concept of being homogeneous. This is done in terms of the Apéry set $\mathrm{AP}(S, n_1)$. We then show that any homogeneous numerical semigroup S with Cohen-Macaulay tangent cone $G(S)$ is of homogeneous type, that is, the Betti numbers of S and $G(S)$ coincide. We also study with some more detail the cases of small emebdding dimensions and the behavior of the homogeneous property under gluing. Finally, we prove that the property of being homogeneous and having Cohen-Macaulay tangent cone fulfills asymptotically under shifting, providing as a consequence a new proof of a result by J. Herzog and S. Stamate (2014) stating that for a monomial curve, the property of being of homogeneous type fulfills asymptotically under shifting.

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