

SANO, décima reunión
Seminario de Álgebra No Conmutativa
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**Non-commutative Gröbner basics: from the theory to the
implementation in the Computer Algebra System SINGULAR:PLURAL**

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12h.-12h.50 m.

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Following Sturmfels, who once proposed to treat the most fundamental applications of Gröbner bases in the commutative case as a collection of *Gröbner basics*, we introduce non-commutative Gröbner basics over G -algebras (a.k.a. PBW algebras). We go through the various (ring-theoretical, homological etc) properties of G -algebras and point out the properties, which can be proved by constructive means.

Among the non-commutative Gröbner basics, there are

- Ideal (resp. module) membership problem
- Intersection with subrings (elimination of variables)
- Intersection of ideals (resp. submodules)
- Quotient and Saturation of ideals
- Kernel of a module homomorphism
- Kernel of a ring homomorphism
- Algebraic relations between polynomials

All these applications are implemented in SINGULAR:PLURAL, we will comment some aspects of their use and compute some examples live. Moreover, there are several methods for computing syzygies and free resolutions of modules implemented.

However, there are important applications, arising in the non-commutative case, such as

- Two-sided Gröbner basis for bimodules
- Gel'fand-Kirillov dimension for modules
- Operations with opposite and enveloping algebras
- Preimage of left ideal under a morphism of GR -algebras
- Annihilators of finite-dimensional modules

We show, how these algorithms are implemented and which difficulties one has with related algorithms.

We will present some open theoretical and algorithmical problems as well as directions of future research.

Proyecto de Investigación MTM2004-1406, MEC
"Métodos algebraicos en Geometría no conmutativa"
Proyecto de Investigación MTM2004-8125, MEC
"Aplicaciones del Álgebra a la Geometría no conmutativa"