Topological methods for the study of differential equations with singularities

Boundary value problems for ordinary differential equations with singularities (both in the time and the phase variable) arise in applications, especially in physics; therefore this topic has been of substantial interest of scientists and engineers for decades.

In our course, we focus on the periodic boundary value problem for second-order ordinary differential equations with singularities in the phase variable, in particular, to find a positive solution to the problem

$$u''(t) + f(u(t))u'(t) + g(u(t)) = h(t, u(t)) \qquad \text{for a. e. } t \in [0, \omega], \tag{1}$$

$$u(0) = u(\omega), \qquad u'(0) = u'(\omega), \tag{2}$$

where h is a Carathéodory function, and f, g are continuous functions with possible singularities at zero.

We begin with the particular case of Eq. (1),

$$u''(t) + g(u(t)) = h(t), \tag{3}$$

mentioning the history of that equation, methods used to prove its solvability, and problems connected with. Then we pass to the more general case, Eq. (1) to talk about the results which have been obtained recently and methods of their proofs.